Variable Speed Limit Control for Mixed Powered Two-Wheelers and Cars Traffic

Sosina Gashaw¹, Jérôme Härri¹, Paola Goatin²

Abstract—This paper proposes a Variable Speed Limit (VSL) control strategy for a mixed flow of cars and Powered Two-Wheelers (PTWs). Due to the difference in physical and maneuvering characteristics of PTWs and cars, their impact on the traffic flow dynamics is different. Therefore, a control measure adapted to each vehicle class is required. Accordingly, we propose a vehicle-class specific VSL control scheme that regulates the speed limit for each vehicle class according to traffic efficiency and safety objectives, namely minimizing the total travel time and the speed difference between the two vehicle classes, respectively. The dynamics of the mixed traffic flow is formulated in Lagrangian coordinates, which allows vehicle-class specific group/platoon based speed limitations.

The proposed VSL control scheme is analyzed through simulation experiments. The results show that vehicle specific control is beneficial both in terms of traffic efficiency and safety.

I. INTRODUCTION

With the growth of traffic in urban and freeway roads, traffic congestion has become the biggest and the recurring challenge. Due to the multifaceted impact of congestion on traffic flow efficiency and safety, congestion mitigation has been under scrutiny for many years. The only practicable way of addressing traffic congestion is by managing and controlling the incoming traffic, as increasing road capacity is usually impractical. Based on the traffic condition, dynamic rules that influence drivers/traffic behavior are induced to avoid the occurrence of congestion or at least to minimize its effects. An appropriate control of traffic congestion in turn contributes to accident and emission reduction.

Variable Speed Limit (VSL) control is one of the widely implemented traffic control strategies [1], and it has showed a promising potential in creating stable flow [2], improving safety [3], [4] and mitigating pollution [5]. Depending on the objectives, VSL can be applied in different ways. A VSL intended to reduce speed variations aims at avoiding the occurrence of congestion [6] or traffic safety issues [7] due to speed inhomogeneity. Another application of VSL is to manage congestion at freeway bottlenecks, for example at lane merge locations or lane blocking incidents. By regulating the flow rate upstream of the bottleneck with VSL, traffic delay can be reduced [2], [8]. Similarly, VSL is implemented to control the upstream propagation of shock wave [9], [10]. In essence, the speed is controlled in such a way to reduce inflow to the congestion area so that the congestion dissolves rapidly.

Most of the existing VSL control systems are designed for homogeneous traffics, i.e. the traffic flows are assumed to be composed of vehicles with identical characteristics. In reality, traffic flows comprise vehicles with varied physical and maneuvering characteristics. The collective traffic flow dynamics is the result of the property of the individual vehicle class and the interaction among the classes. Moreover, each class has a different effect on the traffic flow characteristics. Thus, applying indistinguishable control and management actions in such heterogeneous conditions limits the efficiency of the control system because, first, the system fails to predict the traffic state accurately. Second, identical control action is applied irrespective of the impact the vehicle classes have on the traffic flow. There are only very few studies addressing control strategies for heterogeneous traffic flows. For traffic flows consisting of cars and trucks, attempts have been made to incorporate the difference between vehicle types. To take into account the heterogeneity of traffic flow, a multi-class model based freeway traffic control is introduced in [11], [12]. Similarly, Deo et al. [13] propose a model predictive (MPC) ramp metering and VSL control that utilizes multi-class model and show the performance improvement obtained by incorporating the heterogeneity in prediction model. A multi-class model based route guidance presented in [14] further shows the advantage of adapting a class specific control.

The aforementioned studies address the multi-class aspect in the context of slow moving trucks and fast moving cars. Despite having different length and speed, cars and truck have similar driving characteristics. The driving dynamics and characteristics of powered two wheelers (PTWs), such as motorcycles and mopeds, are however largely different, as they may share the same lane or filter through rows of traffic. Although lane filtering by PTWs is not legally accepted everywhere, due to safety concerns, it is a common practice on most of the European roads. Because of these unique maneuvering behaviors, PTWs uniquely impact traffic flows and also have a fully different perception of traffic conditions from cars (e.g., a road jammed for cars may not be necessarily jammed for PTWs, see Figure 1(a)).

