[ORAL PRESENTATION]Joint Angle and Delay Estimation (JADE) by Partial Relaxation

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Localization by JADE

- Node positioning in a wireless system requires the gathering of location information from radio signals traveling between the target node and one, or multiple, reference anchors.
- The JADE (Joint Angle and Delay Estimation) approach measures delays/angles between an intended node and anchors to estimate the location of the former, with the help of the position of the latter.
- High accurarcy (in terms of MSE) is needed to reliably estimate location parameters for sub-meter accuracy.

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System model

Model

$$\mathbf{x}(\ell) = \mathbf{H}(\boldsymbol{\theta}, \boldsymbol{\tau})\boldsymbol{\gamma}(\ell) + \mathbf{n}(\ell) \tag{1}$$

$$\mathbf{H}(\boldsymbol{\theta}, \boldsymbol{\tau}) = \begin{bmatrix} h(\theta_1, \tau_1) & \dots & h(\theta_q, \tau_q) \end{bmatrix} \tag{2}$$

$$\boldsymbol{\gamma}(\ell) = \begin{bmatrix} \gamma_1(\ell) & \dots & \gamma_q(\ell) \end{bmatrix} \tag{3}$$

$$\mathbf{x}(\ell) = \begin{bmatrix} \mathbf{x}_1^\top(\ell) & \dots & \mathbf{x}_N^\top(\ell) \end{bmatrix}^\top \tag{4}$$

$$\mathbf{x}_n(\ell) = \begin{bmatrix} x_{n,1}(\ell) & \dots & x_{n,M}(\ell) \end{bmatrix}^\top \tag{5}$$

- $\mathbf{H}(\boldsymbol{\theta}, \boldsymbol{\tau})$ contains the multipath spatiotemporal signatures.
- $\boldsymbol{\gamma}(\ell)$ denotes multipath complex gains in ℓ^{th} frame.
- $x_{n,m}(\ell)$ denotes data on m^{th} sub-carrier received by n^{th} antenna in ℓ^{th} frame.

Data Collection

$$\mathbf{X} = \mathbf{H}(\boldsymbol{\theta}, \boldsymbol{\tau})\mathbf{G} + \mathbf{N} \tag{6}$$

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JADE by Partial Relaxation (PR)

Objective

Solve

$$(\hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\tau}}) = \arg\min_{\boldsymbol{\theta}, \boldsymbol{\tau}} \operatorname{tr}\left\{\mathcal{P}^{\perp}_{\mathbf{H}(\boldsymbol{\theta}, \boldsymbol{\tau})} \hat{\mathbf{R}}\right\} \quad (7)$$

PR Approach

Reformulate the problem as

$$(\hat{\boldsymbol{\theta}}_{PR}, \hat{\boldsymbol{\tau}}_{PR}) = \arg\min_{\theta, \tau, \mathbf{B}} \operatorname{tr}\left\{\mathcal{P}^{\perp}_{[\mathbf{h}(\theta,\tau) \;\; \mathbf{B}]} \hat{\mathbf{R}}\right\} \quad (8)$$

Algo 1: DML

Unstructured Signatures Optimization Problem

Projection decomposition:

$$\mathcal{P}^{\perp}_{\mathbf{H}} = \mathbf{I} - \mathcal{P}_{[\mathbf{h}\,\mathbf{B}]} = \mathbf{I} - (\mathcal{P}_{\mathbf{h}} + \mathcal{P}_{\mathcal{P}^{\perp}_{\mathbf{h}}\mathbf{B}}) = \mathcal{P}^{\perp}_{\mathbf{h}} - \mathcal{P}_{\mathcal{P}^{\perp}_{\mathbf{h}}\mathbf{B}}$$

Focus on optimization of unstructured JADE signatures $\mathbf{B}$:

$$\mathcal{P}_{\mathcal{P}^{\perp}_{\mathbf{h}}\mathbf{B}} = \mathcal{P}^{\perp}_{\mathbf{h}}\mathbf{B}(\mathbf{B}^H\mathcal{P}^{\perp}_{\mathbf{h}}\mathbf{B})^{-1}\mathbf{B}^H\mathcal{P}^{\perp}_{\mathbf{h}} = \mathbf{U}\,\mathbf{U}^H\,,\ \mathbf{U}^H\mathbf{U} = \mathbf{I},\ \mathbf{U}^H\mathbf{h} = 0$$

Now

$$\begin{aligned} &\operatorname{tr}\{\mathcal{P}_{\mathcal{P}^{\perp}_{\mathbf{h}}\mathbf{B}}\hat{\mathbf{R}}\} = \operatorname{tr}\{\mathbf{U}\mathbf{U}^H\hat{\mathbf{R}}\} = \operatorname{tr}\{\mathbf{U}^H\hat{\mathbf{R}}\mathbf{U}\} \\ &= \operatorname{tr}\{\mathbf{U}^H(\mathcal{P}_{\mathbf{h}} + \mathcal{P}^{\perp}_{\mathbf{h}})\hat{\mathbf{R}}(\mathcal{P}_{\mathbf{h}} + \mathcal{P}^{\perp}_{\mathbf{h}})\mathbf{U}\} = \operatorname{tr}\{\mathbf{U}^H\mathcal{P}^{\perp}_{\mathbf{h}}\hat{\mathbf{R}}\mathcal{P}^{\perp}_{\mathbf{h}}\mathbf{U}\} \end{aligned}$$

