Modeling and Performance Analysis of the Main MAC and PHY Features of the 802.11ac Standard: A-MPDU Aggregation vs Spatial Multiplexing

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***Abstract*—The Very High Throughput (VHT) supplied by the IEEE 802.11ac standard is the outcome of several amendments incorporated at its both Medium Access Control (MAC) and PHYsical (PHY) layers. In particular, the Aggregate-MAC Protocol Data Unit (A-MPDU) frame aggregation allows, at the MAC level, increasing the amount of data to be transmitted and the spatial multiplexing technique serves at the PHY level to accelerate the rate at which such data are forwarded. Therefore, the goals we are seeking to achieve in this paper are, on one hand, to demonstrate that A-MPDU aggregation and spatial multiplexing may, respectively, lead to a decrease in throughput and an increase in overhead, and on the other hand, to highlight that A-MPDU aggregation and spatial multiplexing have mutual needs that could mutually satisfy operational challenges. For this purpose, we model the 802.11ac station enabling A-MPDU aggregation and spatial multiplexing, and we compute the throughput and the overhead of the 802.11ac network. After that, we run Monte Carlo simulations for validating the accuracy of the mathematical model, and we calculate 95% confidence intervals to establish credibility to the simulation results.**

***Index Terms*—VHT IEEE 802.11ac, A-MPDU aggregation, spatial multiplexing, modeling, performance analysis, simulation and validation.**

NOMENCLATURE

Acronyms

The acronyms used throughout the paper are summarized here. The meaning of an acronym is usually indicated once, upon first occurrence, in this paper.

A-MPDU	Aggregate-MAC Protocol Data Unit.
ACK	ACKnowledgment.
BEB	Binary Exponential Backoff.
BER	Bit Error Rate.
BPSK	Binary Phase Shift Keying.
CRC	Cyclic Redundancy Check.
CSI	Channel State Information.
CSMA/CA	Carrier Sense Multiple Access/Collision Avoidance.
CTI	Cross Talk Interference.
DCF	Distributed Coordination Function.
IFSs	Inter-Frame Spaces.
L-LTF	Legacy-Long Training Field.
L-SIG	Legacy-Signaling field.
L-STF	Legacy-Short Training Field.
MAC	Medium Access Control.
MCS	Modulation and Coding rates Schemes.
MIMO	Multiple-Input Multiple-Out.
MPDU	MAC Protocol Data Unit.
MT	Mobile Terminal.
PHY	PHYsical.
PPDU	PHY Protocol Data Unit.
PSDU	PHY Service Data Unit.
RTS/CTS	Request To Send/Clear To Send.
VHT	Very High Throughput.
VHT-LTF	VHT-Long Training Field.
VHT-SIG-A	VHT-Signaling part A.
VHT-SIG-B	VHT-Signaling part B.
VHT-STF	VHT-Short Training Field.
WLANs	Wireless Local Area Networks.
256-QAM	256-Quadrature Amplitude Modulation.

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Notations

In this paper notations are generally confined to the section in which they are used. The main notations are given here.

$AMPDU$	Size of an A-MPDU frame.
BA	Size of a Block ACK.
$DIFS$	Duration of distributed inter-frame space.
DLT	Size of the MPDU delimiter in an A-MPDU.
$E[HT]$	Mean channel holding time.
$E[payload]$	Average amount of payload bits successfully transmitted on the transmission channel.
FCS	Size of the frame check sequence of a MPDU.
H_{MAC}	Size of the MAC header of a MPDU.
HDR	Size of management information of a MPDU in an A-MPDU.
HTR	HDR transmission rate per spatial stream.

m	Maximum backoff stage.
$MSDU$	Size of a MAC service data unit.
MTR	MSDU transmission rate per spatial stream.
n	Number of stations in the network.
$Nmpdu$	Number of MPDUs contained in an A-MPDU.
Nss	Number of spatial streams.
OH	Overhead of the 802.11ac network.
P_B	Probability that the channel is busy.
P_C	Collision probability of an A-MPDU.
P_{SUC}	Probability that a 802.11ac station wins the transmission channel.
P_{TR}	Probability that there is at least one station transmitting data.
PAD	Size of the padding in an A-MPDU.
$SIFS$	Duration of short inter-frame space.
T_{BA}	Transmission time of a BA.
T_{SUC}	Sensing time of a successful transmission.
T_{UNS}	Sensing time of an unsuccessful transmission.
TH	Throughput of the 802.11ac network.
$Tmpdu$	Transmission time of a MPDU on a single spatial stream.
$Tampdu$	Transmission time of an A-MPDU of $Nmpdu$ MPDUs on Nss spatial streams.
$Tpayload$	Transmission time of payload bits of an A-MPDU frame.
TS	Duration of a time slot.
$TSampdu$	Transmission time, in number of time slots, of an A-MPDU.
T_{PHY}	Transmission time of the PHY header.
W_0	Minimum contention window.
W_i	Contention window size at the ith transmission attempt.
W_m	Maximum contention window.
δ	Signal propagation time.
τ	Transmission probability of an A-MPDU.

I. Introduction

THE advent of the new IEEE 802.11ac standard has completely revolutionized the world of Wireless Local Area Networks (WLANs) by providing new WiFi controllers labeled "ac" whose performances are almost similar to their Ethernet counterparts in the wired conventional networks. Indeed, with these new wireless cards, the transmission rate has taken a historical jump from 600 Mbps in IEEE 802.11n standard to 7000 Mbps; that is why we are talking about Very High Throughput (VHT) 802.11ac. This has prompted the development and deployment of multimedia and real-time applications that are throughput-hungry and requiring low latency. Accordingly, the daily use of WLANs has become easier and especially more comfortable.

For achieving a VHT in a WLAN network, several mechanisms, methods and techniques have been introduced either at MAC or PHY layer of the IEEE 802.11ac standard [1]–[4]. We particularly mention two basic and mandatory characteristics that a 802.11ac station can handle locally for improving its throughput. The first characteristic is about the formation of long frames capable of reaching 1048575 bytes by using the Aggregate-MAC Protocol Data Unit (A-MPDU) frame aggregation mechanism of the MAC layer. The second characteristic is regarding the use of multiple radio antennas, where it is possible to simultaneously transmit up to eight spatial streams by employing the spatial multiplexing technique of the PHY layer.

At the MAC level, the A-MPDU frame aggregation involves forming very large frames, which should in principle increase the throughput. However, in the event of unsuccessful transmission (collision), which is a frequent phenomenon in 802.11 networks, the larger the size of the A-MPDU frame, the higher the time of unsuccessful transmission; which will dramatically lower the throughput. At the PHY level, the role of the spatial multiplexing is to accelerate data transmission speed. Nevertheless, such acceleration concerns only the MAC frame and not the information (such as: headers, acknowledgments, etc.) and time delays (such as: inter-frame spaces, backoff time, etc.) for managing the transmission of the latter, known as: overhead. Consequently, the greater the number of streams to be multiplexed, the lower the channel efficiency is.

The main contributions of the present work are: (i) to show the two issues of decreasing the throughput and increasing the overhead resulting respectively from A-MPDU aggregation and spatial multiplexing; (ii) to highlight that a cross-layer communication between the A-MPDU frame aggregation mechanism and the spatial multiplexing technique is needed, so that each mechanism could benefit from the advantages of the other. That is, to allow using a very large frames at the MAC layer, it is necessary to ensure that a minimum number of radio antennas is available at the PHY level aiming at increasing the throughput. Furthermore, we demonstrate that an optimal MAC frame size exist, which allows maximizing the channel usage when using the spatial multiplexing technique.

To achieve the above objectives, we are opting for stochastic mathematical modeling in order to propose a new mathematical model based on a Markov chain reproducing the functioning of a 802.11ac station with A-MPDU aggregation and spatial multiplexing. The purpose of the mathematical model is to compute the throughput and the overhead of the 802.11ac network. Furthermore, we will conduct Monte-Carlo simulations for validating the accuracy of our model. We will also calculate 95% confidence intervals to give more credibility to simulation results.

The remainder of this paper is organized as follows. In Sections II and III, we will respectively present the operating principle and the work done in the field of A-MPDU aggregation and spatial multiplexing. The mathematical modeling and performance analysis of the 802.11ac network will be respectively provided in Section IV and V. The accuracy of our model will be discussed in Section VI. Finally, we will conclude the paper in Section VII.