PTWs represent a growing class of traffic, between the year 2002 and 2011 the fleet of PTWs increased by 17% in Europe [15]. The ability of PTWs to ride between lanes of traffic contributes to congestion reduction and cutting travel time [16], [17]. Nonetheless, unless PTWs are included in VSL and other traffic control systems, the control action could impair the potential benefits. Besides, given the high vulnerability of PTWs, safety issues should be taken into

¹Sosina Gashaw and Jérôme Härri are with EURECOM, 06904 Sophia Antipolis, France
²Paola Goatin is with Inria Sophia Antipolis Méditerranée, Université Côte d’Azur, Inria, CNRS, LIAD, 06904 Sophia Antipolis, France
account. For instance, VSL control is often implemented to manage congested or close to congestion traffic situations. In moderate to high traffic levels, PTWs highly engage in lane filtering [18], [19], which is one of the factors that increase the risk of accident. PTWs appear to be traveling faster than other vehicles [18] while accidents occur during lane filtering events. For this reason, the speed control decisions have to be adjusted in order not to further escalate accidents that arise from the speed difference. To ensure efficient and safe operation accordingly, it is important to integrate PTWs in the control systems.

In this paper, we therefore propose a VSL control for PTWs and cars following a Model Predictive Control (MPC) approach. The VSL system determines the speed limit for each vehicle type based on traffic efficiency and safety objectives, namely minimizing the total travel time and minimizing the speed difference between PTWs and cars. In previous work, we introduced a lagrangian representation of mixed traffic flow of PTWs and cars [20]. The proposed control system uses this multi-class model as a traffic state prediction model. We choose the Lagrangian representation because of the flexibility it gives to apply vehicle group/platoon based speed limitation. We analyze the vehicle class specific control approach with simulation experiments. The advantage of class-specific control is discussed by comparing with no-control and single control cases. Our proposed strategy notably illustrates the need to optimize speed differences between PTWs and cars in a multi-class VSL control to keep the peculiar advantage of PTWs and yet to mitigate safety risks.

This paper is organized as follows. In Section 2 we describe the prediction model and the control scheme. The simulation experiments as well as the discussion on the results are presented in Section 3. Finally, Section 4 concludes the paper with a brief summary and a discussion of future directions.

II. MOTIVATION AND METHODOLOGY

We consider a VSL system that regulates the incoming traffic to minimize congestion. The system predicts the onset of congestion and a proper speed limit is selected to avoid the occurrence of congestion. In an inevitable situations, the propagation of congestion to the upstream direction is suppressed through VSL.

The traffic flow is composed of two vehicle classes, PTWs and cars. The two vehicle classes have different maneuvering behaviors, e.g., PTWs filter between lanes, maintain smaller gaps, etc. Hence, the two classes perceive the traffic conditions differently, the speed-density relation of the two classes shown in Figure 1(a) illustrates this. Furthermore, the traffic properties, such as critical and jam density, for each vehicle class vary with the traffic composition, see Figure 1(b).

Under this kind of traffic flow applying identical speed limit for each vehicle class may impact the traffic flow efficiency. As there may be conditions where PTWs should be controlled, at certain traffic conditions the impact of two-wheeler is minimal and imposing a speed limit is unnecessary (see Figure 2(a)). Depending on the proportion of PTWs the impact they have on escalating congestion varies. For example, when there is a high number of PTWs, in the event of congestion, PTWs can filter between slow moving cars and enter to the congested area (see Figure 2(b)). Thereby, the outflow from the congested area for cars decreases, which in consequence prolongs the time required to resolve the traffic jam. Nevertheless, at low proportions of PTWs, imposing a speed limit on PTWs has an insignificant impact to the congestion clearance/minimization, rather it may increase travel time of PTWs and apparently PTW riders less like be obedient to the speed limit. Moreover, the decision of the speed limit should take into account safety issues since the speed difference between the two vehicle classes could possibly increase the risk of accident.

![Fig. 1: The fundamental properties of the traffic flow (a) Speed-density relationship for cars and PTWs (b) Flow-density diagram for cars, with different proportions of PTWs](image)

(a) Small number of PTWs  
(b) Large number of PTWs

Fig. 2: An illustrative example of traffic conditions at different PTWs proportions

Therefore, we apply vehicle class specific variable speed limit that takes into account traffic efficiency and safety objectives. Different from the common link-based controls, we implement a platoon-based speed limit.