Then

$$\max_{\mathbf{U}^H\mathbf{U}=\mathbf{I}} \operatorname{tr}\{\mathbf{U}^H\mathcal{P}^{\perp}_{\mathbf{h}}\hat{\mathbf{R}}\mathcal{P}^{\perp}_{\mathbf{h}}\mathbf{U}\} = \sum_{k=1}^{q-1}\lambda_k\left(\mathcal{P}^{\perp}_{\mathbf{h}}\hat{\mathbf{R}}\mathcal{P}^{\perp}_{\mathbf{h}}\right)$$

Algo 1: DML

DML Optimization Problem

Combining, we get

$$\min_{\mathbf{B}} \operatorname{tr}\left\{\mathcal{P}^{\perp}_{[\mathbf{h}(\theta,\tau)\ \ \mathbf{B}]}\hat{\mathbf{R}}\right\} = \operatorname{tr}\left\{\mathcal{P}^{\perp}_{\mathbf{h}}\hat{\mathbf{R}}\right\} - \sum_{k=1}^{q-1}\lambda_k\left(\mathcal{P}^{\perp}_{\mathbf{h}}\hat{\mathbf{R}}\right)$$

Solution is to search the peaks of the following 2D spectrum

$$f_{\mathrm{DML}}(\theta,\tau) = \frac{1}{\sum_{k=q}^{NM}\lambda_k\left(\mathcal{P}^{\perp}_{\mathbf{h}(\theta,\tau)}\hat{\mathbf{R}}\right)} \tag{9}$$

Algo 2: Weighted Subspace Fitting (WSF)

WSF Optimization Problem

Weigh the covariance signal subspace by an appropriate matrix $\mathbf{W}$,

$$\underset{\mathbf{U},\theta,\tau}{\text{maximize}} \quad \text{tr}\left\{\mathbf{U}^H \mathcal{P}^{\perp}_{\mathbf{h}(\theta,\tau)} \hat{\mathbf{U}}_s \mathbf{W} \hat{\mathbf{U}}_s^{\mathrm{H}} \mathbf{U}\right\} \tag{10a}$$

$$\text{subject to} \quad \mathbf{U}^H \mathbf{U} = \mathbf{I}, \tag{10b}$$

Solution is to search the peaks of the following 2D spectrum

$$f_{\mathrm{WSF}}(\theta,\tau) = \frac{1}{\sum_{k=q}^{NM} \lambda_k \left(\mathcal{P}^{\perp}_{\mathbf{h}(\theta,\tau)} \hat{\mathbf{U}}_s \mathbf{W} \hat{\mathbf{U}}_s^{\mathrm{H}}\right)} \tag{11}$$

Algo 3: Covariance Fitting (CF)

CF Optimization Problem

Fit the covariance of the data according to the model

$$\mathbf{R} = \sigma_k^2 \mathbf{h}(\theta_k, \tau_k)\mathbf{h}^{\mathrm{H}}(\theta_k, \tau_k) + \mathbf{J}\mathbf{J}^{\mathrm{H}} \qquad (12)$$

$$\begin{aligned} \underset{\theta_k, \tau_k, \sigma_k^2, \mathbf{J}}{\text{minimize}} \quad & \left\| \hat{\mathbf{R}} - \sigma_k^2 \mathbf{h}(\theta_k, \tau_k)\mathbf{h}^{\mathrm{H}}(\theta_k, \tau_k) - \mathbf{J}\mathbf{J}^{\mathrm{H}} \right\|^2 \\ \text{subject to} \quad & \hat{\mathbf{R}} - \sigma_k^2 \mathbf{h}(\theta, \tau)\mathbf{h}^{\mathrm{H}}(\theta, \tau) \succcurlyeq \mathbf{0} \end{aligned} \qquad (13)$$

Solution is to search the peaks of the following 2D spectrum

$$f_{\mathrm{CF}}(\theta, \tau) = \frac{1}{\sum_{k=q}^{NM} \lambda_k^2 \left(\hat{\mathbf{R}} - \frac{\mathbf{h}(\theta,\tau)\mathbf{h}^{\mathrm{H}}(\theta,\tau)}{\mathbf{h}^{\mathrm{H}}(\theta,\tau)\hat{\mathbf{R}}^{-1}\mathbf{h}(\theta,\tau)} \right)} \qquad (14)$$