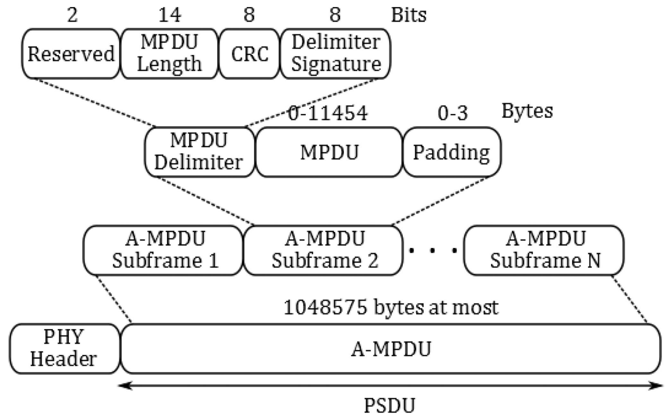

Fig. 1. A-MPDU frame aggregation.

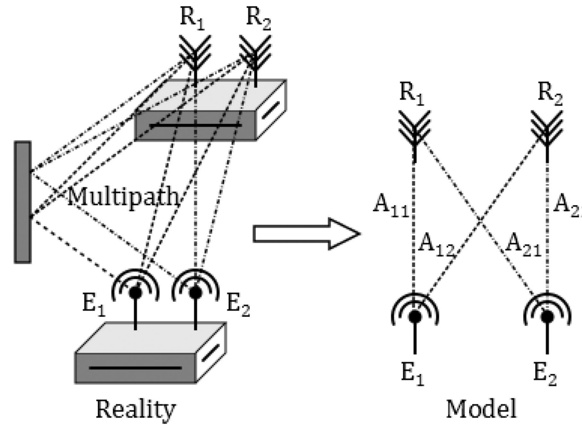

Fig. 2. An example illustrating the spatial multiplexing.

II. Background

This section is devoted to the presentation of two basic characteristics of the 802.11ac standard allowing to increase the wireless network throughput up to 7 Gbits/s; namely, frame aggregation of the MAC layer and spatial multiplexing of the PHY layer. At the end of this section, we will also present the frame format of the 802.11ac standard.

A. A-MPDU Frame Aggregation

At the MAC layer, the 802.11ac standard intends to use different frame aggregation schemes. More precisely, it suggests mandatory use of frame aggregation via an A-MPDU, which was introduced in 802.11n [5]. A-MPDUs are improved in 802.11ac by expanding their length, thereby packing multiple MPDUs within the same PPDU (PHY Protocol Data Unit); which in turn enhances throughput and MAC efficiency [6].

The principle of A-MPDU aggregation is to join several MPDU subframes with an individual PHY header. Nevertheless, all the MPDUs within an A-MPDU must be transmitted to the same receiver address. Moreover, the number of MPDUs to be aggregated relies on the number of frames already in the transmission queue, i.e., there is no waiting/holding time to constitute an A-MPDU [5]. The maximum size that an A-MPDU may obtain, i.e., the maximum size of the PSDU (PHY Service Data Unit) that can be received is 1048575 bytes [7]. In addition, aggregate exchange sequences are enabled with a protocol that acknowledges multiple MPDUs with only one Block ACK (ACKnowledgment).

The fundamental structure of the A-MPDU aggregation is presented in Figure 1. A set of of fields, called MPDU delimiter are introduced before each MPDU and padding bits varied from 0 to 3 bytes are inserted at the queue. The basic functioning of the MPDU delimiter is to specify the MPDU position and size within the A-MPDU frame. The padding bytes are inserted so that each MPDU is a multiple of four bytes in size, which may assist subframe delineation at the receiver. In simple terms, the MPDU delimiter and padding bytes establish the structure of the A-MPDU frame. After the A-MPDU is received, a de-aggregation process starts. Firstly, it checks the MPDU delimiter for any errors on the basis of the CRC (Cyclic Redundancy Check) value. If it is correct, the MPDU is extracted, and it continues with the next subframe until it reaches the end of the PSDU. In the opposite case, it checks every four bytes until it locates a valid MPDU delimiter or the end of the PSDU. The delimiter signature has a single pattern to help the de-aggregation process while scanning the MPDU delimiter.

B. Spatial Multiplexing Technique

The most used MIMO (Multiple-Input Multiple-Out) technique, especially with products respecting the 802.11ac standard is the spatial multiplexing. The data to be transmitted is split into N streams, and each stream is transmitted over a separate antenna [8]. This will have taken N times less time that it would have taken if only one antenna was used to transmit the data from beginning to end. The throughput is therefore N times higher. When these N streams reach the receiver antennas, provided they have distinctive spatial signatures, the receiver is then able to distinguish them, to receive them successfully and thus reconstruct the original data. The spatial multiplexing is therefore more efficient when there are many obstacles and reflections, notably inside buildings.

The principle of spatial multiplexing is as follows. Let us consider the example shown in Figure 2, which consists of one sender and one receiver, each have two antennas in the context of the spatial multiplexing. The Figure 2 shows the real situation and its representation in the context of the spatial multiplexing. The signal transmitted by the antenna 1 of the sender is denoted as E_1. It is captured by the antenna 1 of the receiver after having undergone an attenuation denoted as A_{11}. In the same way, the signal E_2 transmitted by the antenna 2 of the sender is captured by the antenna 1 of the receiver after having undergone an attenuation denoted as A_{21}. The signal captured by the antenna 1 of the receiver, denoted as R_1, is defined by Equation (1). Similarly, the signal captured by the antenna 2 of the receiver is given by Equation (2). The receiver having received R_1 and R_2, if it could also estimate the value of different attenuations

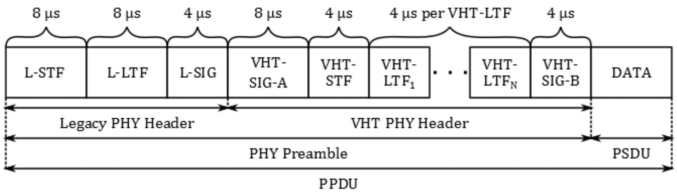

Fig. 3. 802.11ac PPDU format.

TABLE I
FIELDS OF THE 802.11AC PPDU

Field	*Description*
L-STF	Legacy-Short Training Field.
L-LTF	Legacy-Long Training Field.
L-SIG	Legacy-Signaling field.
VHT-SIG-A	Very High Throughput-Signaling part A.
VHT-STF	Very High Throughput-Short Training Field.
VHT-LTF	Very High Throughput-Long Training Field.
VHT-SIG-B	Very High Throughput-Signaling part B.
DATA	The data field carries the PSDU.

(A_{11}, A_{12}, A_{21}, and A_{22}), then it can retrieve E_1 and E_2. This is a resolution of a system of two equations with two unknowns.

$$R_1 = E_1 \times A_{11} + E_2 \times A_{21}. \quad (1)$$

$$R_2 = E_1 \times A_{12} + E_2 \times A_{22}. \quad (2)$$

The number of streams simultaneously transmitted is constrained by the minimum number of antennas of the receiver or the sender. Indeed, the sender obviously cannot transmit more simultaneous streams than the number of antennas it uses, and the receiver is not be able to decode more streams than the number of antennas it uses; because the system of equations would have fewer equations than unknowns, which is impossible to resolve. The 802.11ac standard foresees at most 8 spatial streams, which assumes that the sender and receiver have at most 8 antennas and are compatible with this mode [9].

C. 802.11ac PPDU Format

During a transmission, a PSDU is processed and appended to the PHY preamble to create the PPDU. At the receiver, the PHY preamble is processed to help in the detection, demodulation and delivery of the PSDU [10]. Figure 3 shows the 802.11ac PPDU format. The fields of the 802.11ac PPDU format are summarized in Table I [11]. L-STF, L-LTF and L-SIG fields are all for the purpose of compatibility with legacy 802.11a/n devices. VHT-SIG-A is important for multi-user MIMO to inform the participating stations of the parameters of the directional portion of the frame. VHT-STF is a training field whose primary purpose is for MIMO data power computation. VHT-LTFs are used to periodically compute the CSI (Channel State Information), through a channel sounding protocol [12], needed for MIMO transmission. Finally, VHT-SIG-B includes information regarding MCS (Modulation and Coding rates Schemes) and per user data field lengths. The detailed description of the 802.11ac PHY preamble is provided in [12].