The proposed MPC based variable speed control has two basic building blocks, a multi-class prediction model, and a multi-objective and class specific control algorithm. The control action produced by the control algorithm depends on the measured current traffic state and the future traffic state anticipated by the prediction model.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s )</td>
<td>average spacing</td>
</tr>
<tr>
<td>( v )</td>
<td>speed of vehicles</td>
</tr>
<tr>
<td>( \Delta t )</td>
<td>traffic simulation time step</td>
</tr>
<tr>
<td>( \Delta n )</td>
<td>platoon size (vehicles)</td>
</tr>
<tr>
<td>( \rho )</td>
<td>density of vehicles</td>
</tr>
<tr>
<td>( v_{ctrl} )</td>
<td>speed limit (m/s)</td>
</tr>
<tr>
<td>( v_f )</td>
<td>free flow speed</td>
</tr>
<tr>
<td>( J )</td>
<td>objective function</td>
</tr>
</tbody>
</table>

TABLE I: Table of symbols
A. Lagrangian Mixed Traffic State (Prediction) Model

A multi-class LWR model is used to describe the evolution of the traffic state. In LWR model, the equation representing the traffic flow dynamics can be formulated in Eulerian or Lagrangian coordinates. We apply here the Lagrangian representation, the detailed explanation can be found in [20].

There are two approaches to derive the conservation equation for multi-class flows. In the first method, the conservation equation for a given class is described with respect to a Lagrangian reference frame that moves with that class. Accordingly, the conservation equation for each class is written as:

\[
\frac{\partial s_u(x(t),t)}{\partial t} + \frac{\partial v_u(n,t)}{\partial n} = 0 \quad u = 1, 2 \tag{1}
\]

where \( s \) and \( v \) represent, respectively, the average spacing and the average speed. The spacing \( s = \frac{1}{\rho} \), where \( \rho \) is density.

The second method constructs the conservation equation with respect to a single Lagrangian reference frame, which moves with one of the vehicle classes (named as reference vehicle class). The conservation equation is therefore written differently for the reference and the other vehicle classes. The equation for the reference vehicle class is given by:

\[
\frac{\partial s_r(x(t),t)}{\partial t} + \frac{\partial v_r(n,t)}{\partial n} = 0, \tag{2}
\]

and for the other vehicle classes:

\[
\frac{\partial s_u}{\partial t} + \frac{\partial ((v_r(n,t) - v_u(n,t))/s_o(n,t))}{\partial n} = 0 \tag{3}
\]

In this paper, we employ the second approach, and cars-vehicle class is selected as the reference class.

Cars and PTWs have different driving behavior. Further, since we are considering a mixed flow of the two classes, the speed of each class depends on the density (or the spacing) of both vehicle classes. Thus, to capture these characteristics, the speed-density (or speed-spacing) relation is described differently for cars and PTWs.

\[
v_u = V_u(\rho_1, \rho_2) = V_u^{\max}(1 - F_u(\rho_1, \rho_2)^2) \tag{4}
\]

where \( V_u^{\max} \) denotes the maximum speed of class \( u \), and \( F_u(\rho_1, \rho_2) \) represents the proportion of inaccessible free space for vehicle class \( u \). For a given traffic state, the proportion of inaccessible free space for PTWs and cars is different (see reference [21]).

![Cluster 2: Speed limit](image)

**Fig. 3:** The schematic representation of platoon-based speed limitation

In Lagrangian discretization, rather than dividing the road stretch into segments, the vehicles are grouped into clusters (see Figure 3). In the first approach (equation (1)), each vehicle class is individually grouped into clusters. On the contrary, in the second approach (equation (2)-(3)), the clusters are formed according to the reference vehicle class. In our case the reference class is cars-vehicle class.