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Simulation Parameters

- $q = 2$ sources (multipath components)
- $\theta_1 = 6^\circ$ and $\theta_2 = 66^\circ$
- $\tau_1 = 5$ nsec and $\tau_2 = 10$
- number of subcarriers $M = 32$, number of antennas $N = 2$
- Source covariance

$$\mathbf{P} = \begin{bmatrix} 1 & 0.2e^{-j\frac{\pi}{6}} \\ 0.2e^{j\frac{\pi}{6}} & 1 \end{bmatrix}$$

Computer Simulations

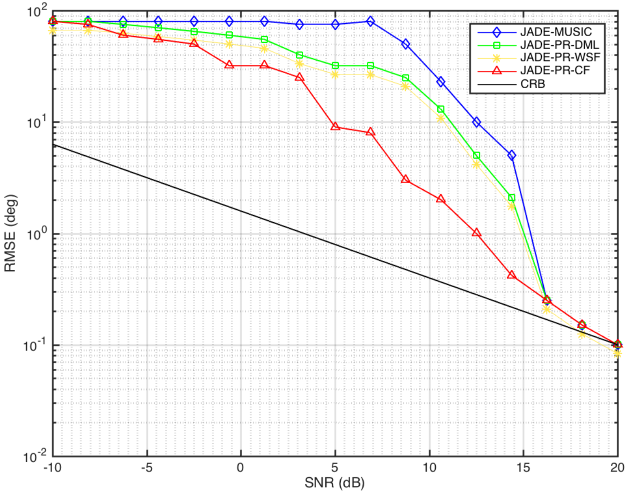

Figure: The RMSE of AoA estimates $\hat{\theta}_k$ per method vs SNR and the CRB.

Computer Simulations

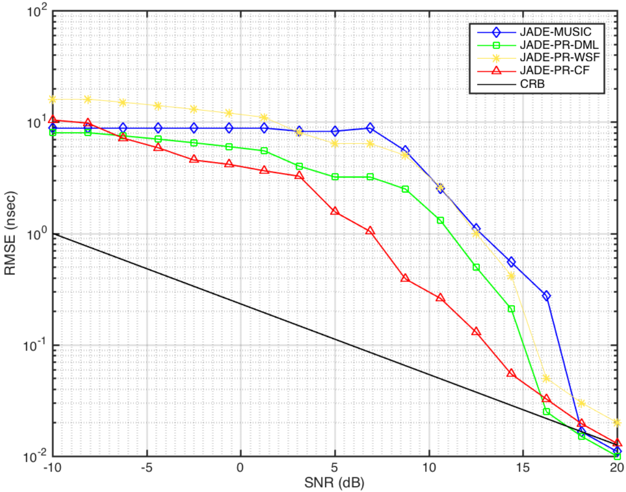

Figure: The RMSE of ToA estimates $\hat{\tau}_k$ per method vs SNR and the CRB.

Computer Simulations

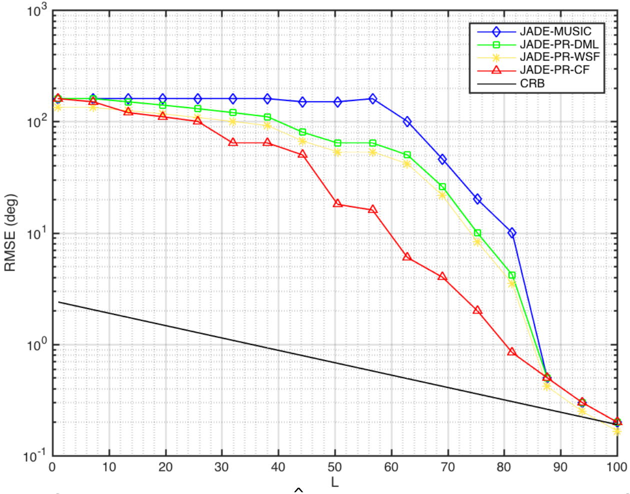

Figure: The RMSE of AoA estimates $\hat{\theta}_k$ per method vs Number of Snapshots and the CRB.

Computer Simulations

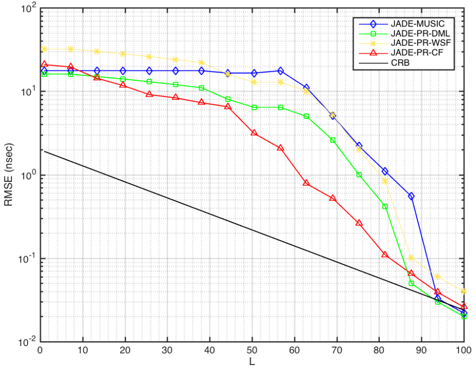

Figure: The RMSE of ToA estimates $\hat{\tau}_k$ per method vs Number of Snapshots and the CRB.

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Conclusions

- We have introduced three JADE estimators to the partially relaxed model: DML, WSF, CF.
- Simulation results demonstrate the Mean-Squared-Error convergence towards the Cramér-Rao Bound of each of these methods, either in SNR or in number of snapshots.

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