III. RELATED WORK

In this section, we give a literature review of the main research that has been done on the fundamental characteristics of the 802.11ac standard, including: A-MPDU frame aggregation at the MAC layer and spatial multiplexing technique (or simply multi-antenna transmission) at the PHY layer. Indeed, we distinguish in this area of research three different kinds of paper: the research focusing on spatial multiplexing technique, the research that optimises the operation of the A-MPDU frame aggregation, and finally, the research that considers both A-MPDU aggregation and spatial multiplexing.

In the context of spatial multiplexing technique, *Redieteab et al.* in [13] proposed an approach based on the modeling of both MAC and PHY related aspects to make a comparison between single-user MIMO and multi-user MIMO. The authors highlighted that it is important to consider the CTI (Cross Talk Interference) when assessing the performance of multi-user MIMO transmission schemes. Indeed, multi-user MIMO transmission can sometimes have less throughput gain and be much less stable than single-user MIMO. *Oh et al.* in [14] implemented a IEEE 802.11ac channel sounding simulator and presented the relative results including signal representation, frequency response and constellation. The authors demonstrated that, the CSI is strongly needed for enhancing the 802.11ac spatial multiplexing capacity. Firstly, the CSI should be well estimated by the receiving station. Secondly, the estimated CSI has to be transmitted to the transmitting station with a minimum time delay. *Zubow et al.* in [15] demonstrated that the down-link of 802.11ac-based wireless networks with unequal network load can be considerably improved when performing a combination of two physical layer MIMO techniques, namely: multi-user MIMO and interference nulling. The former mitigates interference within a cell, whereas the latter reduces inter-cell interference. *Kampeas et al.* in [16] proposed and evaluated two distributed opportunistic user selection algorithms for down-link multi-user MIMO, which select a group of users to transmit concurrently, in a single iteration, without coordinating between the users and with very limited CSI exchange between the users and the access point. The proposed algorithms not only select users with preferable channel conditions for transmission, but also minimize the interference among the selected users.

Regarding A-MPDU frame aggregation, *Bellalta et al.* in [17] proposed and analyzed a method for performing frame aggregation. The obtained results show that, if the number of active stations is excessively high, the heterogeneity of destinations in the frames contained in the queue makes it hard to reap the full benefits of the frame aggregation. According to the authors, this issue may be mitigated by increasing the queue size, which enhances the opportunity of scheduling a considerable number of frames at each transmission; thereby providing improved system throughput to the detriment of a higher delay. *Nomura et al.* in [18] proposed a frame aggregation method that uses the receiving MT (Mobile Terminal) selection providing higher priority to MTs with higher throughput in the next transmission, while minimizing signaling overhead. The transmission performance and its fairness between MTs have been analyzed by means of system-level simulation. From the presented results, the proposed method significantly improves both performances, compared with the traditional frame aggregation methods.

Finally, the most critical studies addressing both A-MPDU aggregation and spatial multiplexing are: (i) *Cha et al.* in [19], who addressed a trade-off problem between multi-user MIMO of the 802.11ac standard; (ii) multi-user frame aggregation proposed by *Singh* in [20]. The authors drew two conclusions from their study: if each user's encoded data stream has a similar length, the multi-user MIMO gives better average throughput than the multi-user frame aggregation; otherwise, the frame aggregation performs better than multi-user MIMO regarding the average throughput. *Chung et al.* in [21] introduced an A-MPDU frame aggregation scheme using fragmented MPDUs with a VHT compressed Block ACK. The authors showed through computer simulation that the use of fragmented MPDUs within the A-MPDU frame alleviates the waste of spatial medium resources in terms of meaningless A-MPDU paddings. Consequently, the proposed method reaches a multi-user throughput improvement, over 28% at low data rates and up to 3% at high data rates. *Abdallah et al.* in [22] implemented a throughput-maximization framework for the joint adaptation of data rate, MIMO mode and frame aggregation configuration. The authors provided approximate throughput expressions for different data rates and MIMO modes by using approximate expressions for BER (Bit Error Rate). They further underlined that, for each BER value, there is an optimal frame size, which maximizes throughput. However, the proposed framework is based only on point-to-point transmission, i.e., it does not consider the impact of collisions on the achieved throughput. *Khan et al.* in [23] demonstrated via computer simulation and theoretical analysis that the choice of a particular modulation and coding scheme, number of spatial streams, and channel size can strongly compromise the 802.11ac throughput. Although 802.11ac increases throughput several fold due to high-order modulation scheme, more bandwidth and spatial streams, the performance can degrade substantially in an error-prone channel. However, the conducted analysis has completely ignored the effect of large sizes of the A-MPDU frame on the 802.11ac performance.

Whilst much research has been devoted to studying and enhancing both A-MPDU frame aggregation and spatial multiplexing technique, no studies have focused specifically on the following issues:

- The impact of the A-MPDU frame size on the decrease of the 802.11ac throughput;
- The impact of the number of spatial streams on the increase of channel overhead.

The core contribution of our research work lies in proposing a new mathematical model based on a Markov chain, which models the behavior of a 802.11ac station taking into account both A-MPDU frame aggregation and spatial multiplexing technique. The mathematical model will be then used to highlight the following phenomena:

- Increasing the size of the A-MPDU frame cannot always increase the throughput because of collisions;
- Increasing the number of spatial streams leads to increase of the throughput, but it causes an inefficient use of channel due to overheads;
- Finally, we show in what way both A-MPDU frame aggregation and spatial multiplexing technique can be used so that they could mutually satisfy operational challenges.

IV. Modeling 802.11ac With A-MPDU Aggregation and Spatial Multiplexing

In this section, we propose a new three-dimensional discrete time Markov chain that models the functioning of a 802.11ac station enabling essentially the following two mechanisms, namely: the A-MPDU frame aggregation mechanism at the MAC layer and the spatial multiplexing mechanism at the PHY layer. Resolving the equations of the stationary probabilities of this Markov chain will allow us to determine the A-MPDU transmission probability τ of a 802.11ac station. This probability will then be used for developing mathematical models, in order to compute the throughput and the overhead of the 802.11ac network.

A. Hypothesis of the 802.11ac Mathematical Model

In what follows, we provide a detailed listing of all of the basic underlying hypothesis for the purposes of modeling the 802.11ac network with A-MPDU aggregation and spatial multiplexing. Tables II and III respectively define all the parameters and the basic probabilities necessary for modeling the 802.11ac network.

1) The network is composed by a constant number of 802.11ac stations, where their transmission queues always contain MPDUs to be transmitted on the network.
2) Each 802.11ac station considers a single traffic class with the same contention parameters, namely: the inter-frame spaces and the contention window. This means that the 802.11ac stations perform the Distributed Coordination Function (DCF) [10], which allows a single traffic class to gain access to the channel when its backoff time counter is zero for transmitting one A-MPDU frame.
3) The transmission channel is ideal, which means that collisions are the only reason of data loss in the network.
4) At the MAC level, all stations in the network implement the A-MPDU frame aggregation mechanism. Thus, prior to any transmission, A-MPDU frames of the same size are formed from simple MPDUs.
5) At the PHY level, all stations in the network enable the spatial multiplexing mechanism with the same number of spatial streams for sending and receiving A-MPDU frames.

TABLE II
PARAMETERS OF THE 802.11AC MATHEMATICAL MODEL

Parameter	*Description*
n	Number of stations in the network.
m	Maximum backoff stage.
W_0	Minimum contention window.
W_m	Maximum contention window.
W_i	Contention window size at the i^{th} transmission attempt.
$MSDU$	Size of a MAC service data unit (payload bits of a MPDU).
H_{MAC}	Size of the MAC header of a MPDU.
FCS	Size of the frame check sequence of a MPDU.
DLT	Size of the MPDU delimiter in an A-MPDU.
PAD	Size of the padding in an A-MPDU.
HDR	Size of management information of a MPDU in an A-MPDU.
$AMPDU$	Size of an A-MPDU frame.
MTR	MSDU transmission rate per spatial stream.
HTR	HDR transmission rate per spatial stream.
$Tmpdu$	Transmission time of a MPDU on a single spatial stream.
$Nmpdu$	Number of MPDUs contained in an A-MPDU.
Nss	Number of spatial streams.
$Tampdu$	Transmission time of an A-MPDU of $Nmpdu$ MPDUs on Nss spatial streams.
$Tpayload$	Transmission time of payload bits of an A-MPDU frame.
TS	Duration of a time slot.
$TSampdu$	Transmission time, in number of time slots, of an A-MPDU.
BA	Size of a Block ACK.
T_{BA}	Transmission time of a BA.
T_{PHY}	Transmission time of the PHY header.
$DIFS$	Duration of distributed inter-frame space.
$SIFS$	Duration of short inter-frame space.
δ	Signal propagation time.