Assuming number of vehicles in the clusters remains unchanged, i.e there are no on-ramps and off-ramps, the discretized form of the conservation equation for cars is given by (Hereafter, unless specified, index 1 and 2 denote car and PTW vehicle classes, respectively).

\[
s_{1,i}(k+1) = s_{1,i}(k) - \frac{\Delta t}{\Delta n} (v_{1,i}(s_{1,i}(k), s_{2,i}(k)) - v_{1,i-1}(s_{1,i-1}(k), s_{2,i-1}(k))) \tag{5}
\]

Where \( i \), \( \Delta n \), respectively, denote the cluster index, the number of cars in the cluster, \( k \) denotes the time step counter and it has the following relation with the simulation time \( t \) and the model update time step \( \Delta t, t = k \Delta t \).

The average spacing \( (s) \) of PTWs inside the clusters of cars is written as.

\[
\left(\frac{s_{1,i}}{s_{2,i}}\right)(k+1) = \frac{s_{1,i}}{s_{2,i}}(k) - \frac{\Delta t}{\Delta n} (f_{2,i+1/2}(k) - f_{2,i-1/2}(k)) \tag{6}
\]

where \( f_{2,i+1/2} \) and \( f_{2,i-1/2} \) are the flow rates of PTWs at cluster \( i \) boundaries. Let \( n_2 \) be the number of PTWs inside cluster \( i \), \( \frac{1}{s_{2,i}} = \frac{n_2}{\Delta n} \), where \( s_{2,i} \) is the average spacing of PTWs (note that \( s_{2,i} \) represents the density) in cluster \( i \) and \( s_{1,i} \Delta n \) is the length of the cluster. Substituting this into equation (6), we obtain

\[
n_{2,i}(k+1) = n_{2,i}(k) - \Delta t (f_{2,i+1/2}(k) - f_{2,i-1/2}(k)) \tag{7}
\]

The flows at the boundaries are defined as follows:

If \( v_{1,i} < v_{2,i} \),

\[
\begin{align*}
f_{2,i+1/2} &= \min \left( 0, \frac{(v_{r,i} - v_{o,i+1})}{s_{o,i+1}} \right) \\
f_{2,i-1/2} &= \min \left( 0, \frac{(v_{r,i} - v_{o,i})}{s_{o,i}} \right) - \min \left( 0, \frac{(v_{r,i} - v_{o,i-1})}{s_{o,i-1}} \right)
\end{align*}
\]

If \( v_{1,i} > v_{2,i} \),

\[
\begin{align*}
f_{2,i+1/2} &= \frac{(v_{r,i} - v_{o,i})}{s_{o,i}} - \max \left( 0, \frac{(v_{r,i} - v_{o,i+1})}{s_{o,i+1}} \right) \\
f_{2,i-1/2} &= \max \left( 0, \frac{(v_{r,i} - v_{o,i-1})}{s_{o,i-1}} \right)
\end{align*}
\]

The simulation time step \( \Delta t \) should be restricted to Courant-Friedrichs-Lewy (CFL) condition, i.e.

\[
\Delta t \leq \frac{\Delta n}{\max(\lambda_1, \lambda_2)} \tag{8}
\]

where \( \lambda \) stands for information propagation speed (in vehicles per unit time).
In the presence of on-ramps and off-ramps, the equations for the reference class can be formulated following [22]. For PTWs, equation (6) is rewritten as:

\[
\left(\begin{array}{c}
v_{1,i}^{\text{ctrl}}(k+1) \\
v_{2,i}^{\text{ctrl}}(k+1)
\end{array}\right) = \left(\begin{array}{c}
v_{1,i}^{\text{ctrl}}(k) \\
v_{2,i}^{\text{ctrl}}(k)
\end{array}\right) - \frac{\Delta t}{\Delta n} \left(\begin{array}{c}
f_{2,i+1/2}(k) \\
f_{2,i-1/2}(k) - r_2(x(i), k) + l_2(x(i), k)
\end{array}\right)
\]

where \(r_2(x(i))\) and \(l_2(x(i))\) are the on-ramp and off-ramp PTWs’ flows rate at the location \(x\) of cluster \(i\), respectively.

B. MPC-based VSL Controller

The variable speed limit control problem is solved in a model predictive control (MPC) scheme. The MPC approach implements a receding optimization strategy. At each time instant, an optimal control sequence is solved over the prediction interval \([k_cT_c, T_c(N_p + k_c - 1)]\), where \(N_p\) is the prediction horizon.

