Massive MISO IBC Reduced Order Zero Forcing Beamforming - a Multi-Antenna Stochastic Geometry Perspective

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***Abstract*—In a multi-user multi-antenna interfering broadcast channel (IBC) system, optimal linear receive and transmit beamformers are of the Maximum Signal-to-Interference-plus-Noise (MaxSINR) type. These designs make an optimal compromise between noise enhancement and interference suppression and reduce to matched filters at low SNR and zero-forcing at high SNR . The novel beamformers are here optimized for the Expected Weighted Sum Rate (EWSR) for the case of Partial Channel State Information at the Transmitters (CSIT). We consider a Gaussian partial CSIT model, combining channel estimates and covariance information. We constrain the transceiver to either zero-force or ignore each interference term. This leads to a reduced-order zero-forcing (RO-ZF) design in which the number of interference terms being zero-forced increases with SNR. We extend a recently introduced large system analysis for optimized beamformers with partial CSIT, by a stochastic geometry inspired randomization of the channel covariance eigen spaces, leading to much simpler analytical results which depend only on some essential channel characteristics. RO-ZF designs lead to variable reductions of computational complexity and CSI requirements.**

I. Introduction

In this paper, Tx may denote transmit/transmitter /transmission and Rx may denote receive/receiver/reception. By deploying large number of antennas at the base station (BS) [1], Massive MIMO (MaMIMO) can enhance the network throughput. However, in order to fully exploit the capabilities, it has precise requirements of CSIT. Optimal BF design with limited feedback information or partial CSIT has been proposed in [2].

Recently introduced large system analysis for MaMISO systems [3] facilitate the analysis and design of wireless systems in the massive MISO limit. E.g. it may allow to evaluate beamforming performance without computing explicit beamformers by obtaining deterministic (instead of channel realization dependent) expressions for various scalar quantities. Few extensions can be found in [4]–[6]. [2] proposes a large system analysis for optimized BF with partial CSIT as considered here. Furthermore, the channel, channel estimate and channel error covariances can be arbitrary and different for all users. However, the resulting deterministic analysis is quite cumbersome and does not allow much analytical insight. In stochastic geometry based methods [7], the location of the users is assumed to be random, their geographic distribution then inducing a certain probability distribution for the channel attenuations. Whereas most stochastic geometry work focuses on the distribution of the attenuations, here we consider an extension to multi-antenna systems. The multipath propagation for the various users leads to randomized angles of arrival at the BS which can be translated into spatial channel response contributions that depend on the antenna array response. In the massive MIMO regime in which the number of BS antennas gets very large, it has been observed and exploited that despite complex multipath propagation, the channel covariance matrix tends to be low rank. Exploiting the randomized nature of the user and scatterer positions and making abstraction of the antenna array response, we proposed to model the user channel subspaces as isotropically randomly oriented. This allows us to assume the eigen vectors of the channel covariance matrix to be Haar distributed, and this identically and independently for all users.

A. Contributions of this paper

In this paper:

- We first review optimal zero forcing beamformer (ZF BF) for the expected weighted sum rate (EWSR) criterion in the MaMIMO limit.
- We extend the concept of reduced-order ZF BF [8] for partial CSIT and propose a greedy approach to optimize the reduced ZF orders.
- We evaluate the ergodic sum rate performance for Least-Squares (LS), LMMSE and subspace projection channel estimators. Numerical results suggest that there is substantial gain by exploiting the channel covariance information compared to just using the LS estimates.
- New large system analysis for various cases of BF with partial CSIT is proposed, with a randomized analysis of the covariance subspaces, leading to much simpler results. This constitutes a marriage between large system analysis and multi-antenna stochastic geometry.
- Simulation results indicate that large system approximations are very accurate even for small system dimensions and reveal the deterministic dependence of the system performance on several important scalar parameters, such as the channel attenuation, signal powers and SNR (whereas [2] doesn't lead to any tractable analytical solutions).

Notation: In the following, boldface lower-case and upper-case characters denote vectors and matrices respectively. The operators $\mathrm{E}(\cdot)$, $tr(\cdot)$, $(\cdot)^H$, $(\cdot)^T$ represents expectation, trace , conjugate transpose and transpose respectively. $\mathrm{diag}(\cdot)$ represents the diagonal matrix formed by the elements $(\cdot)$. $(\cdot)^*$ represents conjugate of a complex scalar. A circularly complex Gaussian random vector with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Theta}$ is distributed as $\boldsymbol{x} \sim \mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Theta})$. $\mathbf{V}_{max}(\mathbf{A}, \mathbf{B})$ or $\mathbf{V}_{max}(\mathbf{A})$ represents (normalized) dominant generalized eigenvector of $\mathbf{A}$ and $\mathbf{B}$ or (normalized) dominant eigenvector of $\mathbf{A}$ respectively and $\lambda_{max}(\mathbf{A})$ is the corresponding max eigen value.

