BLIND AND SEMI-BLIND SINGLE USER RECEIVER TECHNIQUES FOR ASYNCHRONOUS CDMA IN MULTIPATH CHANNELS*

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ABSTRACT

We consider multiple users in an asynchronous DS-CDMA system operating in a multipath environment. The received cyclostionary spread signal sampled at the chip rate is converted to a stationary vector signal, leading to a linear multichannel model. Linear receivers for multiple access interference (MAI) suppression are studied with emphasis on computationally simple algorithms. The desired user channel estimate is obtained by a new *blind* technique using the spreading sequence properties and second-order statistics. A blind MMSE-ZF receiver is subsequently obtained. Equivalence to the *anchored* MOE receiver is shown. Since the blind receiver relies on the inversion of the signal covariance matrix $\boldsymbol{R}_{YY}$, a consistent estimate of which requires a large number of data points if a large number of users are concurrently active, a *semi-blind* alternative for the estimation of the interference canceling filter is presented. Iterative improvements of this estimate based upon exploitation of the finite-alphabet are investigated. Performances of different interference cancellation schemes are compared in terms of the output signal-to-interference-plus-noise ratio (SINR).

I. INTRODUCTION AND PREVIOUS WORK

Blind solutions for DS-CDMA systems have received considerable attention since the pioneering work of [1], which is based upon an *anchored* minimum output energy (MOE) criterion. The anchored receiver constrains the inner product of the receiver signal with the spreading sequence to be fixed, thus restricting the optimization problem to within the constrained space. The desirable feature of such a scheme is that its informational complexity is the same as that of a matched filter detector, i.e., only the desired user signature waveform and timing information are required for its operation. Besides, it is desirable in some applications, like at the mobile terminal, to employ an algorithm that banks simply on single user information.

The problem addressed in [1] was that of DS-CDMA communications over a channel without multipath. A constrained optimization scheme was proposed in [2] for multipath channels where the receiver's output energy is minimized subject to appropriate constraints. Connections with the *Capon* philosophy were drawn in that paper. The above mentioned receivers can be shown to converge asymptotically (SNR$\rightarrow\infty$) to the zero-forcing (ZF) or decorrrelating solution. It was shown in [3] that in order to accommodate a number of users approaching code space dimensions, longer receivers are required for the ZF solution to be achievable. Moreover, we presented in [3] the optimal MMSE receiver for multipath channels and asynchronous conditions, obtained by applying multichannel linear prediction to the received cyclostationary signal. Direct estimation of the MMSE receiver was introduced in [4] following the observation that the MMSE receiver lies in the signal subspace. MMSE receiver constrained to the signal subspace in the case of channels longer than a symbol period was investigated in [5], where a singular-value decomposition (SVD) was used to determine the orthogonal subspaces. The channel estimate in this work was obtained as a generalization to longer delay spreads of the subspace technique originally proposed in [6]. Identifiability issues under long delay spread conditions were however not elaborated upon. Moreover, the above mentioned schemes have high complexity since an estimate of subspaces is required.

Semi-blind approaches, on the other hand, have recently kicked off with the intuitively attractive idea of employing as much *a priori* knowledge as is available. Forthcoming third generation mobile cellular systems like the European UMTS Wideband CDMA and TDMA/CDMA [7][8] standards both anticipate the use of a training sequence integrated within the signal frame. It is worth mentioning that in the context of blind estimation, CDMA systems possess the most desirable characterstics of all existing multiple access systems with the necessary (extra) bandwidth and integrated *a priori* knowledge in terms of spreading sequences. Any further information, like known training data, should provide further gains resulting in more efficient interference suppression and reduced computational complexity.

Although scant, the CDMA literature on *semi-blind* has had the term employed with varying significations. Semi-blindness to some comes from known spreading codes of intracell users, with the inter-cell co-channel users contributing to the blind part. In our problem, we shall consider knowledge of only the spreading sequence of the user of interest, with known training symbols for this user (thus a semi-blind problem).

We propose, in this work, a new blind MMSE zero-forcing receiver for DS-CDMA systems in multipath channels. This receiver expoits spreading sequence properties to estimate the desired user channel at a low cost. This channel estimate compensates somewhat for estimation errors in $\boldsymbol{R}_{YY}$. Further improvements are obtained by employing semi-blind and finite-alphabet information. The delay spread is assumed to be possibly more than a symbol period, and channel lengths of different users can be unequal depending upon the type of service. Long delay spreads can occur in UMTS TDMA/CDMA and in the case of high-rate users in UMTS W-CDMA.

II. MULTIUSER DATA MODEL

The p users are assumed to transmit linearly modulated signals over a linear multipath channel with additive Gaussian noise. It is assumed that the receiver employs a single antenna to receive the mixture of signals from all users, although the model can easily be extended to the case of multiple antennas. Oversampling is inherent to CDMA systems due to the large (extra) bandwidth and the need to resolve chip pulses. The received

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continuous time signal can be written in baseband notation as

$$y(t)=\sum_{j=1}^{p}\sum_{k}a_j(k)g_j(t-kT_s)+v(t), \quad (1)$$

where $a_j(k)$ are the transmitted symbols from the user j, T_s is the common symbol period, $g_j(t)$ is the overall channel impulse response for the jth user. Assuming the $\{a_j(k)\}$ and $\{v(t)\}$ to be jointly wide-sense stationary, the process $\{y(t)\}$ is wide-sense cyclostationary with period T_s. At the sampler output, we obtain the wide-sense stationary $m \times 1$ vector signal $\boldsymbol{y}(k)$ at the symbol rate. The overall channel impulse response for jth user's signal, $g_j(t)$, is the convolution of the spreading code and $h_j(t)$, itself the convolution of the chip pulse shape and the actual channel (assumed to be FIR) representing the multipath fading environment. This can be expressed as

$$g_j(t)=\sum_{s=0}^{m-1}c_j(s)h_j(t-sT), \quad (2)$$

where T is the chip duration. We consider that the FIR channel length for the jth user is m_jT. Let $k_j \in [0, m-1]$ be the chip-delay index for the jth user: $h_j(k_jT)$ is the first non-zero chip-rate sample of $h_j(t)$. The parameter N_j is the duration of $g_j(t)$ in symbol periods. It is a function of m_j and k_j. We consider user 1 as the user of interest and assume that $k_1 = 0$ (synchronization to user 1). Let $N=\sum_{j=1}^{p}N_j$. The vectorized chip-rate samples lead to a discrete-time $m \times 1$ vector signal at the symbol rate that can be expressed as