For the sake of minimizing the computation complexity, a control horizon \(N_c \leq N_p\) is selected. Consequently, from the control horizon onward the control variable becomes constant. Then, only the first values of the control sequences is applied to the system, and the horizon is moved to the future by \(T_c\) step.

We implement a general control framework similar to [23], where vehicle class-specific speed control is applied. The vehicle class-specific speed limit is determined for each cluster. The clustering of vehicles is performed based on cars, i.e. cars are grouped into \(\Delta N\) clusters. When the speed limit is specified for cars in a given cluster, the speed limit for PTWs inside that cluster is also specified separately (see Figure 3). Moreover, the number of cars in a cluster remains invariant, however the number of PTWs may change. When PTWs move from one cluster to another, they adopt the speed limit specified for PTWs in the current cluster.

The control variables for \(N_c\) number of clusters and two vehicle classes is written as

\[
v_{1}^{\text{ctrl}}(k) = [v_{1,1}^{\text{ctrl}}(k), ..., v_{1,N}^{\text{ctrl}}(k)]
\]

\[
v_{2}^{\text{ctrl}}(k) = [v_{2,1}^{\text{ctrl}}(k), ..., v_{2,N}^{\text{ctrl}}(k)]
\]

\[
u^{\text{ctrl}}(k) = [v_{1}^{\text{ctrl}}(k), v_{2}^{\text{ctrl}}(k)]^T
\]

and the state variables, i.e. the average spacing and the average speed

\[
s_{1}(k) = [s_{1,1}(k), ..., s_{1,N}(k)]
\]

\[
s_{2}(k) = [s_{2,1}(k), ..., s_{2,N}(k)]
\]

\[
v_{1}(k) = [v_{1,1}(k), ..., v_{1,N}(k)]
\]

\[
v_{2}(k) = [v_{2,1}(k), ..., v_{2,N}(k)]
\]

Then, the state equation becomes

\[
x(k) = [s_{1}(k), s_{2}(k), v_{1}(k), v_{1}(k)]^T
\]

\[
x(k+1) = f(x(k), u^{\text{ctrl}}(k))
\]

In other words, the traffic state at time \(k + 1\) is a function of the traffic state and the control input, which is the speed limit, at time \(k\). When the speed limit control is applied, the speed for each vehicle class becomes

\[
v_{u,i} = \min\{v_{u,i}, (1 + \alpha_i)v_{ctrl}\}
\]

where \(v_{u,i}\) is the speed derived from the fundamental relation and \(\alpha_i\) is the driver non-compliance factor, i.e. the disobedience of drivers to the speed limit. The equation in equation (11) implies that at 100\% compliance, vehicles may drive lower than the speed limit due to the traffic condition, but the maximum speed is limited to \(v_{ctrl}\).

To disseminate the speed limit information V2X communication can be used. The VSL control system broadcasts the class-specific speed limits together with the cluster identification number. Since cars do not change cluster, they can decode the speed limit information in a straightforward manner. However, PTWs move from one cluster to another. In order to determine the speed limit in the current cluster, PTWs need to know the identification number of the cluster.

For this purpose, the cluster heads periodically broadcast an updated information such as location, cluster identification, speed limit for PTWs. Consequently, PTWs can infer the current cluster and the speed limit using the information received from the cluster heads and their location.

Objective function: Given the initial conditions, the control objective is to minimize the total time spent (TTS) by all vehicle classes in the freeway mainline via the adjustment of the speed limit. Moreover, we include a safety objective that minimizes the speed difference (SD) between vehicle classes. Thus, our objective function has the following form.

\[
J = \alpha_{\text{TTS}} \sum_{k_c=1}^{N_c} \sum_{i=1}^{N} (n_{1,i}(k_c) + n_{2,i}(k_c)) \Delta t + \alpha_{\text{SD}} \sum_{k_c=1}^{N_c} \sum_{i=1}^{N} \left(\frac{v_{1,i}^{\text{ctrl}}(k_c) - v_{2,i}^{\text{ctrl}}(k_c)}{v_1 - v_2}\right)^2, v_1 - v_2 \neq 0
\]

(12)

Where \(n_{1,i}\) and \(n_{2,i}\) are number of class 1 and class 2 vehicles, respectively. Correspondingly, \(v_{1,i}^{\text{ctrl}}(k_c)\), \(v_{2,i}^{\text{ctrl}}(k_c)\) stands for the speed limit of class 1 and 2. The weighting factors for TTS \((\alpha_{\text{TTS}})\) and SD \((\alpha_{\text{SD}})\) are tuned depending on the control policy. To decide the weighting factors different aspects such as the traffic composition can be considered.