II. IBC Signal Model

We consider an IBC with C cells with a total of K single antenna users. We shall consider a system-wide numbering of the users. User k is served by BS b_k. The received signal at user k in cell b_k is

$$\boldsymbol{y}_k = \underbrace{\mathbf{h}_{k,b_k}^H \mathbf{g}_k x_k}_{\text{signal}} + \underbrace{\sum_{\substack{i \neq k \\ b_i = b_k}} \mathbf{h}_{k,b_k}^H \mathbf{g}_i x_i}_{\text{intracell interf.}} + \underbrace{\sum_{j \neq b_k} \sum_{i: b_i = j} \mathbf{h}_{k,j}^H \mathbf{g}_i x_i}_{\text{intercell interf.}} + \boldsymbol{v}_k \quad (1)$$

where x_k is the intended (white, unit variance) scalar signal stream, $\mathbf{h}_{k,b_i}$ is the $M_{b_k} \times 1$ channel from BS b_i to user k. The Rx signal (and hence the channel) is assumed to be scaled so that we get for the noise $v_k \sim \mathcal{CN}(0,1)$. BS b_k serves $K_{b_k} = \sum_{i:b_i=b_k} 1$ users. The $M_{b_k} \times 1$ spatial Tx filter or beamformer (BF) is $\mathbf{g}_k$.

III. Channel and CSIT Model

For simplicity, we omit all the user indices k. We start from a deterministic Least-Squares (LS) channel estimate

$$\widehat{\mathbf{h}}_{LS} = \mathbf{h} + \widetilde{\mathbf{h}}, \quad (2)$$

where $\mathbf{h}$ is the true MISO channel, and the error is modeled as circularly symmetric white Gaussian noise $\widetilde{\mathbf{h}} \sim \mathcal{CN}(0, \widetilde{\sigma}^2 \mathbf{I})$. Now each MISO channel is modeled according to a correlation structure (Karhunen-Loeve representation [9]) as follows,

$$\mathbf{h} = \mathbf{C}\,\mathbf{c}, \;\; \mathbf{c} = \mathbf{D}^{1/2}\mathbf{c}', \quad (3)$$

where $\mathbf{c}' \sim \mathcal{CN}(0, \mathbf{I}_L)$ and $\mathbf{D}$ is diagonal. Here $\mathbf{C}$ is the $M \times L$ eigen vector matrix of the reduced rank channel covariance $\boldsymbol{R}_{\mathbf{hh}} = \mathbf{CDC}^H$. The total sum rank across all users $N_p = \sum_{k=1}^{K} L_{k,c}$ is assumed to be less than M_c, where $L_{k,c}$ is the channel rank between user k and BS c. Assuming the channel covariance subspace is known, the LMMSE channel estimate can be written as $\widehat{\mathbf{h}} = \mathbf{CDC}^H \left(\mathbf{CDC}^H + \widetilde{\sigma}^2 \mathbf{I}\right)^{-1} \widehat{\mathbf{h}}_{LS}$. Applying the matrix inversion lemma and exploiting $\mathbf{C}^H\mathbf{C} = \mathbf{I}_L$, this simplifies to

$$\widehat{\mathbf{h}} = \mathbf{C}\left(\widetilde{\sigma}^2 \mathbf{D}^{-1} + \mathbf{I}\right)^{-1} \mathbf{C}^H \widehat{\mathbf{h}}_{LS} = \mathbf{C}\widehat{\mathbf{D}}^{1/2}\widehat{\mathbf{c}}, \quad (4)$$

where $\widehat{\mathbf{D}} = \left(\widetilde{\sigma}^2 \mathbf{D}^{-1} + \mathbf{I}\right)^{-1} \mathbf{D}$ and $\widehat{\mathbf{c}} = \mathbf{D}^{-1/2}(\sigma^2 \mathbf{D}^{-1} + \mathbf{I})^{-1/2} \mathbf{C}^H \widehat{\mathbf{h}}_{LS}$. The posterior error covariance becomes

$$\boldsymbol{R}_{\widetilde{\mathbf{h}}\widetilde{\mathbf{h}}} = \mathbf{CDC}^H - \mathbf{CDC}^H \left(\mathbf{CDC}^H + \widetilde{\sigma}^2 I\right)^{-1} \mathbf{CDC}^H, \quad (5)$$

which the matrix inversion lemma allows to simplify to,

$$\boldsymbol{R}_{\widetilde{\mathbf{h}}\widetilde{\mathbf{h}}} = \mathbf{C}\left[\mathbf{D} - \left(\widetilde{\sigma}^2 \mathbf{D}^{-1} + \mathbf{I}\right)^{-1} \mathbf{D}\right] \mathbf{C}^H = \mathbf{C}\widetilde{\mathbf{D}}\mathbf{C}^H. \quad (6)$$

So we can write for $\mathbf{S} = \mathrm{E}_{\mathbf{h}|\widehat{\mathbf{h}}}\left(\mathbf{h}\mathbf{h}^H\right) = \widehat{\mathbf{h}}\,\widehat{\mathbf{h}}^H + \boldsymbol{R}_{\widetilde{\mathbf{h}}\widetilde{\mathbf{h}}} = \mathbf{CWC}^H$, where $\mathbf{W} = \widehat{\mathbf{D}}^{1/2}\widehat{\mathbf{c}}\widehat{\mathbf{c}}^H\widehat{\mathbf{D}}^{1/2} + \widetilde{\mathbf{D}}$.

IV. Partial CSIT BF based on Different Channel Estimates

In the MaMIMO limit, BF design with partial CSIT will depend on the quantities $\mathbf{S} = \mathrm{E}_{\mathbf{h}|\widehat{\mathbf{h}}}\left(\mathbf{h}\mathbf{h}^H\right) = \widehat{\mathbf{h}}\,\widehat{\mathbf{h}}^H + \boldsymbol{R}_{\widetilde{\mathbf{h}}\widetilde{\mathbf{h}}}$. We shall consider three possible channel estimates.