TABLE III
PROBABILITIES OF THE 802.11AC MATHEMATICAL MODEL

Probability	*Definition*
τ	Transmission probability of an A-MPDU by any station of the network.
P_C	Collision probability of an A-MPDU of any station of the network.
P_B	Probability that the channel is busy for any station of the network.

6) The 802.11ac stations access the transmission channel by following the rules of Carrier Sense Multiple Access/Collision Avoidance (CSMA/CA) protocol [24], i.e., without using the Request To Send/Clear To Send (RTS/CTS) exchange sequence. This enables a clear evaluation of the role played by A-MPDU aggregation and spatial multiplexing in optimizing the 802.11ac network performance.
7) The collision probability of an A-MPDU frame is constant for all 802.11ac stations, and it is also independent of the number of transmission failures.

B. Transmission Probability τ of an A-MPDU Frame

For estimating the transmission probability τ that a 802.11ac station transmits an A-MPDU frame in an arbitrary time slot, we propose to model the operation of a single 802.11ac station by a Markov chain. Thereafter, this probability will be needed to compute the throughput and the overhead of the 802.11ac network.

Let $X(t)$ be the stochastic process modeling the status s ($s \in \{B, T\}$) of a 802.11ac station at a given time slot t. Indeed, a 802.11ac station may find itself in two different situations, namely: the status noted B which means that the 802.11ac station is in the process of decrementing its backoff time counter, and the status noted T which means that the 802.11ac station is in the process of transmitting its A-MPDU frame.

Let $Y(t)$ be the stochastic process modeling, at a given time slot t, either the backoff stage i (in the case of the status B), or the ith retransmission attempt of an A-MPDU frame (in the case of the status T). The values of i vary from 0 to m.

Let $Z(t)$ be the stochastic process modeling, at a given time slot t, either the remaining backoff time j to decrement (in the case of the status B), or the elapsed time k of transmission of an A-MPDU frame (in the case of the status T). The possible values of j and k vary from 1 to $W_i - 1$ and from 0 to $TSampdu - 1$, respectively.

For each 802.11ac station:

- The size of the contention window W_i at the backoff stage i is determined using Equation (3). Indeed, W_i follows a binary exponential algorithm.

$$W_i = 2^i \times W_0. \quad (3)$$

- The transmission time $TSampdu$, in number of time slots, of an A-MPDU frame is determined using Equation (4). It is obtained by dividing the transmission time $Tampdu$, in number of microseconds, of an A-MPDU frame by the duration of a time slot TS, which means the minimum duration of detecting a transmission over the radio channel. This division operation is required, because the stochastic process $Z(t)$ (previously defined) evolves following instants of time expressed in time slots. This requirement originally is imposed by the backoff time counter [10], which is expressed in terms of time slots.

$$TSampdu = \frac{Tampdu}{TS}. \quad (4)$$

- The transmission time $Tampdu$ of an A-MPDU frame is determined using Equation (5). It is computed by multiplying the transmission time $Tmpdu$ of a MPDU (on a single spatial stream) by the number $Nmpdu$ of MPDUs aggregated into the A-MPDU frame, then divided by the number Nss of spatial streams. This is how A-MPDU frame aggregation and spatial multiplexing mechanisms have been taken into account in developing our mathematical model.

$$Tampdu = \frac{Nmpdu \times Tmpdu}{Nss}. \quad (5)$$

- The transmission time $Tmpdu$ of a MPDU on a single spatial stream is determined using Equation (6). It is calculated by dividing the size HDR of management information of a MPDU and the size $MSDU$ of a MAC service data unit (data payload) by their respective transmission rates, namely: the transmission rate HTR of management information and the transmission rate MTR of data payload. Indeed, management information and data payload are usually transmitted using different modulation techniques.

$$Tmpdu = \frac{HDR}{HTR} + \frac{MSDU}{MTR}. \quad (6)$$

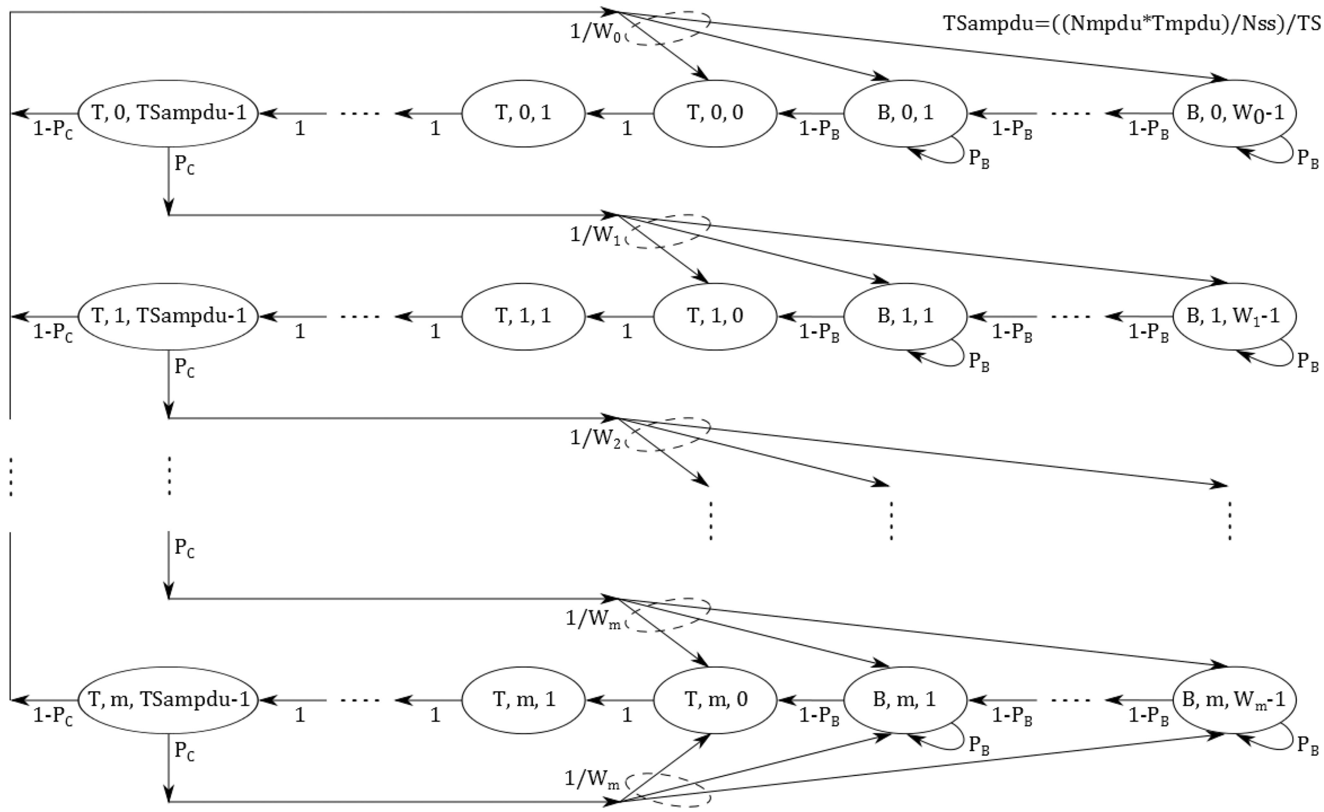

Fig. 4. Markov chain model of a 802.11ac station enabling A-MPDU frame aggregation and spatial multiplexing mechanisms.

- The size HDR of management information of a MPDU in an A-MPDU frame is determined using the Equation (7). It is obtained by adding the size H_{MAC} of the MAC header, the size FCS of the frame check sequence, the size DLT of the MPDU delimiter and the size PAD of the padding.

$$HDR = H_{MAC} + FCS + DLT + PAD. \qquad (7)$$

The three-dimensional process $\{X(t), Y(t), Z(t)\}$ can be modeled by the discrete-time Markov chain represented in Figure 4, upon adoption of the well-known hypothesis of *Bianchi's* model [25]. This hypothesis is defined as follows: at each transmission attempt, and regardless of the number of retransmissions, each packet collides with constant and independent probability.