Previous studies indicate the relation between speed variance and accident rate [24], [25]. For instance, in [25] the speed variance and accident rate for homogeneous traffic flow is formulated as follow

\[
\text{AccRate} = c_1 + c_2 \ast (v_{\text{var}})
\]

(13)

where \(\text{AccRate}\) and \(v_{\text{var}}\) are the number of accidents per vehicle-miles and the speed variance, respectively. \(c_1\) and \(c_2\) are constant parameters. A similar principle can be applied to heterogeneous traffic flow. In fact, in a mixed cars and PTWs flow, due to the divergent behavior of PTWs, a range of other factors also may impact the accident rate. Nonetheless,
it is reasonable to assume that accident rate and speed variance have direct relation. Therefore, by integrating the minimization of the difference between the speed limits for the two vehicle classes in equation (12), the accident rate can be reduced.

Furthermore, in the objective function the number of vehicles in each vehicle class is included. The optimization result thus is adapted to the proportion of PTWs (or cars).

Constraints: The control variable is constrained by the following conditions: The difference between the speed limits in a consecutive control steps should be less than the maximum allowed speed change, which is related to the deceleration/acceleration capability of the vehicle class and safety.

\[
\Delta v^\text{ctrl}_{u,i} \leq v^\text{maxdiff}_u
\]  

(14)

The control speed for each class \( u \) should be bounded by the minimum speed limit and the free flow speed of the respective class.

\[
v^\text{ctrl}_{u,i} \in [v^\text{min}_u, v^\text{max}_u]
\]  

(15)

III. SIMULATION EXPERIMENTS

In order to evaluate the performance of the proposed VSL control, we compare results from the following three approaches. No control: no speed limit is imposed on any of the two vehicle classes; Single control (1-VSL): the speed limit applies only for cars or identical speed limit is applied for both vehicle classes; Class specific control (2-VSL): separate speed limit for cars and PTWs, which is the proposed approach.

A. Simulation setup

![Simulation scenario](freeway_link)

The proposed vehicle class specific VSL control scheme is evaluated in the following simulation scenario. We consider a 3 km long freeway link with no off-ramp and on-ramp. At the initial state, congestion is created at the middle section of the freeway segment, and we have a free flow condition in the upstream and downstream directions (Figure 4). The VSL is applied to control the flow in the upstream of the congested section. The initial densities are given in Table II. The initial densities are chosen such that free-flow conditions are created upstream and downstream of the central location, which is the congested section. The value of the initial densities at the upstream free-flow region are changed so that to increase PTWs proportions, and thereby can have a significant effect.

In the optimization, the control speeds \((v^\text{ctrl}_{1,i}, v^\text{ctrl}_{2,i})\) are chosen from the discrete set of VSLs specified for each vehicle class, where \(v^\text{ctrl}_{1,i} \in \{15, 12, 9, 6\}\) and \(v^\text{ctrl}_{2,i} \in \{20, 17, 14, 11\}\). Furthermore, the speed difference between two consecutive speed limits is constrained to 3m/s (≈ 10km/hr). For the traffic simulation, the Lagrangian coordinate moves with cars and the platoon/cluster size equals \(\Delta n = 50\). The MPC parameters are set to the following values, \(N_c = 3 (1.5 \text{ min}), N_p = 5 (2.5 \text{ min}), T_e = 30s (T_e = 36*\Delta t)\). The value of \(N_p\) is set to the time needed by cars to cross the road segment in free-flow condition.

The implemented MPC generates possible sequences of \(v^\text{ctrl}_{1,i}, v^\text{ctrl}_{2,i}\) combinations, which conform to the constraints specified, from the discrete speed sets and searches for the sequence that optimize the objective function.