$$\begin{aligned}\boldsymbol{y}(k) &= \sum_{j=1}^{p}\sum_{i=0}^{N_j-1}\boldsymbol{g}_j(i)a_j(k-i)+\boldsymbol{v}(k)\\ &=\sum_{j=1}^{p}\boldsymbol{G}_{j,N_j}A_{j,N_j}(k)+\boldsymbol{v}(k)=\boldsymbol{G}_N\boldsymbol{A}_N(k)+\boldsymbol{v}(k)\end{aligned} \quad (3)$$

where,

$$\boldsymbol{y}(k)=\begin{bmatrix}y_1(k)\\ \vdots\\ y_m(k)\end{bmatrix}, \boldsymbol{g}_j(k)=\begin{bmatrix}g_{1j}(k)\\ \vdots\\ g_{mj}(k)\end{bmatrix}, \boldsymbol{v}(k)=\begin{bmatrix}v_1(k)\\ \vdots\\ v_m(k)\end{bmatrix}$$

$$\boldsymbol{G}_{j,N_j}=\left[\boldsymbol{g}_j(N_j-1)\ldots\boldsymbol{g}_j(0)\right], \boldsymbol{G}_N=\left[\boldsymbol{G}_{1,N_1}\ldots\boldsymbol{G}_{p,N_p}\right]$$

$$A_{j,N_j}(k)=\left[a_j^H(k-N_j+1)\ldots a_j^H(k)\right]^H$$

$$\boldsymbol{A}_N(k)=\left[A^H_{1,N_1}(k)\ldots A^H_{p,N_p}(k)\right]^H,$$

and the superscript ^{H} denotes Hermitian transpose. The matrix $\boldsymbol{G}_{1,N_1}$ (for user 1) can be written in terms of the spreading code and the channel vector $\boldsymbol{h}_1$ as $\boldsymbol{G}_{1,N_1}=[\boldsymbol{g}_1(N_1-1)\cdots\boldsymbol{g}_1(0)]$ with $\boldsymbol{g}_1(i)=\boldsymbol{C}_1(i)\boldsymbol{h}_1$, and, the matrices $\boldsymbol{C}_1(i)$ are shown in figure 1, and the band consists of the column vector $\left[c_0^H\cdots c_{m-1}^H\right]^H$ shifted and displaced successively to the right. For the interfering users, we have a similar setup except that owing to asynchrony, the band in fig. 1 is shifted down by k_j positions and is no longer conincident with the top left edge of the box. We denote by $\boldsymbol{C}_1$, the concatenation of the code matrices given above for user 1: $\boldsymbol{C}_1=[\boldsymbol{C}_1^H(0)\cdots\boldsymbol{C}_1^H(N_1-1)]^H$.

It is clear that the signal model above addresses a multiuser setup with a possiblity of joint interference cancellation for all sources simultaneously [9] provided the timing information and spreading codes of all of them are available. As we shall see later, it is possible to decompose the problem into single user ones thus making the implementation suitable for applications such as at mobile terminals or as suboptimal processing stage at the base station.

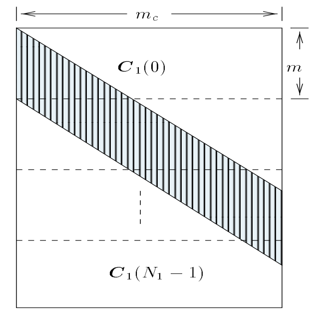

Figure 1. The Code Matrix $\boldsymbol{C}_1$

III. BLIND MMSE ZERO FORCING RECEIVER

We stack M successive $\boldsymbol{y}(k)$ vectors in a super vector

$$\boldsymbol{Y}_M(k)=\mathcal{T}_M(\boldsymbol{G}_N)\boldsymbol{A}_{N+p(M-1)}(k)+\boldsymbol{V}_M(k), \quad (4)$$

where, $\mathcal{T}_M(\boldsymbol{G}_N) = \left[\mathcal{T}_{M,1}(\boldsymbol{G}_{1,N_1})\cdots\mathcal{T}_{M,p}(\boldsymbol{G}_{p,N_p})\right]$ and $\mathcal{T}_M(\boldsymbol{x})$ is a banded block Toeplitz matrix with M block rows and $\left[\boldsymbol{x}\ \ \boldsymbol{0}_{n\times(M-1)}\right]$ as first block row (n is the number or rows in $\boldsymbol{x}$), and $\boldsymbol{A}_{N+p(M-1)}(k)$ is the concatenation of user data vectors ordered as $\left[A^H_{1,N_1+M-1}(k), A^H_{2,N_2+M-1}(k)\cdots A^H_{p,N_p+M-1}(k)\right]^H$. Consider the scenario depicted in fig. 2 for a single user. Due

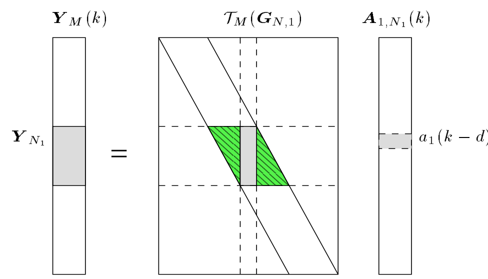

Figure 2. The ISI and MAI for Desired Symbol

to the limited delay spread, the effect of a particular symbol, $a_1(k-d)$, propagates to the next N_1-1 symbol periods, rendering the channel a moving average process of order N_1-1 [9]. For the other users, the matrices $\mathcal{T}_M(\boldsymbol{G}_{N_j})$, where $i=2\cdots p$, have a similar structure and can be viewed as being superimposed over the channel matrix $\mathcal{T}_M(\boldsymbol{G}_{N_1})$ in fig. 2. Same applies for the data vectors $A_{j,N_j+M-1}(k)$, $\forall j \neq i$. The overall effect of the ISI and the MUI is therefore that of engendering the shaded triangles in the figure, which need to be removed from $\boldsymbol{Y}_{N_1}$. To this end, let us introduce the following orthogonal transformation:

$$\boldsymbol{T}_1=\left[\begin{array}{ccc}\boldsymbol{0} & \boldsymbol{C}_1^H & \boldsymbol{0}\end{array}\right], \quad \boldsymbol{T}_2=\begin{bmatrix}\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{C}_1^{\perp} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}\end{bmatrix}, \quad (5)$$

where, $\boldsymbol{C}_1^{\perp H}$ is the orthogonal complement of $\boldsymbol{C}_1$, the tall code matrix given in section II. ($\boldsymbol{C}_1^{\perp}\boldsymbol{C}_1=\boldsymbol{0}$). Then, $\boldsymbol{C}_1^H\boldsymbol{Y}_{N_1}=\boldsymbol{T}_1\boldsymbol{Y}_M$ and the middle (block) row of the matrix $\boldsymbol{T}_2$ acts as a blocking transformation for the signal of interest. Note that $P_{\boldsymbol{T}_1^H}+P_{\boldsymbol{T}_2^H}=\boldsymbol{I}$, where, P_X is the projection operator (projection on the column space of $\boldsymbol{X}$). This gives us a possiblity of estimating the $a_1(k-d)$ contribution in $\boldsymbol{Y}_{N_1}$ blindly. We have,

$$\boldsymbol{Z}(k)=\left[\boldsymbol{T}_1-\boldsymbol{Q}\boldsymbol{T}_2\right]\boldsymbol{Y}_M(k), \quad (6)$$

and the interference cancellation problem settles down to minimization of the trace of the matrix $\boldsymbol{R}_{ZZ}$ for a matrix $\boldsymbol{Q}$, which results in

$$\boldsymbol{Q} = \left(\boldsymbol{T}_1 \boldsymbol{R}^d \boldsymbol{T}_2^H\right) \left(\boldsymbol{T}_2 \boldsymbol{R}^d \boldsymbol{T}_2^H\right)^{-1}, \tag{7}$$

and where, $\boldsymbol{R}^d$ is the noiseless (denoised) data covariance matrix, $\boldsymbol{R}_{YY}$, with the subscript removed for convenience. The output $\boldsymbol{Z}(k)$ can directly be processed by a multichannel matched filter to get the symbol, $\hat{a}_1(k-d)$, the data for the user 1.

$$\hat{a}_1(k-d) = \boldsymbol{F}^H \boldsymbol{Y}_M(k) = \boldsymbol{h}_1^H \left(\boldsymbol{T}_1 - \boldsymbol{Q}\boldsymbol{T}_2\right) \boldsymbol{Y}_M(k) \tag{8}$$

An estimate of the channel $\mathbf{G}_1(z) = \mathbf{C}_1(z)\mathbf{h}_1(z)$ can be ob-

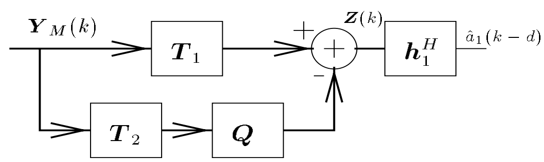

Figure 3. MMSE-ZF Receiver

tained as the by product of the interference cancellation scheme. Notice that the interference canceler is analogous to a MMSE-ZF in the form of a smoother or two-sided linear predictor for the single user case [10] with $\boldsymbol{T}_1 = [\mathbf{0} \;\; \boldsymbol{I} \;\; \mathbf{0}]$ and $\boldsymbol{T}_2$ without the middle (block) row, which when employed in a multiuser scenario is no longer capable of MAI suppression coming from the middle block of $\boldsymbol{Y}_M(k)$ of fig. 2, unless a fair amount of data smoothing is introduced [10]. $\boldsymbol{Z}(k)$ corresponds to the vector of *prediction* errors, and the covariance matrix of the prediction errors is given by

$$\boldsymbol{R}_{ZZ} = \boldsymbol{T}_1 \boldsymbol{R}^d \boldsymbol{T}_1^H - \boldsymbol{T}_1 \boldsymbol{R}^d \boldsymbol{T}_2^H \left(\boldsymbol{T}_2 \boldsymbol{R}^d \boldsymbol{T}_2^H\right)^{-1} \boldsymbol{T}_2 \boldsymbol{R}^d \boldsymbol{T}_1^H, \tag{9}$$

From the above structure of the two-sided (or rather *full dimensional*) linear prediction problem, the key observation is that the matrix $\boldsymbol{R}_{ZZ}$ is rank-1 in the noiseless case! Using this fact, one can identify the composite channel as the maximum eigenvector of the matrix $\boldsymbol{R}_{ZZ}$, since $\boldsymbol{Z}(k) = \boldsymbol{C}_1 \boldsymbol{h}_1 \tilde{a}_1(k-d)$.

A. Relation with Unbiased MOE Approach

Suppose that $\boldsymbol{F}$ is a linear receiver vector applied to the received data $\boldsymbol{Y}_M(k)$. F is unbiased if $\boldsymbol{F}^H \tilde{\boldsymbol{g}}_1 = 1$, where, $\tilde{\boldsymbol{g}}_1 = \boldsymbol{T}_1^H \boldsymbol{h}_1$. Then the following relation holds.

$$\arg \min_{F:F^H \tilde{g}_1 = 1} \mathsf{MSE}_{\text{unbiased}} = \arg \min_{F:F^H \tilde{g}_1 = 1} \mathsf{OE} = \arg \max_{F} \mathsf{SINR}, \tag{10}$$

This simply implies that the minimum mean-squared error (MMSE), and the minimum output energy (MOE)[1], are interchangeable criteria under the unbiased constraint, and are equivalent to the maximization of the output SINR.

The unbiased MOE criterion proposed in [2], which is a generalization of the instantaneous channel case of [1], is in principle a max/min problem solved in two steps with,

[1] also known as minimum variance distortionless response (MVDR), a particular instance of the linearly constrained minimum-variance (LCMV) criterion.