(i) *LS Channel Estimate*

We have $\widehat{\mathbf{h}}_{LS} = \mathbf{h} + \widetilde{\mathbf{h}}$, where $\mathbf{h}$ and $\widetilde{\mathbf{h}}$ are independent. In the LS case, $\boldsymbol{R}_{\widetilde{\mathbf{h}}\widetilde{\mathbf{h}}} = \widetilde{\sigma}^2 \mathbf{I}$.

(ii) *LMMSE Channel Estimate*

We have $\mathbf{h} = \widehat{\mathbf{h}} + \widetilde{\mathbf{h}}$ in which $\widehat{\mathbf{h}}$ and $\widetilde{\mathbf{h}}$ are decorrelated and hence independent in the Gaussian case. In the LMMSE case, $\boldsymbol{R}_{\widetilde{\mathbf{h}}\widetilde{\mathbf{h}}}$ is the posterior covariance. The resulting $\mathbf{S} = \widehat{\mathbf{h}}\widehat{\mathbf{h}}^H + \boldsymbol{R}_{\widetilde{\mathbf{h}}\widetilde{\mathbf{h}}}$ now forms an unbiased estimate of $\mathbf{h}\mathbf{h}^H$: $\mathrm{E}_{\widehat{\mathbf{h}}}\mathbf{S} = \boldsymbol{R}_{\mathbf{hh}}$.

(iii) *Subspace Projection based Channel Estimate*

We also investigate the effect of limiting channel estimation error to the covariance subspace (without the LMMSE weighting, this is a simplification of the LMMSE estimate). The subspace channel estimate is given as,

$$\widehat{\mathbf{h}}_S = \boldsymbol{P}_C \widehat{\mathbf{h}}_{LS} = \mathbf{h} + \boldsymbol{P}_C \widetilde{\mathbf{h}}_{LS}\,, \; \boldsymbol{R}_{\widetilde{\mathbf{h}}_S\widetilde{\mathbf{h}}_S} = \widetilde{\sigma}^2 \boldsymbol{P}_C, \quad (7)$$

where $\boldsymbol{P}_C = \mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{C}^H$ represents the projection onto the covariance subspace. Here, $\mathbf{S} = \widehat{\mathbf{h}}_S\widehat{\mathbf{h}}_S^H + \boldsymbol{R}_{\widetilde{h}_S\widetilde{h}_S} = \mathbf{C}(\widehat{\mathbf{c}}\widehat{\mathbf{c}}^H + \widetilde{\sigma}^2\mathbf{I})\mathbf{C}^H$.

A. BF with Partial CSIT

Three types of BF design with partial CSIT can be analyzed. In the case of partial CSIT we get for the Rx signal,

$$\begin{aligned} y_k = {} & \widehat{\mathbf{h}}_{k,b_k}^H \mathbf{g}_k x_k + \underbrace{\widetilde{\mathbf{h}}_{k,b_k}^H \mathbf{g}_k x_k}_{\text{sig. ch. error}} \\ & + \sum_{i=1,\neq k}^{K} (\widehat{\mathbf{h}}_{k,b_i}^H \mathbf{g}_i x_i + \underbrace{\widetilde{\mathbf{h}}_{k,b_i}^H \mathbf{g}_i x_i}_{\text{interf. ch. error}}) + v_k\,. \end{aligned} \quad (8)$$

1) Naive BF EWSR: just replace $\mathbf{h}$ by $\widehat{\mathbf{h}}$ in a perfect CSIT approach. Ignore $\widetilde{\mathbf{h}}$ everywhere. 2) Optimal BF EWSR: accounts for covariance CSIT in the signal and interference terms.

B. Max EWSR ZF BF in the MaMISO limit (ESEI-WSR)

The scenario of interest here is to design optimal beamformers when there is only partial CSIT. Once the CSIT is imperfect, various optimization criteria such as outage capacity can be considered. Here the design is based on expected weighted sum rate (EWSR) (and in a first instance with LMMSE channel estimates). The actual EWSR represents two rounds of averaging. In a first stage, the WSR is averaged over the channels given the channel estimates and covariance information (i.e. the partial CSIT), leading to a cost function that can be optimized by the Tx. The optimized result then needs to be averaged over the channel estimates to obtain the final ergodic WSR. In the MaMISO limit, due to the law of large numbers, a number of scalars converge to their expected value, facilitating averaging the WSR. From the law of total expectation and motivated from the ergodic capacity formulations [10] (point to point MIMO systems), [11] (multi user MISO systems),

$$
\begin{aligned}
&EWSR = \mathrm{E}_{\widehat{\mathbf{h}}} \max_{\mathbf{g}} EWSR(\mathbf{g}),\\
&EWSR(\mathbf{g}) = \mathrm{E}_{\mathbf{h}|\widehat{\mathbf{h}}} WSR(g) = \sum_{k=1}^{K} u_k \, \mathrm{E}_{\mathbf{h}|\widehat{\mathbf{h}}} \ln(s_k/s_{\overline{k}})\\
&\overset{(a)}{=} \sum_{k=1}^{K} u_k \, \ln((\mathrm{E}_{\mathbf{h}|\widehat{\mathbf{h}}} s_k)/(\mathrm{E}_{\mathbf{h}|\widehat{\mathbf{h}}} s_{\overline{k}})) = \sum_{k=1}^{K} u_k \, \ln(r_{\overline{k}}^{-1} r_k),
\end{aligned}
\tag{9}
$$

where transition (a) represents the MaMISO limit leading to ESEI-WSR (Expected Signal Expected Interference WSR), u_k are the rate weights, $\mathbf{g}$ represents the collection of BFs $\mathbf{g}_k$. $s_{\overline{k}}$ is the (channel dependent) interference plus noise power and s_k is the signal plus interference plus noise power. Their conditional expectations are