The performance of the proposed VSL is investigated by comparing results from the three different VSL control scenarios. First, no VSL control is applied, and the improvement obtained from the VSL control is evaluated with respect to this uncontrolled case. Total time spent (TTS) by vehicles is used as a metric for the evaluation. Furthermore, we study two VSL control cases. In the first case, the control speed is derived and applied only for cars (Hereafter we call it 1-VSL), whereas in the second case a specific VSL is applied for the individual vehicle class (Hereafter we call it 2-VSLs).

The first case answers the question whether we need to have VSL control for PTWs or not. On the other hand, the latter case provides an insight on the benefit/need of having VSL control for PTWs.

B. Simulation results

Uncontrolled case: The evolution of cars densities and speed in uncontrolled case are presented in Figure 5. As illustrated by the figures, the congestion propagates backward (from 1500 m to 500 m) and the effect linger for long time. We show the results for cars only because they are more affected by the congestion and the backward propagation of the congestion is more visible.

![Evolution of traffic densities and speed for cars under uncontrolled case](density_speed_profile)

(a) Density profile  (b) Speed profile

Fig. 5: Evolution of the traffic densities and speed for cars under uncontrolled case

Single control (1-VSL): In this case, we apply a VSL control for cars only. The speed limitation applies only to the platoon upstream of the congested area. The density and speed evolution of cars are shown in Figure 6. In addition, the speed limits over the simulation period are shown in Figure

<table>
<thead>
<tr>
<th>Location</th>
<th>[0-1500 m]</th>
<th>[1500-2500 m]</th>
<th>[2500-3000 m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho_1)</td>
<td>0.18</td>
<td>0.45</td>
<td>0.2</td>
</tr>
<tr>
<td>(\rho_2)</td>
<td>0.14</td>
<td>0.2</td>
<td>0.1</td>
</tr>
</tbody>
</table>

**TABLE II: Initial densities (veh/m).** \(\rho_1\) for cars and \(\rho_2\) for PTWs
7, no speed limit is imposed on PTWs. As reported by the result, the backward congestion propagation is suppressed. Furthermore, a 3.01% (relative change) improvement in the TTS is obtained. However, as illustrated on Figure 9, a speed difference of 10 to 14 m/s is created between cars and PTWs, which creates dangerous overtaking situations.

In the above single speed limit control (I-VSL), the VSL controls the flow of cars only, i.e. no speed limit is imposed on PTWs. We examine also the other case of single control, in which identical speed limit is derived for both vehicle classes. According to the result, compared to the uncontrolled case, a 1.91% increase in the TTS is observed. In the tested case, the system fails to meet the required objective, i.e. reducing the TTS. Forcing PTWs to behave like cars results in the decrease of the flow efficiency. This indicates the need to apply different speed limits for each vehicle class.

Class specific control (2-VSLs): In this experiment, similar to the previous experiments, the VSL is imposed only to the vehicles upstream of the congested area. But, we have a VSL specific to each vehicle class. The speed and densities evolution are shown in Figure 8, and Figure 10 depicts the speed limits for cars and PTWs. In addition to the TTS minimization, we add the minimization of the speed difference in the objective function, the weighting factors $\alpha_{\text{TTS}}, \alpha_{\text{SD}}$ set to 1. From the result in Figure 8, it can be seen that the congestion propagation is suppressed. We achieve 3.12% improvement in the TTS, compared to the uncontrolled case. Compared to I-VSLs, the TTS is reduced by 0.115%. However, a major improvement is obtained in terms of minimizing the speed difference between cars and PTWs. Figure 9 depicts the speed difference between PTWs and cars during I-VSLs and 2-VSLs controls. What we can observe is that the speed difference between cars and PTWs is minimized in the 2-VSLs control since the speed limits are optimized considering the speed difference.

The results represent what we observe at the selected traffic proportion and condition. However, we have noticed also for lower proportions of PTWs, if the speed limit is optimized to minimize TTS only, I-VSLs and 2-VSLs produce identical results. The reason for this is that PTWs have insignificant impact and imposing speed limit on PTWs neither improve the TTS of cars nor the TTS of the collective traffic. In this case, the advantage with 2-VSLs is the possibility to control the speed difference between cars and PTWs, and thereby to minimize accident risks.