The mathematical model, for modeling the first versions (a, b and g) of the IEEE 802.11 standard, introduced by *Bianchi* in [25] can always be expanded for modeling the IEEE 802.11ac amendment. We argue this by the fact that the basic functioning of 802.11ac is still conform to the original IEEE 802.11 standard [24]. Indeed, the process of competition between the stations (allowing a station to gain access to the channel) and the error recovery mechanism (allowing a station to retransmit lost data) have remained unchanged, namely: the Carrier Sense Multiple Access/Collision Avoidance (CSMA/CA) protocol, the Binary Exponential Backoff (BEB) algorithm, the Inter-Frame Spaces (IFSs) delays, etc.

Our mathematical model differs from that of the *Bianchi's* model mainly in the transmitting phase. In the *Bianchi's* model, only one MPDU (a simple frame) can be transmitted at once on one spatial stream. But with our model, a series ($Nmpdu$) of MPDUs (an A-MPDU frame) can be transmitted at once on multiple (Nss) spatial steams. In the *Bianchi's* model, the transmission is simply modeled by the states where the Backoff counter is zero. While, in our model, the transmission is modeled by the states whose first value is *T*. The transmission time is not taken into account in the *Bianchi's* model. In our model, the transmission time is calculated by parameter $TSampdu$, which is given by Equation (4).

In the following, we list all non-zero transition probabilities and their meaning of the proposed Markov chain:

- The transition probability $P\{B, i, j/B, i, j+1\}$ from the state $(B, i, j+1)$ to the state (B, i, j) for $i \in [0, m]$ and $j \in [1, W_i - 2]$ is given by the Equation (8). It models the decrementing of the backoff time counter of a 802.11ac station.

$$P\{B, i, j/B, i, j+1\} = 1 - P_B. \qquad (8)$$

- The transition probability $P\{T, i, 0/B, i, 1\}$ from the state $(B, i, 1)$ to the state $(T, i, 0)$ for $i \in [0, m]$ is given by Equation (9). It models the decrementing of the last time slot of the backoff time counter of a 802.11ac station before starting the transmission of its A-MPDU frame.

$$P\{T, i, 0/B, i, 1\} = 1 - P_B. \qquad (9)$$

- The transition probability $P\{B, i, j/B, i, j\}$ from the state (B, i, j) to the same state (B, i, j) for $i \in [0, m]$ and

$j \in [1, W_i - 1]$ is given by Equation (10). It models the freeze of the decrementing of the backoff time counter of a 802.11ac station because of detecting a busy channel.

$$P\{B, i, j/B, i, j\} = P_B. \quad (10)$$

- The transition probability $P\{T, i, k+1/T, i, k\}$ from the state (T, i, k) to the state $(T, i, k+1)$ for $i \in [0, m]$ and $k \in [0, TSampdu - 2]$ is given by Equation (11). It models the transmission of an A-MPDU frame of a 802.11ac station.

$$P\{T, i, k+1/T, i, k\} = 1. \quad (11)$$

- The transition probability $P\{B, 0, j/T, i, TSampdu-1\}$ from the state $(T, i, TSampdu - 1)$ to the state $(B, 0, j)$ for $i \in [0, m]$ and $j \in [1, W_0 - 1]$ is given by Equation (12). It models the selection of a new backoff time counter in the contention window W_0 for transmitting a new A-MPDU frame.

$$P\{B, 0, j/T, i, TSampdu - 1\} = \frac{1 - P_c}{W_0}. \quad (12)$$

- The transition probability $P\{T, 0, 0/T, i, TSampdu - 1\}$ from the state $(T, i, TSampdu - 1)$ to the state $(T, 0, 0)$ for $i \in [0, m]$ is given by Equation (13). It models the transmission of a new A-MPDU frame with a zero backoff time counter.

$$P\{T, 0, 0/T, i, TSampdu - 1\} = \frac{1 - P_c}{W_0}. \quad (13)$$

- The transition probability $P\{B, i, j/T, i-1, TSampdu-1\}$ from the state $(T, i-1, TSampdu - 1)$ to the state (B, i, j) for $i \in [1, m]$ and $j \in [1, W_i - 1]$ is given by Equation (14). It models the selection of a new backoff time counter in the contention window W_i for retransmitting a lost A-MPDU frame.

$$P\{B, i, j/T, i - 1, TSampdu - 1\} = \frac{P_c}{W_i}. \quad (14)$$

- The transition probability $P\{T, i, 0/T, i-1, TSampdu-1\}$ from the state $(T, i-1, TSampdu - 1)$ to the state $(T, i, 0)$ for $i \in [1, m]$ is given by Equation (15). It models the ith retransmission with zero backoff time counter of a lost A-MPDU frame.

$$P\{T, i, 0/T, i - 1, TSampdu - 1\} = \frac{P_c}{W_i}. \quad (15)$$

- The transition probability $P\{B, m, j/T, m, TSampdu - 1\}$ from the state $(T, m, TSampdu - 1)$ to the state (B, m, j) for $j \in [1, W_m - 1]$ is given by Equation (16). It models the selection of a new backoff time counter in the contention window W_m for retransmitting a lost A-MPDU frame.

$$P\{B, m, j/T, m, TSampdu - 1\} = \frac{P_c}{W_m}. \quad (16)$$

- The transition probability $P\{T, m, 0/T, m, TSampdu - 1\}$ from the state $(T, m, TSampdu - 1)$ to the state $(T, m, 0)$ is given by the Equation (17). It models the selection of a zero backoff time counter in the backoff stage m for retransmitting a lost A-MPDU frame.

$$P\{T, m, 0/T, m, TSampdu - 1\} = \frac{P_c}{W_m}. \quad (17)$$

The proposed Markov chain model represents in a comprehensive manner the decrementing of the backoff time counter (across the states (B, i, j) for $i \in [0, m]$ and $j \in [1, W_i - 1]$) and the transmission of the A-MPDU frame (across the states (T, i, k) for $i \in [0, m]$ and $k \in [0, TSampdu - 1]$). Therefore, the 802.11ac station randomly selects a backoff time counter which is decremented as long as the transmission channel is idle. When the last time slot of the backoff time counter (which is represented by the state $(B, i, 1)$ for $i \in [0, m]$) is decremented, the 802.11ac station stats the transmission of its A-MPDU frame. At the end of the last time slot of the A-MPDU transmission time (which is represented by the state $(T, i, TSampdu - 1)$ for $i \in [0, m]$), two events (in terms of probabilities) are possible regardless of the minimum sensing time of a collision:

1) Successful transmission modeled by the probability $1 - P_C$. The 802.11ac station will therefore choose a new backoff time counter in the initial contention window (W_0) to transmit a new A-MPDU frame.
2) Unsuccessful transmission modeled by the probability P_C. The 802.11ac station will first double its current contention window (W_i) while the maximum backoff stage (m) is not reached, then it chooses a new backoff counter to retransmit the lost A-MPDU frame.

We will now calculate for each state (s, i, h) of the proposed Markov chain its stationary probability $\pi_{s,i,h}$, where: $s \in \{B, T\}$, $i \in [0, m]$, $h \in \{j, k\}$, $j \in [1, W_i - 1]$ and $k \in [0, TSampdu - 1]$. The mathematical formula of $\pi_{s,i,h}$ is given by Equation (18). It represents the probability of being in the state (s, i, h) at infinity, i.e., when the system will have sufficiently functioned. In fact, we will give more attention to these stationary probabilities in the remainder of this section, in order to determine the transmission probability τ.

$$\pi_{s,i,h} = lim_{t \to \infty} P\{X(t) = s, Y(t) = i, Z(t) = h\} \quad (18)$$

In order to find the stationary distribution of our Markov chain, we apply the well-known *Balance Equation* [26] given by Equation (19), where: S is the state space of the Markov chain, π_i is the stationary probability of being in the state i, and p_{ij} is the transition probability from the state i to the state j. The *Balance Equation* has the following interpretation: it says that in the Markov chain, the flow from the state i to the state j is equal to that from the state j to the state i. For more details about the *Balance Equation*, please refer to [26].

$$\pi_i \times p_{ij} = \pi_j \times p_{ji}, \quad \forall i, j \in S. \quad (19)$$

After applying the *Balance Equation* on our Markov chain, we have obtained the following stationary probabilities:

- The stationary probability $\pi_{B,i,j}$ of being in the state (B, i, j) for $i \in [0, m - 1]$ and $j \in [1, W_i - 1]$ is given by Equation (20).

$$\pi_{B,i,j} = \frac{W_i - j}{W_i} \times \frac{P_C^i}{1 - P_B} \times \pi_{T,0,0}. \quad (20)$$

- The stationary probability $\pi_{B,m,j}$ of being in the state (B, m, j) for $j \in [1, W_m - 1]$ is given by Equation (21).