$$
\begin{aligned}
&r_{\overline{k}} = 1 + \sum_{i \neq k} \mathrm{E}_{\mathbf{h}|\widehat{\mathbf{h}}} |\mathbf{h}_{k,b_i}^H \mathbf{g}_i|^2 = 1 + \sum_{i \neq k} \mathbf{g}_i^H \mathbf{S}_{k,b_i} \mathbf{g}_i,\\
&r_k = r_{\overline{k}} + \mathbf{g}_k^H \mathbf{S}_{k,b_k} \mathbf{g}_k, \ \mathbf{S}_{k,b_k} = \mathbf{C}_{k,b_k} \mathbf{W}_{k,b_k} \mathbf{C}_{k,b_k}.
\end{aligned}
\tag{10}
$$

For optimal ZF BF, all the interfering powers $\mathbf{g}_i^H \mathbf{S}_{k,b_i} \mathbf{g}_i = 0$ and thus $\mathbf{g}_k$ should belong to the orthogonal complement of the eigen vector subspace of all the interfering users. For this purpose, we define $\mathbf{C}_{\overline{k}}$ as the eigen vector space of all the users (except k) channel from b_k, $\mathbf{C}_{\overline{k}} = [\mathbf{C}_{1,b_k}, ..., \mathbf{C}_{k-1,b_k}, \mathbf{C}_{k+1,b_k}, .., \mathbf{C}_{K,b_k}]$. Further we split $\mathbf{g}_k = \mathbf{g}'_k p_k^{1/2}$, where p_k is the power allocated to user k, and $||\mathbf{g}'_k|| = 1$. By adding the Lagrange terms for the BS power constraints, $\sum_{c=1}^{C} \mu_c (P_c - \sum_{k:b_k=c} ||\mathbf{g}_k||^2)$, to the EWSR in (9), we get the gradient (with $\alpha_k = \frac{u_k}{r_k}$),

$$
\frac{\partial EWSR}{\partial \mathbf{g}_k^*} = \alpha_k \mathbf{S}_{k,b_k} \mathbf{g}_k - \mu_{b_k} \mathbf{g}_k = 0, \tag{11}
$$

leading to $\mathbf{g}_k \propto \mathbf{V}_{max}(\mathbf{S}_{k,b_k})$. Finally we obtain the ZF BF as, $\mathbf{g}'_k = \boldsymbol{P}_{\mathbf{C}_{\overline{k}}}^{\perp} \mathbf{V}_{max}(\mathbf{S}_{k,b_k})$, where $\boldsymbol{P}_{\mathbf{C}_{\overline{k}}}^{\perp}$ represents the projection onto the orthogonal complement of $\mathbf{C}_{\overline{k}}$. To further simplify, consider the eigen decomposition of $\mathbf{W}_{k,b_k} = \mathbf{V}_{k,b_k} \mathbf{\Lambda}_{k,b_k} \mathbf{V}_{k,b_k}^H$. Then we can write $\mathbf{S}_{k,b_k} = \mathbf{C}_{k,b_k} \mathbf{V}_{k,b_k} \mathbf{\Lambda}_{k,b_k} \mathbf{V}_{k,b_k}^H \mathbf{C}_{k,b_k}$. Multiplication of the semi-unitary matrix $\mathbf{C}_{k,b_k}$ with the unitary matrix $\mathbf{V}_{k,b_k}$ results in a semi-unitary matrix itself and thus the eigen values of $\mathbf{S}_{k,b_k}$ are same as that of $\mathbf{W}_{k,b_k}$ and the corresponding eigen vectors become same as that of $\mathbf{W}_{k,b_k}$ left multiplied by $\mathbf{C}_{k,b_k}$. Finally we rewrite $\mathbf{g}_k$ as,

$$
\mathbf{g}'_k = \boldsymbol{P}_{\mathbf{C}_{\overline{k}}}^{\perp} \mathbf{C}_{k,b_k} \mathbf{V}_{max}(\mathbf{W}_{k,b_k}) \, . \tag{12}
$$

Further optimizing w.r.t p_k leads to the following water filling solution for the power,

$$
p_k = \left(\frac{u_k}{\mu_{b_k}} - \frac{1}{\mathbf{g}'^H_k \mathbf{S}_{k,b_k} \mathbf{g}'_k} \right)^+, \tag{13}
$$

where $(x)^+ = \max\{0, x\}$ and the Lagrange multipliers μ_c are adjusted (e.g. by bisection) to satisfy the power constraints.

V. Reduced Order ZF with Partial CSIT

In this section, we consider the BF to be a reduced order ZF (RO-ZF) which is introduced in [8]. This can be interpreted as the number of interfering channels to be zero-forced for a user k is much less than K. The RO-ZF BF $\mathbf{g}_k$ can be written as, $\mathbf{g}_k = \frac{\boldsymbol{P}_{\mathbf{C}_{I_k}}^{\perp} \mathbf{C}_{k,b_k} \mathbf{V}_{max}(\mathbf{W}_{k,b_k})}{||\boldsymbol{P}_{\mathbf{C}_{I_k}}^{\perp} \mathbf{C}_{k,b_k} \mathbf{V}_{max}(\mathbf{W}_{k,b_k})||}$. Here, $\boldsymbol{P}_C = \mathbf{C}(\mathbf{C}^H \mathbf{C})^{\#} \mathbf{C}^H$ represent the projection onto the column space of $\mathbf{C}$, $\boldsymbol{P}_{\mathbf{C}}^{\perp} = \mathbf{I} - \boldsymbol{P}_C$ is the projection onto its orthogonal complement (# represents the Moore-Penrose pseudo-inverse). For the convenience of analysis, we define the following: I_k denotes the set of user indices for which the ZF is done. $\mathbf{C}_{I_k}$ represents the matrix of all the user eigen vector space in I_k. Complexity in the RO-ZF case will be about half of that of full ZF (multiplying the $M \times LK$ $\mathbf{C}$ by a triangular $LK \times LK$ instead of a full $LK \times LK$, computation of the $LK \times LK$ inverse or triangular factor takes $O((LK)^3)$ operations, with a smaller factor if only a triangular factor is needed and not a full inverse).