<u>step:1</u> *Unbiased MOE*

$$\min_{F:F^H \tilde{g}_1 = 1} \boldsymbol{F}^H \boldsymbol{R}_{YY} \boldsymbol{F} \Rightarrow \boldsymbol{F} = \frac{1}{\tilde{\boldsymbol{g}}_1^H \boldsymbol{R}_{YY}^{-1} \tilde{\boldsymbol{g}}_1} \boldsymbol{R}_{YY}^{-1} \tilde{\boldsymbol{g}}_1, \tag{11}$$

with $\mathsf{MOE}(\hat{\boldsymbol{h}}_1) = \frac{1}{\tilde{\boldsymbol{g}}_1 \boldsymbol{R}_{YY}^{-1} \tilde{\boldsymbol{g}}_1}$, followed by,

<u>step:2</u> *Capon's Method*

$$\max_{\hat{\boldsymbol{h}}_1 : \|\hat{\boldsymbol{h}}_1\| = 1} \mathsf{MOE}(\hat{\boldsymbol{h}}_1) \Rightarrow \min_{\hat{\boldsymbol{h}}_1 : \|\hat{\boldsymbol{h}}_1\| = 1} \hat{\boldsymbol{h}}_1^H \left(\boldsymbol{T}_1 \boldsymbol{R}_{YY}^{-1} \boldsymbol{T}_1^H\right) \hat{\boldsymbol{h}}_1, \tag{12}$$

from where, $\hat{\boldsymbol{h}}_1 = V_{min}(\boldsymbol{T}_1 \boldsymbol{R}_{YY}^{-1} \boldsymbol{T}_1^H)$. It can be shown that if $\boldsymbol{T}_2 = \boldsymbol{T}_1^{\perp}$, then

$$\boldsymbol{T}_1 \boldsymbol{R}_{YY}^{-1} \boldsymbol{T}_1^H = \left(\boldsymbol{T}_1 \boldsymbol{T}_1^H\right) \boldsymbol{R}_{ZZ}^{-1} \left(\boldsymbol{T}_1 \boldsymbol{T}_1^H\right), \tag{13}$$

where, $\boldsymbol{R}_{ZZ}$ is given by (9), and $\boldsymbol{Q}$, given by (7), is optimized to minimize the prediction error variance. $\boldsymbol{R}^d$ replaces $\boldsymbol{R}_{YY}$ in the above developments. From this, we can obtain $\hat{\boldsymbol{h}}_1$ as $\hat{\boldsymbol{h}}_1 = V_{max}\{\left(\boldsymbol{T}_1 \boldsymbol{T}_1^H\right)^{-1} \boldsymbol{R}_{ZZ} \left(\boldsymbol{T}_1 \boldsymbol{T}_1^H\right)^{-1}\}$. In order to evaluate the quality of the blind receiver obtained from the above criterion, we consider the noiseless received signal ($v(t) \equiv 0$). We have the following two cases of interest.

1. Uncorrelated symbols

In the absence of noise, with *i.i.d.* symbols, the stochastic estimation of $\boldsymbol{T}_1 \boldsymbol{Y}$ from $\boldsymbol{T}_2 \boldsymbol{Y}$ is the stochastic estimation of $\boldsymbol{T}_1 \mathcal{T}_M(\boldsymbol{G}_{1:p}) \boldsymbol{A}$ from $\boldsymbol{T}_2 \mathcal{T}_M(\boldsymbol{G}_{1:p}) \boldsymbol{A}$ with $\boldsymbol{R}_A = \sigma_a^2 \boldsymbol{I}$. Hence, it is equivalent to the deterministic estimation of $\mathcal{T}_M^H(\boldsymbol{G}_{1:p}) \boldsymbol{T}_1^H$ from $\mathcal{T}_M^H(\boldsymbol{G}_{1:p}) \boldsymbol{T}_2^H$: $\|\mathcal{T}_M^H(\boldsymbol{G}_{1:p}) \boldsymbol{T}_1^H - \mathcal{T}_M^H(\boldsymbol{G}_{1:p}) \boldsymbol{T}_2^H \boldsymbol{Q}^H\|_2^2$. Then, given the condition

$$\begin{aligned}
&\mathsf{span}\{\boldsymbol{T}_1^H\} \cap \mathsf{span}\{\mathcal{T}_M(\boldsymbol{G}_{1:p})\} = \mathsf{span}\{\mathcal{T}_M(\boldsymbol{G}_{1:p}) \boldsymbol{e}_d'\} \\
&\Rightarrow \mathsf{span}\{\mathcal{T}_M(\boldsymbol{G}_{1:p})\} \subset \mathsf{span}\{\boldsymbol{T}_2^H\} \oplus \mathsf{span}\{\tilde{\boldsymbol{g}}_1\} \\
&\because \mathcal{T}_M(\boldsymbol{G}_{1:p}) \boldsymbol{e}_d' = \mathcal{T}_M(\boldsymbol{G}_1) \boldsymbol{e}_d = \tilde{\boldsymbol{g}}_1 = \boldsymbol{T}_1 \boldsymbol{h}_1,
\end{aligned} \tag{14}$$

and where, $\boldsymbol{e}_d'$ and $\boldsymbol{e}_d$ are vectors of appropriate dimensions with all zeros and one 1 selecting the desired column in $\mathcal{T}_M(\boldsymbol{G}_{1:p})$ and $\mathcal{T}_M(\boldsymbol{G}_1)$ respectively. We can write the channel convolution matrix $\mathcal{T}_M(\boldsymbol{G}_{1:p})$ as

$$\mathcal{T}_M(\boldsymbol{G}_{1:p}) = \tilde{\boldsymbol{g}}_1 \boldsymbol{e}_d'^H + \mathcal{T}_M(\boldsymbol{G}_{1:p}) P_{\boldsymbol{e}_d'^{\perp}} = [\tilde{\boldsymbol{g}}_1 \;\; \boldsymbol{T}_2^H] \boldsymbol{A}, \tag{15}$$

for some $\boldsymbol{A}$. Then we can write,

$$\begin{aligned}
&\mathcal{T}_M^H(\boldsymbol{G}_{1:p}) \left(\boldsymbol{T}_1^H - \boldsymbol{T}_2^H \boldsymbol{Q}^H\right) = \\
&\boldsymbol{e}_d' \boldsymbol{h}_1^H \boldsymbol{T}_1 \boldsymbol{T}_1^H + \boldsymbol{A}^H \begin{bmatrix} \tilde{\boldsymbol{g}}_1 \boldsymbol{T}_1^H \\ \mathbf{0} \end{bmatrix} - \boldsymbol{A}^H \begin{bmatrix} \mathbf{0} \\ \boldsymbol{T}_2 \boldsymbol{T}_2^H \end{bmatrix} \boldsymbol{Q}^H \\
&= \boldsymbol{e}_d' \boldsymbol{h}_1^H \boldsymbol{T}_1 \boldsymbol{T}_1^H + \boldsymbol{A}_1^H \tilde{\boldsymbol{g}}_1^H \boldsymbol{T}_1^H - \boldsymbol{A}_2^H \left(\boldsymbol{T}_2 \boldsymbol{T}_2^H\right) \boldsymbol{Q}^H.
\end{aligned} \tag{16}$$

Note that $\boldsymbol{e}_d'^H \boldsymbol{A}_i^H = 0, i \in \{1, 2\}$. This implies that the first term on the R.H.S. of (16) is not predictable from the third. Therefore, if the second term is perfectly predictable from the third, then the two terms cancel each other out and $\boldsymbol{R}_{ZZ}$ turns out to be rank-1, and $\hat{\boldsymbol{h}}_1 = \left(\boldsymbol{T}_1 \boldsymbol{T}_1^H\right)^{-1} \mathcal{V}_{max}\left(\boldsymbol{R}_{ZZ}\right)$.