$$\pi_{B,m,j} = \frac{W_m - j}{W_m} \times \frac{P_C^m}{(1 - P_B)(1 - P_C)} \times \pi_{T,0,0}. \quad (21)$$

- The stationary probability $\pi_{T,i,k}$ of being in the state (T, i, k) for $i \in [0, m - 1]$ and $k \in [0, TSampdu - 1]$ is given by Equation (22).

$$\pi_{T,i,k} = \pi_{T,i,0} = P_C^i \times \pi_{T,0,0}. \quad (22)$$

- The stationary probability $\pi_{T,m,k}$ of being in the state (T, m, k) for $k \in [0, TSampdu - 1]$ is given by Equation (23).

$$\pi_{T,m,k} = \pi_{T,m,0} = \frac{P_C^m}{1 - P_C} \times \pi_{T,0,0}. \quad (23)$$

According to the equations (20), (21), (22) and (23), the stationary probabilities $\pi_{s,i,h}$ are expressed as a function that dependents on the following probabilities: the stationary probability $\pi_{T,0,0}$, the collision probability P_C of an A-MPDU frame, and the probability P_B that the channel is sensed busy by a 802.11ac station. $\pi_{T,0,0}$ can then be found by imposing the normalization condition on our Markov chain. It is obtained as follows:

$$1 = \sum_{i=0}^{m} \sum_{j=1}^{W_i - 1} \pi_{B,i,j} + \sum_{i=0}^{m} \sum_{k=0}^{TSampdu-1} \pi_{T,i,k},$$
$$= \pi_{T,0,0} \times \frac{\begin{bmatrix} W_0 \times (1 - P_C - P_C \times (2P_C)^m) \\ +(1 - 2P_C) \\ \times (2 \times TSampdu \times (1 - P_B) - 1) \end{bmatrix}}{2 \times (1 - P_C) \times (1 - 2P_C) \times (1 - P_B)}. \quad (24)$$

Hence, we have:

$$\pi_{T,0,0} = \frac{2 \times (1 - P_C) \times (1 - 2P_C) \times (1 - P_B)}{\begin{bmatrix} W_0 \times (1 - P_C - P_C \times (2P_C)^m) \\ +(1 - 2P_C) \\ \times (2 \times TSampdu \times (1 - P_B) - 1) \end{bmatrix}}. \quad (25)$$

Now, the transmission probability τ of an A-MPDU frame can be expressed as the sum of stationary probabilities $\pi_{T,i,k}$ of transmission states (T, i, k), where $i \in [0, m]$ and $k \in [0, TSampdu - 1]$. It is obtained as follows:

$$\tau = \sum_{i=0}^{m} \sum_{k=0}^{TSampdu-1} \pi_{T,i,k} = \frac{TSampdu}{1 - P_C} \cdot \pi_{T,0,0},$$
$$= \frac{2 \times TSampdu \times (1 - 2P_C) \times (1 - P_B)}{\begin{bmatrix} W_0 \times (1 - P_C - P_C \times (2P_C)^m) \\ +(1 - 2P_C) \\ \times (2 \times TSampdu \times (1 - P_B) - 1) \end{bmatrix}}. \quad (26)$$

The transmission probability τ of an A-MPDU frame, in turn, is based on the following probabilities:

- P_C (*the collision probability of an A-MPDU frame*): the probability that an A-MPDU frame of a 802.11ac station is lost because of a collision at a given time slot, is expressed as the probability that at least one other station among $n - 1$ stations of the 802.11ac network transmits. It is obtained as follows:

$$P_C = 1 - (1 - \tau)^{n-1}. \quad (27)$$

- P_B (*the probability that the channel is busy*): the probability that the transmission channel is sensed busy by a 802.11ac station at a given time slot, is expressed as the probability that at least one other station among $n - 1$ stations of the 802.11ac network transmits. It is obtained as follows:

$$P_B = P_C = 1 - (1 - \tau)^{n-1}. \quad (28)$$

The resolution of the equations (26), (27) and (28), which together form a system of nonlinear equations, by means of numerical methods, allows us to obtain the numerical values of the transition probabilities and stationary probabilities.

C. *Throughput* (TH)

The throughput (TH) that may be achieved in a 802.11ac network, is determined from the various events happening on the transmission channel during an arbitrary time slot. In the following, we provide the basic parameters required for developing TH:

- Let P_{TR} be the probability that there is at least one station transmitting data at a given time slot. Since n stations contend on the channel, and each transmits with probability τ, P_{TR} is given as follows:

$$P_{TR} = 1 - (1 - \tau)^n. \quad (29)$$

- Let P_{SUC} be the probability that a 802.11ac station wins the transmission channel at a given time slot. P_{SUC} is expressed by the fact that exactly one 802.11ac station transmits on the transmission channel. This probability is also conditioned by the fact that there is at least one station transmitting on channel. It is given as follows:

$$P_{SUC} = \frac{n\tau(1 - \tau)^{n-1}}{1 - (1 - \tau)^n}. \quad (30)$$

- Let T_{SUC} be the sensing time of a successful transmission of an A-MPDU frame of a 802.11ac station on the transmission channel. T_{SUC} is given a follows:

$$T_{SUC} = T_{PHY} + Tampdu + SIFS + T_{BA}$$
$$+ DIFS + 2 \times \delta. \quad (31)$$

- Let T_{UNS} be the sensing time of an unsuccessful transmission of an A-MPDU frame of a 802.11ac station on the transmission channel. T_{UNS} is given a follows:

$$T_{UNS} = T_{PHY} + Tampdu + \delta + DIFS. \quad (32)$$

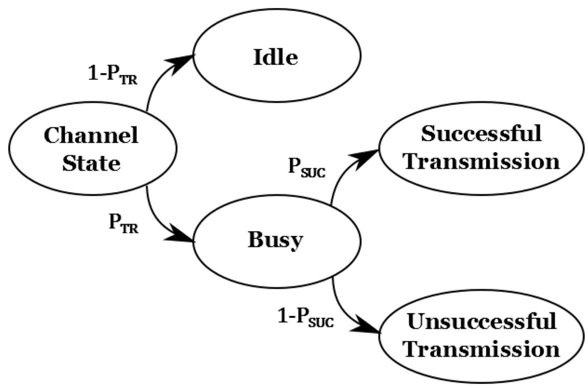

Fig. 5. A schematic depiction of transmission channel state changes, during an arbitrary time slot.

- $Tampdu, T_{PHY}$ and T_{BA}, are respectively determined by the Equations (5), (33) and (34).

$$T_{PHY} = 36\,\mu s + 4\,\mu s \times Nss. \tag{33}$$

$$T_{BA} = \frac{BA}{HTR}. \tag{34}$$

To determine TH, let us analyze the different events which can happen in a randomly chosen time slot (these events are illustrated in Fig. 5):

- With the probability $(1 - P_{TR})$, the transmission channel is idle (P_{TR} is given by Equation (29)). The time duration of this event is TS, and the amount of payload bits successfully transmitted during this event is equal to 0.
- With the probability $P_{TR} \times P_{SUC}$, the transmission channel contains a successful transmission (P_{SUC} is given by Equation (30)). The time duration of this event is T_{SUC} (T_{SUC} is given by the Equation (31)), and the amount of payload bits successfully transmitted during this event is equal to $Nmpdu \times MSDU$.
- With the probability $P_{TR} \times (1 - P_{SUC})$, the transmission channel contains an unsuccessful transmission. The time duration of this event is T_{UNS} (T_{UNS} is given by Equation (32)), and the amount of payload bits successfully transmitted during this event is equal to 0.