VI. Large System Analysis for RO-ZF and Full Order ZF

In this section we consider the large system analysis for the RO-ZF scheme proposed in this paper and also the full order ZF (full order means $|I_k| = K - 1, \forall k$). We assume that the LS channel estimation error $\widetilde{\sigma^2}$ remains finite with SNR. If for instance the error variance on the channel estimate would be inversely proportional to SNR, then at high SNR the channel estimate becomes exact and the covariance information does not bring any improvements. The channel estimation error remaining finite can be representative of the UL power being much less than the DL power (channel estimation from UL pilots and using TDD reciprocity). The ESEINR (Expected Signal to Expected Interference plus Noise Ratio) can be written as,

$$
\begin{aligned}
&\gamma_k^{RO-ZF} = \frac{P_{S,k}}{P_{I,k}+1} = \frac{p_k \mathbf{g}'^H_k \mathbf{S}_{k,b_k} \mathbf{g}'_k}{\sum_{i=1, i \neq k}^{K} p_i \mathbf{g}'^H_i \mathbf{S}_{k,b_i} \mathbf{g}'_i + 1},\\
&\implies \mathbf{g}'^H_k \mathbf{S}_{k,b_k} \mathbf{g}'_k = \frac{\boldsymbol{v}_{k,b_k}^H \mathbf{C}_{k,b_k}^H \boldsymbol{P}_{\mathbf{C}_{\overline{k}}}^{\perp} \mathbf{S}_{k,b_k} \boldsymbol{P}_{\mathbf{C}_{\overline{k}}}^{\perp} \mathbf{C}_{k,b_k} \boldsymbol{v}_{k,b_k}}{\left\| \boldsymbol{P}_{\mathbf{C}_{\overline{k}}}^{\perp} \mathbf{C}_{k,b_k} \boldsymbol{v}_{k,b_k} \right\|^2}.
\end{aligned}
\tag{14}
$$

Consider the eigen decomposition of $\mathbf{W}_{k,b_k} = \mathbf{V}_{k,b_k} \mathbf{\Lambda}_{k,b_k} \mathbf{V}_{k,b_k}^H$ and we denote $\mathbf{V}_{max}(\mathbf{W}_{k,b_k}) = \boldsymbol{v}_{k,b_k}$,

$$
\begin{aligned}
&\boldsymbol{v}_{k,b_k}^H \mathbf{C}_{k,b_k}^H \boldsymbol{P}_{\mathbf{C}_{\overline{k}}}^{\perp} \mathbf{S}_{k,b_k} \boldsymbol{P}_{\mathbf{C}_{\overline{k}}}^{\perp} \mathbf{C}_{k,b_k} \boldsymbol{v}_{k,b_k} \overset{(a)}{=} \frac{1}{M_{b_k}^2} \mathrm{tr}\{\boldsymbol{P}_{\mathbf{C}_{\overline{k}}}^{\perp}\}^2\\
&\boldsymbol{v}_{k,b_k}^H \mathbf{W}_{k,b_k} \boldsymbol{v}_{k,b_k} \overset{(b)}{=} \frac{1}{M_{b_k}^2} \mathrm{tr}\{\boldsymbol{P}_{\mathbf{C}_{\overline{k}}}^{\perp}\}^2 \lambda_{max}(\mathbf{W}_{k,b_k}),\\
&\mathbf{g}'^H_k \mathbf{S}_{k,b_k} \mathbf{g}'_k = \frac{1}{M_{b_k}} (M_{b_k} - \sum_{i=1, i \in I_k}^{K} L_{i,b_k}) \lambda_{max}(\mathbf{W}_{k,b_k}),
\end{aligned}
\tag{15}
$$