2. *Correlated symbols*

In the case of correlated symbols, with a finite amount of data, given the conditions in (14), it still holds that $\mathsf{span}\{\mathcal{T}_M^H(\boldsymbol{G}_{1:p})\boldsymbol{T}_2^H\} = \mathsf{span}\{P_{e_d^{'\perp}}\mathcal{T}_M(\boldsymbol{G}_{1:p})\}$. Now, we can write the received vector $\boldsymbol{Y}_M(k)$ as

$$\boldsymbol{Y}_M(k) = \mathcal{T}_M(\boldsymbol{G}_{1:p})\boldsymbol{A} = \mathcal{T}_M(\boldsymbol{G}_{1:p})\boldsymbol{e}_d^{'}a_1(k-d) + \overline{\mathcal{T}}_M\bar{\boldsymbol{A}}. \tag{17}$$

Now, the estimation of $\boldsymbol{T}_1\boldsymbol{Y}$ in terms of $\boldsymbol{T}_2\boldsymbol{Y} = \boldsymbol{T}_2\mathcal{T}_M(\boldsymbol{G}_{1:p})\boldsymbol{A} = \boldsymbol{T}_2\overline{\mathcal{T}}_M\bar{\boldsymbol{A}}$ is equivalent to estimation in terms of $\bar{\boldsymbol{A}}$.

$$\begin{aligned}
\widetilde{\boldsymbol{T}_1\boldsymbol{Y}}|_{\boldsymbol{T}_2\boldsymbol{Y}} &= \boldsymbol{T}_1\boldsymbol{Y} - \widehat{\boldsymbol{T}_1\boldsymbol{Y}} \\
&= \boldsymbol{T}_1\boldsymbol{Y} - \left(\boldsymbol{T}_1\boldsymbol{R}_{YY}^d\boldsymbol{T}_2^H\right)\left(\boldsymbol{T}_2\boldsymbol{R}_{YY}^d\boldsymbol{T}_2^H\right)^{-1}\boldsymbol{T}_2\boldsymbol{Y} \\
\widetilde{\boldsymbol{T}_1\boldsymbol{Y}}|_{\bar{\boldsymbol{A}}} &= \boldsymbol{T}_1\mathcal{T}_M(\boldsymbol{G}_{1:p})\boldsymbol{e}_d^{'}\tilde{a}_1(k-d) \\
&= \boldsymbol{T}_1\boldsymbol{T}_1^H\boldsymbol{h}_1\tilde{a}_1(k-d)|_{\bar{\boldsymbol{A}}}.
\end{aligned} \tag{18}$$

This results in,

$$\left(\boldsymbol{T}_1\boldsymbol{R}_{YY}^{-d}\boldsymbol{T}_1^H\right)^{-1} = \sigma^2_{\tilde{a}_1(k-d)|\bar{A}}\boldsymbol{h}_1\boldsymbol{h}_1^H, \tag{19}$$

The rank-1 results in a normalized estimate of the channel. It must however be noted that the estimation error variance of the desired symbol is now smaller ($\sigma^2_{\tilde{a}_1(k-d)} < \sigma_a^2$).

B. *Identifiability Conditions for Blind MMSE-ZF Receiver*

Continuing with the noiseless case, or with the denoised version of $\boldsymbol{R}_{YY}$, i.e., $\boldsymbol{R}_{YY}^d = \sigma_a^2\mathcal{T}_M(\boldsymbol{G}_{1:p})\mathcal{T}_M^H(\boldsymbol{G}_{1:p})$,

$$\min_{F:F^H\tilde{g}_1=1}\boldsymbol{F}^H\boldsymbol{R}_{YY}^d\boldsymbol{F} = \sigma_a^2, \quad \textit{iff} \quad \boldsymbol{F}^H\mathcal{T}_M(\boldsymbol{G}_{1:p}) = \boldsymbol{e}_d^{'H}, \tag{20}$$

i.e., the zero-forcing condition must be satisfied. Hence, the unbiased MOE criterion corresponds to ZF in the noiseless case. This implies that $\mathsf{MOE}(\hat{\tilde{\boldsymbol{g}}}_1) < \sigma_a^2 \;\; \textit{if} \;\; \hat{\tilde{\boldsymbol{g}}}_1 \not\sim \tilde{\boldsymbol{g}}_1$. We consider that:

(**i**). *FIR zero-forcing conditions are satisfied*, and
(**ii**). $\mathsf{span}\{\mathcal{T}_M(\boldsymbol{G}_{1:p})\} \cap \mathsf{span}\{\boldsymbol{T}_1^H\} = \mathsf{span}\{\boldsymbol{T}_1^H\boldsymbol{h}_1\}$.