Let $E[HT]$ denotes the mean channel holding time. It depends on the events illustrated in Figure 5, namely: idle channel, successful transmission, and unsuccessful transmission. It is obtained as follows:

$$E[HT] = (1 - P_{TR}) \times TS + P_{TR} \times P_{SUC} \times T_{SUC} + P_{TR} \times (1 - P_{SUC}) \times T_{UNS}. \tag{35}$$

Let $E[payload]$ denotes the average amount of payload bits successfully transmitted on the transmission channel in a randomly chosen time slot. It is obtained as follows:

$$E[payload] = P_{TR} \times P_{SUC} \times Nmpdu \times MSDU. \tag{36}$$

Finally, the throughput (TH) of the 802.11ac network is obtained as the ratio of average amount of payload bits successfully transmitted on the transmission channel to mean channel holding time $E[HT]$. It is given as follows:

$$TH = \frac{E[payload]}{E[HT]}. \tag{37}$$

TABLE IV
VALUES OF THE 802.11AC PHY AND MAC PARAMETERS

Parameter	*Numerical value*	*Parameter*	*Numerical value*
$MSDU$	11414 bytes at most	W_0	32
HDR	47 bytes at most	MTR	780 Mbps at most
H_{MAC}	36 bytes	HTR	58,5 Mbps at most
FCS	4 bytes	Nss	8 at most
DLT	4 bytes	TS	9 µs
PAD	3 bytes at most	T_{PHY}	68 µs at most
$AMPDU$	1048575 bytes at most	$DIFS$	34 µs
BA	40 bytes at most	$SIFS$	16 µs
m	5	δ	1 µs

D. Overhead (OH)

The overhead OH of the 802.11ac network can be expressed as the fraction of time that the transmission channel is employed for handling the successful transmission of payload bits of an A-MPDU frame; it means the fraction of time that the transmission channel is not used to successfully transmit the payload bits of an A-MPDU frame. OH is determined using Equation (38), where: $T_{payload}$ denotes the transmission time of payload bits of an A-MPDU frame. $E[HT]$ and $T_{payload}$ are respectively given by Equations (35) and (39).

$$OH = \frac{E[HT] - T_{payload}}{E[HT]} \tag{38}$$

$$T_{payload} = \frac{Nmpdu \times \frac{MSDU}{MTR}}{Nss} \tag{39}$$

V. NUMERICAL RESULTS AND PERFORMANCE ANALYSIS

In this section, the aim is to examine the numerical results obtained after the implementation (under Matlab) of the mathematical model presented in the previous section. These results are generated using various criteria, including: the size of the A-MPDU frame, the number of spatial streams and the number of stations in the network. To generate the figures below, we have used the values presented in Table IV, which are obtained from the IEEE 802.11ac standard [10]. In particular, the values of MTR and HTR are calculated by using 160 MHz channels in the 5 GHz band, and applying respectively the following modulation and coding rates schemes: 256-Quadrature Amplitude Modulation (256-QAM) and Binary Phase Shift Keying (BPSK). The ultimate objectives of this section are organized as follows:

- Firstly, we will show that the size of the A-MPDU frame, which may extend to 1048575 bytes, has a negative impact on the achieved throughput in a 802.11ac network. Indeed, when the A-MPDU frame reaches a certain size, the throughput of the 802.11ac network declines as A-MPDU frame size increases.

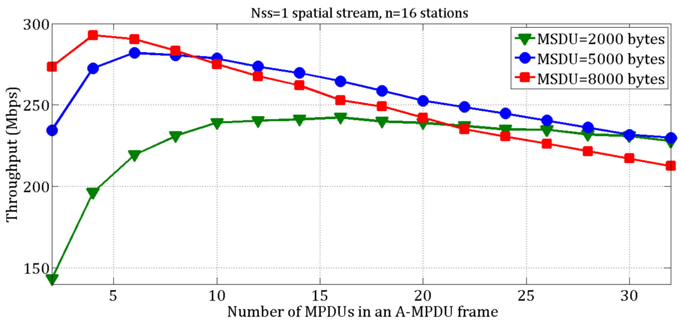

Fig. 6. Throughput variation according to the A-MPDU frame size over one spatial stream.

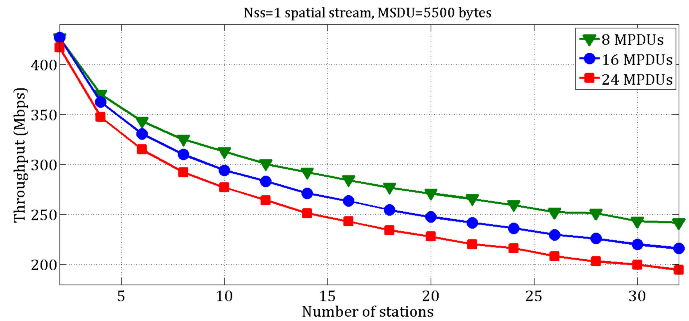

Fig. 7. Throughput variation according to the network size with different sizes of the A-MPDU frame over one spatial stream.

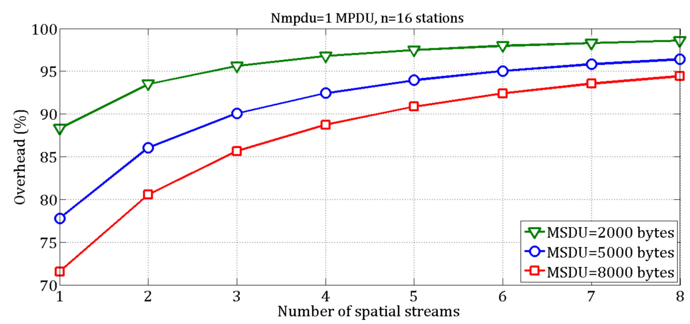

Fig. 8. Overhead variation according to the number of spatial streams with a single MPDU in the A-MPDU frame.

- Secondly, we will show that the use of multiple spatial streams (up to 8) causes an increase in the overhead of the 802.11ac network. As the number of spatial streams increases, the overhead of the 802.11ac network becomes increasingly greater.
- Thirdly, we will show that the use of multiple spatial streams provides a solution to the problem of decrease in the throughput of the 802.11ac network resulting from the size of the A-MPDU frame. Indeed, for each number of spatial streams, there is a maximum size of the A-MPDU frame that maximises the throughput of the 802.11ac network.
- Finally, we will show that the use of the A-MPDU aggregation in turn can lower the overhead of the 802.11ac network caused by the spatial multiplexing. In fact, for each size of A-MPDU frame, there is a maximum number of spatial streams that minimises the overhead of the 802.11ac network.

In Figure 6, we study the variation of the achieved throughput in a midsize 802.11ac network (16 stations) according to the number of MPDUs contained in an A-MPDU frame for different sizes of MSDU (payload bits of a MPDU), namely: 2000 bytes (small-sized MSDU), 5000 bytes (medium-sized MSDU) and 8000 bytes (large-sized MSDU). To obtain the results of this figure, we have considered one spatial stream at the PHY layer. This result will allow us to focus on how A-MPDU aggregation contributes at the MAC layer to the enhancement of the 802.11ac throughput. The Figure 6 shows that 802.11ac stations, which are not equipped with multiple radio antennas (i.e., those which cannot perform spatial multiplexing) will not get the benefits of A-MPDU aggregation, even though the maximum rate of 780 Mbps at which payload bits are transmitted is used. In fact, upwards of a certain size of A-MPDU frame (which is relatively small compared to the maximum size of the A-MPDU frame), the larger the A-MPDU frame size, the smaller the 802.11ac throughput. We argue this by the fact that the collision time of an A-MPDU frame increases with the increase of its size; and since the collision is a frequent phenomena in a 802.11 network, this has a heavy impact on the achieved throughput in a 802.11ac network.

In Figure 7, we study the variation of the throughput over one spatial stream according to the number of stations in the network for different sizes of the A-MPDU frame. Obviously, we remark that the more number of 802.11ac stations in the network, the more collision phenomenon is intensified. Thus, the A-MPDU aggregation is less efficient when considering improving the throughput in a 802.11ac network.

In Figure 8, we analyze the overhead of the 802.11ac network according to the number of spatial streams enabled at the PHY layer. At the MAC layer, we have considered simple MPDU frames (i.e., without A-MPDU aggregation) of different sizes so that we can clearly demonstrate the negative effect of spatial multiplexing on 802.11ac network. The Figure 8 shows that when 802.11ac stations do not have multiple frames (MPDUs) to be aggregated, enabling multiple spatial steams only increases the network overhead. As shown in this figure, as more spatial streams are enabled, the network overhead grows, i.e., the bandwidth is becoming increasingly underutilized. This phenomena is even more critical when the size of data to be sent is too small. This may be explained by the fact that the use of multiple spatial streams accelerates only the transmission of data, while the other parameters (including: headers, inter-frame spaces and acknowledgments) remain as they are. This is why, the overhead of the 802.11ac network increases with the increase of the number of spatial streams.