where we substituted $\left\| \boldsymbol{P}_{\mathbf{C}_{\overline{k}}}^{\perp} \mathbf{C}_{k,b_k} \boldsymbol{v}_{k,b_k} \right\|^2 = \left\| \boldsymbol{v}_{k,b_k}^H \mathbf{C}_{k,b_k}^H \boldsymbol{P}_{\mathbf{C}_{\overline{k}}}^{\perp} \mathbf{C}_{k,b_k} \boldsymbol{v}_{k,b_k} \right\|$ using the property of projection matrices, $\boldsymbol{P}^{\perp}_{\mathbf{C}_{\overline{k}}}\boldsymbol{P}^{\perp}_{\mathbf{C}_{\overline{k}}} = \boldsymbol{P}^{\perp}_{\mathbf{C}_{\overline{k}}}$. Also, (a) in (15) follows from Lemma 4 in Appendix VI of [3], that $\boldsymbol{x}_N^H\mathbf{A}_N\boldsymbol{x}_N \xrightarrow{N\rightarrow\infty} (1/N)tr\mathbf{A}_N$ when the elements of $\boldsymbol{x}_N$ are iid with variance $1/N$ and independent of $\mathbf{A}_N$, and similarly when $\boldsymbol{y}_N$ is independent of $\boldsymbol{x}_N$, that $\boldsymbol{x}_N^H\mathbf{A}_N\boldsymbol{y}_N \xrightarrow{N\rightarrow\infty} 0$. Using this Lemma, $\mathbf{C}^H_{k,b_k}\boldsymbol{P}^{\perp}_{\mathbf{C}_{\overline{k}}}\mathbf{C}_{k,b_k} = \frac{1}{M_{b_k}}\text{tr}\{\boldsymbol{P}^{\perp}_{\mathbf{C}_{\overline{k}}}\}$ and (b) follows from the fact that $\boldsymbol{v}_{k,b_k}$ (max eigen vector from $\mathbf{V}_{k,b_k}$) is orthogonal to all the other columns of $\mathbf{V}_{k,b_k}$ except the one corresponding to $\lambda_{max}(\mathbf{W}_{k,b_k})$. Further, by the law of large numbers, $P_{S,k} - \overline{P}_{S,k} \xrightarrow[a.s]{M\rightarrow\infty} 0$, where,

$$\overline{P}_{S,k} = (1 - \frac{\sum_{i=1,i\in I_k}^{K} L_{i,b_k}}{M_{b_k}})\lambda_{max}(\mathbf{W}_{k,b_k})p_k \tag{16}$$

Next, we consider the terms in $P_{I,k}$,

$$\mathbf{g}'^H_i\mathbf{S}_{k,b_i}\mathbf{g}'_i = \frac{\boldsymbol{v}^H_{i,b_i}\mathbf{C}^H_{i,b_i}\boldsymbol{P}^{\perp}_{\mathbf{C}_{\overline{i}}}\mathbf{S}_{k,b_i}\boldsymbol{P}^{\perp}_{\mathbf{C}_{\overline{i}}}\mathbf{C}_{i,b_i}\boldsymbol{v}_{i,b_i}}{\left\|\boldsymbol{P}^{\perp}_{\mathbf{C}_{\overline{i}}}\mathbf{C}_{i,b_i}\boldsymbol{v}_{i,b_i}\right\|^2}. \tag{17}$$

If $k \in I_i$, then $\boldsymbol{P}^{\perp}_{\mathbf{C}_{\overline{i}}}$ is orthogonal to the columns of $\mathbf{C}_{k,b_i}$ and thus $\mathbf{g}_i^H\mathbf{S}_{k,b_i}\mathbf{g}_i = 0$ else, using Lemma 4, we obtain $\boldsymbol{v}^H_{i,b_i}\mathbf{C}^H_{i,b_i}\boldsymbol{P}^{\perp}_{\mathbf{C}_{\overline{i}}}\mathbf{S}_{k,b_i}\boldsymbol{P}^{\perp}_{\mathbf{C}_{\overline{i}}}\mathbf{C}_{i,b_i}\boldsymbol{v}_{i,b_i} = \frac{1}{L_{i,b_i}}\text{tr}\{\mathbf{C}^H_{i,b_i}\boldsymbol{P}^{\perp}_{\mathbf{C}_{\overline{i}}}\mathbf{S}_{k,b_i}\boldsymbol{P}^{\perp}_{\mathbf{C}_{\overline{i}}}\mathbf{C}_{i,b_i}\}$.

$$\begin{aligned}&\frac{1}{L_{i,b_i}}\text{tr}\{\mathbf{C}^H_{i,b_i}\boldsymbol{P}^{\perp}_{\mathbf{C}_{\overline{i}}}\mathbf{S}_{k,b_i}\boldsymbol{P}^{\perp}_{\mathbf{C}_{\overline{i}}}\mathbf{C}_{i,b_i}\} \overset{(c)}{=} \frac{1}{M_{b_i}}\text{tr}\{\boldsymbol{P}^{\perp}_{\mathbf{C}_{\overline{i}}}\mathbf{S}_{k,b_i}\boldsymbol{P}^{\perp}_{\mathbf{C}_{\overline{i}}}\}\\ &\frac{1}{M_{b_i}}\text{tr}\{\mathbf{W}_{k,b_i}\mathbf{C}^H_{k,b_i}\boldsymbol{P}^{\perp}_{\mathbf{C}_{\overline{i}}}\mathbf{C}_{k,b_i}\} \overset{(d)}{=} \frac{1}{M^2_{b_i}}\text{tr}\{\boldsymbol{P}^{\perp}_{\mathbf{C}_{\overline{i}}}\}\text{tr}\{\mathbf{W}_{k,b_i}\}\\ &= \frac{1}{M_{b_i}}(1 - \frac{\sum_{r=1,r\in I_i}^{K} L_{r,b_i}}{M_{b_i}})\sum_{l=1}^{L_{k,b_i}}\zeta^{(l)}_{k,b_i},\end{aligned} \tag{18}$$

where (c) and (d) are obtained by using Lemma 4 from [4]. Further we obtain $\mathbf{g}'^H_i\mathbf{S}_{k,b_i}\mathbf{g}'_i = \frac{1}{M_{b_i}}\sum_{l=1}^{L_{k,b_i}}\zeta^{(l)}_{k,b_i}$. Finally, we obtain the ESEINR in the large system limit as, $\gamma_k^{RO-ZF} - \overline{\gamma}_k^{RO-ZF} \xrightarrow[a.s]{M\rightarrow\infty} 0$,

$$\overline{\gamma}_k^{RO-ZF} = \frac{(1-\frac{\sum_{i=1,i\in I_k}^{K} L_{i,b_k}}{M_{b_k}})\lambda_{max}(\mathbf{W}_{k,b_k})p_k}{\frac{1}{M_{b_i}}\sum_{i=1,k\notin I_i}^{K}\sum_{l=1}^{L_{k,b_i}}\zeta^{(l)}_{k,b_i}p_i+1} \tag{19}$$