The two step max/min problem boils down to

$$\max_{\hat{h}_1:\|\hat{h}_1\|=1}\hat{\boldsymbol{h}}_1^H\left(\boldsymbol{T}_1\boldsymbol{T}_1^H\right)^{-1}\boldsymbol{T}_1\mathcal{T}_M P^{\perp}_{\mathcal{T}_M^H T_2^H}\mathcal{T}_M^H\boldsymbol{T}_1^H\left(\boldsymbol{T}_1\boldsymbol{T}_1^H\right)^{-1}\hat{\boldsymbol{h}}_1, \tag{21}$$

where, $P_X^{\perp} = \boldsymbol{I} - \boldsymbol{X}(\boldsymbol{X}^H\boldsymbol{X})^{-1}\boldsymbol{X}^H$. Then identifiability implies that $\mathcal{T}_M P^{\perp}_{\mathcal{T}_M^H T_2^H}\mathcal{T}_M^H = \boldsymbol{T}_1^H\boldsymbol{h}_1\boldsymbol{h}_1^H\boldsymbol{T}_1 = \tilde{\boldsymbol{g}}_1\tilde{\boldsymbol{g}}_1^H$, or

$$P^{\perp}_{\mathcal{T}_M^H T_2^H}\mathcal{T}_M^H(\boldsymbol{G}_{1:p}) = P_{e_d^{'}}\mathcal{T}_M^H(\boldsymbol{G}_{1:p}), \tag{22}$$

Condition (i) above implies that $\boldsymbol{e}_d^{'} \in \mathsf{span}\{\mathcal{T}_M^H(\boldsymbol{G}_{1:p})\}$. From condition (ii), since $\boldsymbol{T}_1^H\boldsymbol{h}_1 = \mathcal{T}_M(\boldsymbol{G}_{1:p})\boldsymbol{e}_d^{'}$, we have

$$\begin{aligned}
\mathsf{span}\{\mathcal{T}_M(\boldsymbol{G}_{1:p})\boldsymbol{T}_2^H\} &= \mathsf{span}\{P^{\perp}_{e_d^{'}}\mathcal{T}_M^H(\boldsymbol{G}_{1:p})\} \\
\mathsf{span}\{\mathcal{T}_M^H(\boldsymbol{G}_{1:p})\} &= \mathsf{span}\{\mathcal{T}_M^H(\boldsymbol{G}_{1:p})\boldsymbol{T}_2^H\} \oplus \mathsf{span}\{\boldsymbol{e}_d^{'}\}
\end{aligned} \tag{23}$$

from which, $\mathcal{T}_M^H(\boldsymbol{G}_{1:p}) = P_{\mathcal{T}_M^H T_2^H}\mathcal{T}_M^H(\boldsymbol{G}_{1:p}) + P_{e_d^{'}}\mathcal{T}_M^H(\boldsymbol{G}_{1:p})$, which is the same as (22).

1. *A Note on Sufficiency of Conditions*

We consider first the conditions (**i**). Furthermore, in the following developments, we consider that $p < m$, which is easily achievable when multiple sensors (or oversampling) is employed. The effective number of channels is given by $m_{\text{eff}} = \mathsf{rank}\{\boldsymbol{G}_N\}$, where $\boldsymbol{G}_N$ is given in (3). Let $\mathbf{G}_1(z) = \sum_{k=0}^{N_1-1}\boldsymbol{g}_1(k)z^{-k}$ be the channel transfer function for user 1, with $\mathbf{G}(z) = [\mathbf{G}_1(z)\cdots\mathbf{G}_p(z)]$. Then let us assume the following:

(**a**). $\mathbf{G}(z)$ *is irreducible*, i.e., $\mathsf{rank}\{\mathbf{G}(z)\} = p, \forall z$.
(**b**). $\mathbf{G}(z)$ *is column reduced*:

$$\mathsf{rank}\{[\boldsymbol{g}_1(N_1-1)\cdots\boldsymbol{g}_p(N_p-1)]\} = p.$$

Given that the above two conditions hold, the FIR length M required is given by,

$$M \geq \overline{M} = \left\lceil\frac{N-p}{m_{\text{eff}}-p}\right\rceil. \tag{24}$$

Note that condition (**a**) holds with probability 1 due to the quasi-orthogonality of spreading sequences. As for (**b**), it can be violated in certain limiting cases e.g., in the synchronous case where $\boldsymbol{g}_j(N_j-1)$'s contain very few non-zero elements. Under these circumstances, instantaneous (static) mixture of the sources can zero out some of the $\boldsymbol{g}_j(N_j-1)$ (more specifically, at most $p-1$ of them). Then N gets reduced by at most $p-1$. However, even then, M given by (24) remains sufficient.

The condition (**ii**) can be restated as the following dimensional requirement:

$$\mathsf{rank}\{\mathcal{T}_M(\boldsymbol{G}_{1:p})\} + \mathsf{rank}\{\boldsymbol{T}_1^H\} \leqslant \mathsf{row}\{\mathcal{T}_M(\boldsymbol{G}_{1:p})\} + 1, \tag{25}$$

from where, under the irreducible channel and column reduced conditions,

$$M \geq \underline{M} = \left\lceil\frac{N-p+m_1-1}{m_{\text{eff}}-p}\right\rceil, \tag{26}$$

where, m_1 is the channel length for user 1 in chip periods. If (26) holds, then condition (**ii**) is fulfilled w.p. 1, regardless of the N_j's, i.e., the $\mathsf{span}\{\boldsymbol{T}_1^H\}$ does not intersect with all shifted versions of $\boldsymbol{g}_j$'s, $\forall j > 1$, which further means that no confusion is possible between the channel of the user of interest and those of other users, whether the mixing is static (same lengths) or dynamic (different channel lengths), with lengths measured in symbol periods.

2. *Violation of condition (**ii**)*

If the channel length m_1 is over-estimated, such that N_1 gets over-estimated, then condition (**ii**) is violated w.p. 1. In that case, more than one shifted versions of $\boldsymbol{g}_1$ will fit in the column space of $\boldsymbol{T}_1^H$. The estimated channel in that case can be expressed as $\widehat{\mathbf{G}}_1(z) = \mathbf{G}_1(z)b(z)$, where, $b(z)$ is a scalar polynomial of the order equaling the amount by which the channel has been over-estimated. A solution to this would be to try all orders for N_1 and stop at the correct one.