In general, the results show that I-VSLs and 2-VSLs control have almost the same effect in regards to minimizing TTS. Nonetheless, the 2-VSLs is able to minimize the speed difference between PTWs and cars, thus avoids potential accident...
situations, and this at no impact on TTS compared to 1-VSLs. Implementing a VSL control for both cars and PTW creates a more efficient mobility and jointly reduces accident opportunities. Therefore, vehicle-class based optimization is beneficial from both traffic efficiency and safety aspects.

IV. CONCLUSION AND FUTURE WORK

The growing penetration of PTWs added to their vulnerability and unique maneuverability demands for the integration of PTWs to C-ITS systems. In the response to this emerging need, in this paper we propose a variable speed limit (VSL) application for mixed cars and powered two-wheelers (PTWs) traffic. A separate speed limit has been derived for PTWs and cars according to the objective function and the traffic condition. We consider different optimization objectives such as minimizing TTS and minimizing speed difference between cars and PTWs. Different from the common link-based controls, we implement a platoon-based speed limit. Therefore, the traffic state is formulated in Lagrangian coordinate, which gives a flexibility to apply vehicle group/platoon based speed limitation. Considering that imposing identical VSL for both cars and PTWs would go against the objectives of PTWs to filter through traffic (and would highly not be respected), also imposing VSL only for cars would lead to large speed differences between the two vehicle classes and be source of potential collisions. Accordingly, the proposed VSL control, first still provides an optimal travel time, second maintains the benefit of PTWs to filter through traffic, and finally keeps the speed difference between the classes at the reasonable level, which significantly contributes to enhancing traffic safety in mixed traffic conditions.

The proposed VSL scheme can be extended in different ways. The tendency of PTWs to keep smaller gap with other vehicles is reported to be one of the causes of accidents involving PTWs. Therefore, the maximization of the average spacing in the platoons is one of the interesting aspects we will investigate in the future work. Moreover, the proposed scheme has to be analyzed at different proportions of PTWs and driver compliance rates using empirical data.

ACKNOWLEDGMENT

This work was funded by the French Government (National Research Agency, ANR) through the Investments for the Future Program reference #ANR-11-LABX-0031-01. EURECOM acknowledges the support of its industrial members, namely, Orange, BMW Group, SAP, Monaco Telecom, Symanetc, IABG.

REFERENCES

Sosina Gashaw recently received a Ph.D. from University of Cote d’Azur. Her Ph.D. research focused on modeling heterogeneous vehicular traffic and ITS applications. Her research interests are related to vehicular traffic modeling and simulation, intelligent traffic management, connected vehicles. She holds also an M.Sc. in computer and communication networks engineering and a B.Sc. in electrical engineering from Politecnico di Torino, Italy, and Arba Minch University, Ethiopia, respectively.

Jérôme Härri is a Professor in the Department of Communication Systems at EURECOM, Sophia Antipolis (France), where he leads the Connected Automated Transport System (CATS) team. Previously, He led the Traffic Telematics Junior Research Group at the Institute of Telematics of the Karlsruhe Institute of Technology (KIT), Germany. His research interests include vehicular communication and networking, cooperative ITS strategies, heterogeneous traffic flow modeling, positioning and localization. He has authored and co-authored over 70 international journal and conference papers and has involved in various National and European research projects related to wireless vehicular communications. He received an M.Sc. degree (2002) and a Dr. és sc. (Ph.D.) degree (2007) in telecommunication from the Swiss Institute of Technology (EPFL), Lausanne, Switzerland.

Paola Goatin is a senior researcher (Directeur de Recherche 2me classe) at Inria Sophia-Antipolis (France), where she leads the ACUMES research team focusing on Analysis and Control of Unsteady Models in Engineering Sciences. Her research interests are related to hyperbolic systems of conservation laws, finite volume numerical schemes, macroscopic traffic flow models, PDE-constrained optimization. She received a Ph.D. in Functional Analysis from the International School of Advanced Studies (SISSA-ISAS) in Trieste, Italy and a Laurea (degree) in mathematics from Universita de Padova.