For the full order ZF, the interference power vanishes from the ESEINR terms,

$$\overline{\gamma}_k^{ZF} = (1 - \frac{\sum_{i=1,i\neq k}^{K} L_{i,b_k}}{M_{b_k}})\lambda_{max}(\mathbf{W}_{k,b_k})p_k \tag{20}$$

The power updates for the RO-ZF BF can be shown to be as similar to the interference aware water filling as shown in [8] and the simplified expressions directly follow from the above equations as,

$$p_k = (\frac{u_k}{\mu_{b_k}+\sigma^{(2)}_k} - \frac{1}{\sigma^{(1)}_k})^+, \tag{21}$$

where, $\sigma^{(2)}_k = \frac{1}{M_{b_k}}\sum_{i=1,i\notin I_k}^{K}\beta_i\sum_{l=1}^{L_{i,b_k}}\zeta^{(l)}_{i,b_k}$, $\sigma^{(1)}_k = (1 - \frac{\sum_{i=1,i\in I_k}^{K} L_{i,b_k}}{M_{b_k}})\lambda_{max}(\mathbf{W}_{k,b_k}), \beta_i = u_k(\frac{1}{r_{\overline{k}}} - \frac{1}{r_k})$.

Computation of eigen values $\zeta^{(r)}_{k,b_i}$ of $\mathbf{W}_{k,b_i}$: from Section III,

$$\mathbf{W}_{k,b_i} = \check{\mathbf{c}}_{k,b_i}\check{\mathbf{c}}^H_{k,b_i} + \widetilde{\mathbf{D}}_{k,b_i},\ \ \check{\mathbf{c}}_{k,b_i} = \widehat{\mathbf{D}}^{1/2}_{k,b_i}\widehat{\mathbf{c}}_{k,b_i}, \forall i, k \tag{22}$$

In (3), we assume that all the eigen values are equal and positive, i.e $\mathbf{D}_{k,b_i} = \eta_{k,b_i}\mathbf{I}$, $\widetilde{\mathbf{D}}_{k,b_i} = \widetilde{\eta}_{k,b_i}\mathbf{I}$. Thus the eigen values of $\mathbf{W}_{k,b_i}$ can be shown to be $\zeta^{(1)}_{k,b_i} = \lambda_{max}(\mathbf{W}_{k,b_i}) = \|\check{\mathbf{c}}_{k,b_i}\|^2 + \widetilde{\eta}_{k,b_i}$ and $\zeta^{(2)}_{k,b_i} = ... = \zeta^{(L_{k,b_i})}_{k,b_i} = \widetilde{\eta}_{k,b_i}$, where $\widetilde{\eta}_{k,b_i} = \frac{\widetilde{\sigma}^2_{k,b_i}\eta_{k,b_i}}{\widetilde{\sigma}^2_{k,b_i}+\eta_{k,b_i}}$, using the definition of $\widetilde{\mathbf{D}}_{k,b_i}$ from (6). $\lambda_{max}(\mathbf{W}_{k,b_i})$ is random since $\check{\mathbf{c}}_{k,b_i}$ is random. By the law of large numbers (assuming L_{k,b_i} is large but finite and $<< N_t$) we replace it by the expectation which can be computed as follows. $\text{E}(\lambda_{max}(\mathbf{W}_{k,b_i})) = \text{E}(\widehat{\mathbf{c}}^H_{k,b_i}\widehat{\mathbf{D}}_{k,b_i}\widehat{\mathbf{c}}_{k,b_i}) + \widetilde{\eta}_{k,b_i}$. This gets simplified as, $\text{E}\left(\lambda_{max}(\mathbf{W}_{k,b_i})\right) = L_{k,b_i}\widehat{d}_{k,b_i} + \widetilde{\eta}_{k,b_i}$, where $\widehat{d}_{k,b_i} = \frac{\eta^2_{k,b_i}}{\eta_{k,b_i}+\widetilde{\sigma}^2_{k,b_i}}$ from (4) ($\widehat{\mathbf{D}}_{k,b_i} = \widehat{d}_{k,b_i}\mathbf{I}$) and $\text{E}(\widehat{\mathbf{c}}^H_{k,b_i}\widehat{\mathbf{c}}_{k,b_i}) = L_{k,b_i}$ from (4).

VII. Optimization of the ZF Order

In this section, we consider an alternating optimization algorithm (Algorithm 1) which computes the reduced ZF order for each user (I_k). We define here $\theta_{i,b_j} = \sum_{l=1}^{L_{i,b_j}}\zeta^{(l)}_{i,b_j}$, as the channel strength from BS b_j to user i. Note that at finite dimension MIMO, not only the channel strengths but also the relative orientation of the channel vectors count. However, in MaMIMO with multiple of identity covariances, there is no orientation issue, only the channel strengths count. So the user ordering is simple.