C. *Maximization of SINR*

The signal part in $\boldsymbol{Y}_M(k)$ is $\boldsymbol{Y}_s = \tilde{\boldsymbol{g}}_1 a_{1,k-d}$, whereas the interference (MAI & ISI) plus noise is $\boldsymbol{Y}_{\text{in}} = \overline{\mathcal{T}}_M\bar{\boldsymbol{A}} + \boldsymbol{V}_M$, where, $\overline{\mathcal{T}}_M = \mathcal{T}_M(G_{1:p})$ except for the column $\widetilde{\boldsymbol{g}}_1$. Then, for an arbitrary $\boldsymbol{F}$, assuming uncorrelated symbols, we obtain,

$$\mathsf{SINR} = \frac{\boldsymbol{F}^H\boldsymbol{R}_s\boldsymbol{F}}{\boldsymbol{F}^H\boldsymbol{R}_{\rm in}\boldsymbol{F}} = \frac{\sigma_a^2\boldsymbol{F}^H\widetilde{\boldsymbol{g}}_1\widetilde{\boldsymbol{g}}_1^H\boldsymbol{F}}{\boldsymbol{F}^H\left(\boldsymbol{R}_{YY}-\sigma_a^2\widetilde{\boldsymbol{g}}_1\widetilde{\boldsymbol{g}}_1^H\right)\boldsymbol{F}}, \quad (27)$$

from where,

$$\max_F \mathsf{SINR} \leftrightarrow \min_F \mathsf{SINR}^{-1} \leftrightarrow \quad \min_F \frac{\boldsymbol{F}^H\boldsymbol{R}_{YY}\boldsymbol{F}}{\sigma_a^2|\boldsymbol{F}^H\widetilde{\boldsymbol{g}}_1|^2}$$
$$\Rightarrow \min_{F:F^H\widetilde{g}_1=1} \boldsymbol{F}^H\boldsymbol{R}_{YY}\boldsymbol{F}, \quad (28)$$

which is the unbiased MOE cost function of (11).

IV. SEMI-BLIND RECEIVER

Fig. 4 shows the bit-error rate performance of the MMSE (employing $\boldsymbol{R}_{YY}^{-1}$) and the MMSE-ZF receiver of section III. It can be seen that the receivers are plagued by the finite-data effect. When training data side-information is available, this problem can be partially alleviated. To this end, we proceed with the full-length linear prediction problem described in section III. First, the channel vector, $\hat{\widetilde{\boldsymbol{g}}}_1$ is determined as an initial estimate from the blind problem, and the scale factor can be adjusted by means of the training sequence. Secondly, semi-blind information can be used to improve the estimate of the filter $\boldsymbol{Q}$. To incorporate the training information, we formulate the following *weighted least-squares* (WLS) cost function:

$$\min_{\boldsymbol{Q}}\left\{\frac{1}{\sigma_b^2}\sum_{k\notin\mathrm{T.S.}}\|\boldsymbol{Z}(k)\|_2^2+\frac{1}{\sigma_u^2}\sum_{k\in\mathrm{T.S.}}\|\boldsymbol{Z}(k)-\boldsymbol{T}_1^H\hat{\boldsymbol{h}}_1a_1(k-d)\|_2^2\right\}, \quad (29)$$

where, $a_1(k-d)$ is constrained to lie within the training sequence. The weighting factors σ_b^2 and σ_u^2 can be determined respectively as the ensemble averages of $\|\boldsymbol{Z}(k)\|_2^2$ and $\|\boldsymbol{Z}(k)-\boldsymbol{T}_1^H\hat{\boldsymbol{h}}_1a_1(k-d)\|_2^2$ for the blind and training sequence parts of the given data sequence. The denoised signal covariance matrix $\widehat{\boldsymbol{R}}^d = \widehat{\boldsymbol{R}}_{YY} - \lambda_{min}(\widehat{\boldsymbol{R}}_{YY})\boldsymbol{I}$, where, λ_{min} is the minimum eigenvalue of the estimated covariance matrix, and

$$\widehat{\boldsymbol{R}}_{YY} = \frac{1}{\sigma_b^2}\sum_{k\notin\mathrm{T.S.}}\boldsymbol{Y}_k\boldsymbol{Y}_k^H + \frac{1}{\sigma_u^2}\sum_{k\in\mathrm{T.S.}}\boldsymbol{Y}_k\boldsymbol{Y}_k^H. \quad (30)$$

$\mathrm{T.S.}$ in the above refers to the training sequence.

The algorithm is semiblind for the estimation of the interference canceler $\boldsymbol{Q}$ but involves a blind estimate of the channel. An update of the channel vector, in iterative implementations, is however also possible based upon the knowledge that in the noiseless case $\boldsymbol{R}_{ZZ}$ is rank one.

A. *Exploitation of Finite Alphabet*

An iterative implementation of the MMSE-ZF algorithm is possible when decisions are re-used at each iteration to re-estimate the filter $\boldsymbol{Q}$. We propose to start from a semi-blind cost fuction and make hard-decisions, thus exploiting the finite signal constellation (BPSK in this case). Upon each iteration, more correct decisions are available resulting in improved performance. We compare results with the limiting case where all symbols are known at the receiver and their effect is removed from the estimation of $\boldsymbol{Q}$. The hard-decision algorithm converges to this state in a small number of iterations, as seen in fig. 6.

V. UPLINK CONSIDERATIONS

Consider the situation at the base station of a cell. If we suppose that interfererers are limited to the intracell users, then, given the information available at the base station of timing and spreading sequences of all users, we can build the better estimate of the correlation matrix as

$$\boldsymbol{R}_{YY} = \sum_{j=1}^{p}\mathcal{T}(\boldsymbol{C}_j)\mathcal{T}(\hat{\boldsymbol{h}}_j)\mathcal{T}(\hat{\boldsymbol{h}}_j^H)\mathcal{T}(\boldsymbol{C}_j^H) + \sigma_v^2\boldsymbol{I}, \quad (31)$$

where, the channels $\hat{\boldsymbol{h}}_j, \forall j$ of all users can be estimated by the MMSE-ZF receiver algorithm. The code matrix $\boldsymbol{C}_j$ and its orthogonal complement $\boldsymbol{C}_j^{\perp}$ are known (pre-calculated) at the base-station. It is to be noted that the channel estimation method of section III. has minimal complexity and a single extreme eigenvector is to be determined per user.

A. *Noise Variance Estimate*

In (31), the noise variance σ_v^2 is still to be determined. We propose to determine the noise variance as the minimization of the following Frobenius norm:

$$\min_{\sigma_v^2}\|\widehat{\boldsymbol{R}}_{YY} - \sum_{j=1}^{u}\sigma_a^2\mathcal{T}(\boldsymbol{C}_j)\mathcal{T}(\hat{\boldsymbol{h}}_j)\mathcal{T}(\hat{\boldsymbol{h}}_j^H)\mathcal{T}(\boldsymbol{C}_j^H) + \sigma_v^2\boldsymbol{I}\|_F^2. \quad (32)$$

This mimimization problem results in

$$\sigma_v^2 = \mathsf{avg}\left(\mathsf{diag}\{|\widehat{\boldsymbol{R}}_{YY} - \sum_{j=1}^{u}\sigma_a^2\mathcal{T}(\boldsymbol{C}_j)\mathcal{T}(\hat{\boldsymbol{h}}_j)\mathcal{T}(\hat{\boldsymbol{h}}_j^H)\mathcal{T}(\boldsymbol{C}_j^H)\}\right), \quad (33)$$

and avg stands for the averaging operation. It is to be noted that minimum length, M, vectors need to be used to estimate $\widehat{\boldsymbol{R}}_{YY}$ (as long as a noise subspace exists), since then, a better time-averaged version of the covariance matrix would be available.

VI. SIMULATIONS

We consider 8 asynchronous users in the system with a spreading factor of $m = 16$. The channel for the jth user is modeled as an FIR channel of length m_j ranging from $8-21$ chip periods for different j. The channel delay spread is therefore shorter than one symbol period for some users while longer for others. Near-far conditions prevail in that the interfering users are randomly (ranging from 8 to 10 dB.) stronger than the user of interest. Fig. 4 shows the error-rate performance of the blind MMSE-ZF receiver and the MMSE receiver ($\widehat{\boldsymbol{R}}_{YY}^{-1}$) . It can be seen that the performance depends on the quality of the correlation matrix estimate. Better results are therefore obtained if more data is available. This figure highlights the major drawback in the implementation of blind linear receivers obtained from second order statistics and motivates the use of semi-blind techniques. Under power controlled conditions, with good choice of spreading sequences, a simple rake receiver may outperform the linear receivers, unless a good estimate of $\widehat{\boldsymbol{R}}_{YY}$ is available. On the other hand, as seen in fig. 5, the channel is estimated fairly accurately (normalized mean squared error[2] (NMSE) of the order of -25 dB at 20 dB. SNR) with 70 symbols from the rank-1 $\boldsymbol{R}_{ZZ}$. Performance of the noise-subspace based algorithm [6] is also shown for several input SNR's.

[2] NMSE= $E\frac{\|\boldsymbol{h}_1-\hat{\boldsymbol{h}}_1\|^2}{\|\boldsymbol{h}_1\|^2} = \frac{1}{L}\sum_{i=1}^{L}\frac{\|\boldsymbol{h}_1-\hat{\boldsymbol{h}}_1^{(i)}\|^2}{\|\boldsymbol{h}_1\|^2}$

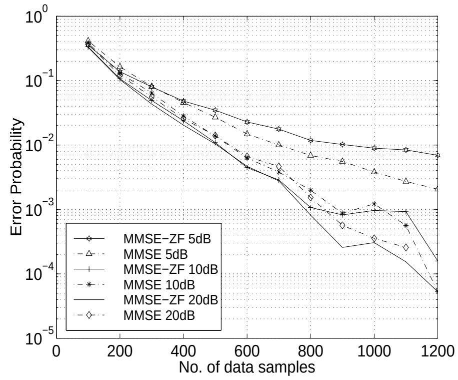

Figure 4. Error rate performance L=16, 8 users

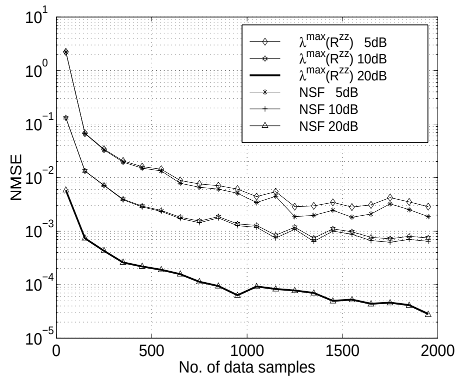

Figure 5. Channel estimation performance

In fig. 6, we show the performace of blind and semi-blind MMSE-ZF receiver and compare it with that of the theoretical MMSE ($\boldsymbol{R}_{YY} = \sigma_a^2 \mathcal{T}_M(\boldsymbol{G}_{1:p})\mathcal{T}_M(\boldsymbol{G}_{1:p}) + \sigma_v^2 \boldsymbol{I}$). It comes as no surprise that the optimal unbiased MMSE is not approached by any of the other receivers due to finite data effect. A theoretical curve for the MMSE-ZF is also provided. It can be seen that the semiblind MMSE-ZF does relatively well. Improved, hard-decision (HD) based receiver converges in a small number of iterations (one or two here) to the case where all symbols are considered known (ASK). In these simulations we considerd 25 training symbols in a packet of 160 symbols. If the number of training symbols is small, slow convergence takes place with a larger number of iterations required.

VII. CONCLUSIONS

The blind MMSE-ZF receiver for DS-CDMA was presented. Its equivalence to the unbiased MOE receiver was shown in terms of optimization criteria. The blind algorithm, like the MMSE linear receiver, requires a large amount of data for the estimation of the channel covariance matrix thus making it rather unpractical for rapidly changing environments and large numbers of users $(m \to p)$. Such algorithms can find their utility in indoor wireless LANs where channel changes at a relatively slow rate and a fair amount of data is available for the estimation of the covariance matrix. A possible implementation can be at the uplink, where, knowledge of spreading codes and timing of all users in the cell can be exploited to obtain a better $\hat{\boldsymbol{R}}_{YY}$. Identifiability conditions for long channels (longer than a symbol period) were given and it was shown that the channel is blindly identifiable w.p.1 (upto a scalar phase factor), unless it is overestimated. The semi-blind algorithm was presented and shown to offer promising gains. An iterative hard decision based algorithm was also proposed which exploits the finite-alphabet property of the signal-constellation to improve the receiver performance.

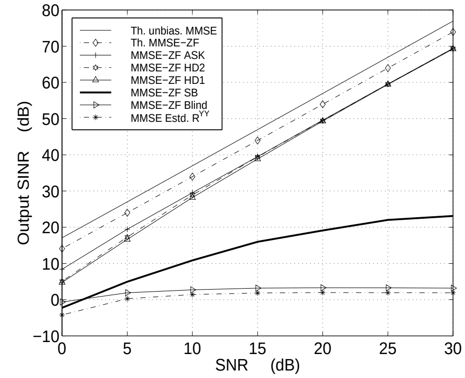

Figure 6. Output SINR performance of different receivers

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