Algorithm 1 Reduced Zero-Forcing Order Determination

Given: $K, M, \sigma^2, \theta_{i,b_j}, \forall i, j$, with ordering $\theta_{1,b_j} \geq \theta_{2,b_j} \geq ... \geq \theta_{K,b_j}$. Start with $I_k = \emptyset, \forall k$, i.e. $\mathbf{g}_k^{(0)} = \mathbf{h}_{k,b_k}$.

for c = 1,..,C

- Compute the interference powers received at all users from BS c. Find the link causing the maximum interference. Let it be BF $\mathbf{g}_k$ to user l.
- Add ZF for the corresponding maximum interference causing channel link. i.e. $I_k = I_k \cap l$.
- Update $\mathbf{g}_k^{(t)}$ ($\mathbf{g}_k$ corresponding to the updated I_k) , such that $b_k = c$.
- Update the user powers p_k using (21).
- Compute the WSR. If the WSR is decreased, exit the loop. Otherwise continue with next iteration $(t + 1)$.

end for

VIII. Simulation Results

In this section, we present the Ergodic Sum Rate Evaluations for BF design for the various channel estimates. Monte

Carlo evaluations of ergodic sum rates are done, where we consider a path-wise or low rank channel model as in section III, with number of paths = channel covariance rank $L = 4$. In

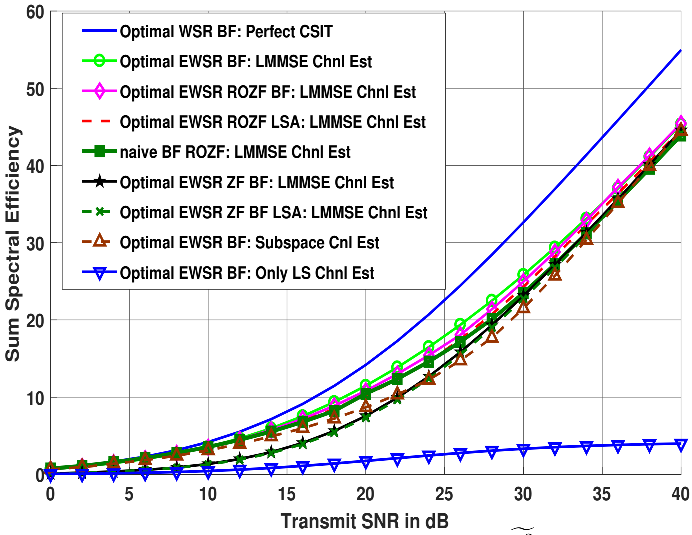

Fig. 1. Sum Rates, $M = 64, K = 10, L = 4, \widetilde{\sigma^2} = 0.1$.

the figures, "LSA" refers to large system approximation and "Chnl Est" refers to channel estimate. In these simulations, the deterministic channel estimation error (ie $\widetilde{\sigma}^2$) does not go to zero as snr $- > \infty$ but remains constant. Otherwise, at high snr it is the channel estimate that dominates, and the partial csit at high snr will just become perfect csit. The simulations in Figure 1 show that exploiting also the channel error covariance information can lead to substantial performance gains compared to just using LS channel estimate. The naive channel estimate based partial CSIT BF approaches are suboptimal. We also compare optimal BF and full and reduced order ZF BF, based on LMMSE channel estimates plus error covariance. Note that in the case of reduced-rank channel covariances considered here, ZF BF may still be possible, even with partial CSIT. At high SNR, ZF BF is

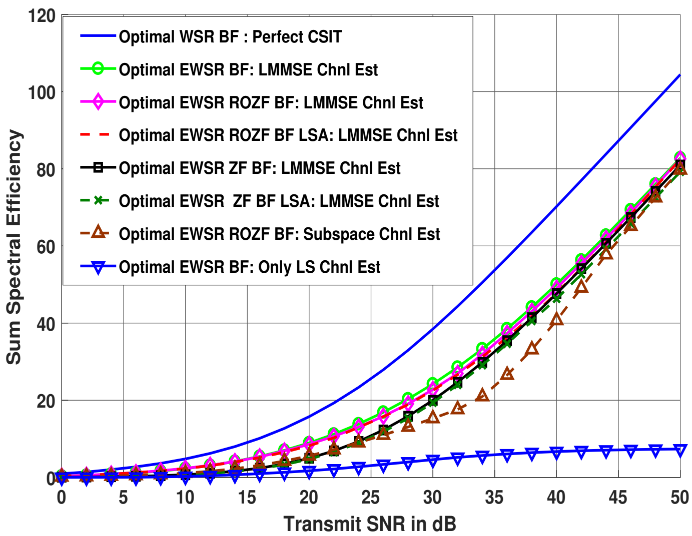

Fig. 2. Sum Rates, $M = 128, K = 15, L = 4, \widetilde{\sigma^2} = 0.1$.

optimal. At low and intermediate SNRs, RO-ZF is able to outperform (full order) ZF and it is quite close to the optimal BF [12]. Figure 2 are for increased dimensions. Further these simulations suggest that the large system approximations for ROZF and ZF are accurate even for finite values for M, K, L.

IX. Conclusions

In this paper, we extend the concept of reduced order ZF BF to partial CSIT. Simulation results indicate that our RO-ZF BF scheme has a performance very close to the optimal BFs, but with much less complexity compared to the full order ZF. We also propose an alternating optimization algorithm which computes the optimal ZF order for each user. Moreover, we show (elsewhere) the improvement in performance by using an LMMSE channel estimate compared to just having LS estimates, and by furthermore properly exploiting all covariance information. Further work will include the exploitation of the large system analysis for the optimization of the reduced order for lesser complexity in RO-ZF BF.

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