In Figure 9, we analyze the variation of the overhead of the 802.11ac network according to the number of stations for

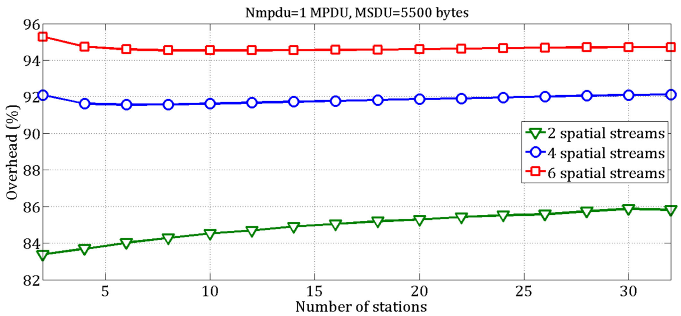

Fig. 9. Overhead variation according to the network size with a single MPDU in the A-MPDU frame over different numbers of spatial stream.

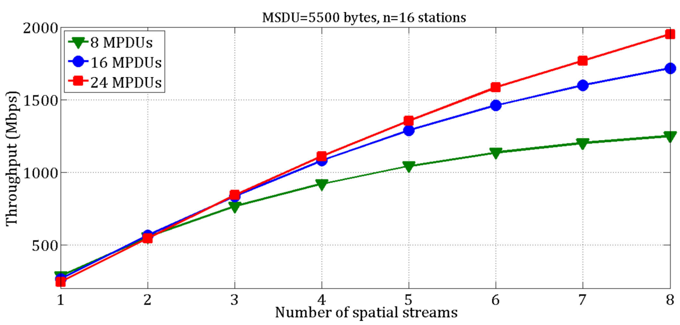

Fig. 10. Throughput variation according to the number of spatial steams with different sizes of the A-MPDU frame.

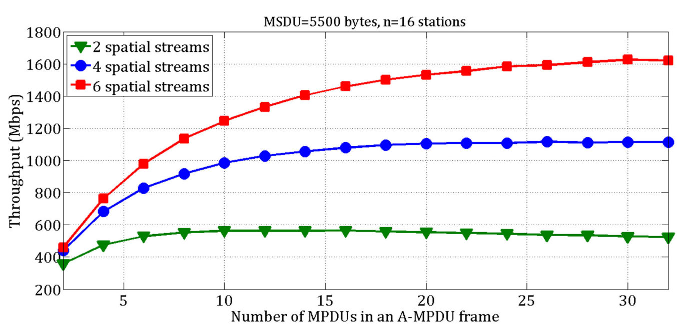

Fig. 11. Throughput variation according to the A-MPDU frame size over different numbers of spatial stream.

different numbers of spatial stream. We clearly demonstrate on this figure that for any number of stations, employing spatial multiplexing at the PHY layer generates a substantial overhead, which increases when the number of spatial streams grow. However, for a fixed number of spatial streams (especially when it is great), the overhead does not vary greatly as a function of number of stations. Although the number of stations increases, the resultant collisions do not have much effect on the network overhead, since the data to be sent are relatively small in size (they are simple MPDUs without aggregation) and the spatial streams are numerous.

In Figures 10, 11 and 12, we demonstrate the role of spatial multiplexing at the PHY layer (the use of multiple spatial streams) for addressing the problem of the decline in the

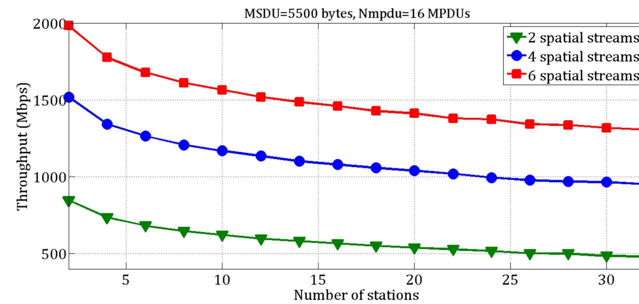

Fig. 12. Throughput variation according to the network size over different numbers of spatial stream.

802.11ac throughput caused by the A-MPDU aggregation at the MAC layer (the generation of long A-MPDU frames).

In Figure 10, we study the variation of the throughput according to the number of spatial streams for different sizes of the A-MPDU frame. On one hand, we can clearly see from this figure that the 802.11ac throughput is proportional to the number of spatial streams. This shows the major need of using multiple spatial streams for achieving very high throughput in a 802.11ac network. In fact, the spatial multiplexing consists in dividing by Nss spatial streams the transmission time of an A-MPDU frame and this makes error recovery faster, i.e., the impact of the collision is not significant. On the other hand, we can see that with a small number of spatial streams (up to 3 spatial streams), the 802.11ac throughput does not increase according to the A-MPDU frame size because the collision time has not been sufficiently lowered. In contrast, with a large number of spatial streams (from 4 spatial streams), the 802.11ac throughput remarkably increases with increasing the size of A-MPDU frames. In fact, the use of a large number of spatial streams greatly increases the overhead caused by the MAC layer (information and time delays for managing the transmission of the A-MPDU frame). This is why, any increase in the size of the A-MPDU frame is reflected in an increase in the 802.11ac throughput.

In Figure 11, we study the variation of the throughput according to the size of the A-MPDU frame over different numbers of spatial stream. From this figure, we infer that for each number of spatial streams, there is an optimal size of the A-MPDU frame which guarantees a maximum throughput in a 802.11ac network. In fact, the 802.11ac throughput firstly increases, and then it stabilizes at a maximum value. This is due to the fact that, for a given number of spatial stream, the effect of collisions on the throughput will again appear. In the case of 2 spatial streams, the 802.11ac throughput will soon start to drop from 18 MPDUs.

In Figure 12, we study the variation of the throughput according to the number of stations in the network for different numbers of spatial stream. By attenuating the effect of repetitive collisions due to the increase of the network size, the use of multiple spatial streams allows for significant improvements of the achieved throughput.

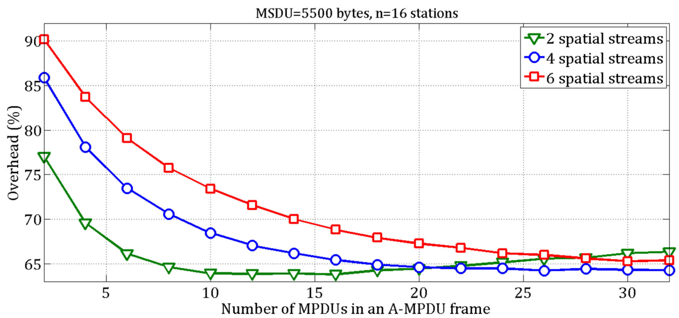

Fig. 13. Overhead variation according to the A-MPDU frame size and for different numbers of spatial stream.

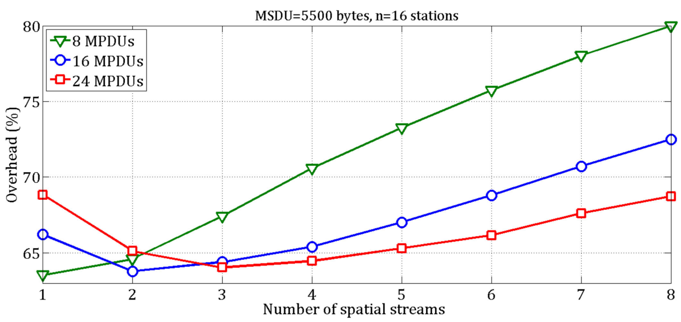

Fig. 14. Overhead variation according to the number of spatial streams and different sizes of the A-MPDU frame.

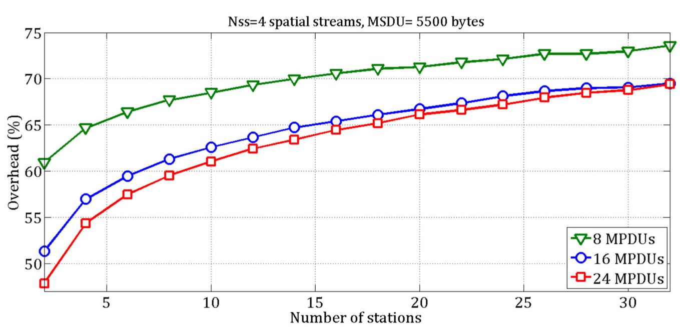

Fig. 15. Overhead variation according to the network size and different numbers of spatial stream.

The objective of the Figures 13, 14 and 15 is to highlight the necessity of using A-MPDU frame aggregation at the MAC layer in order to efficiently utilize the available bandwidth, i.e., to lessen the overhead of the 802.11ac network caused by the spatial multiplexing at the PHY layer.

In Figure 13, we analyse the variation of the overhead of the 802.11ac network according to the size of the A-MPDU frame over different numbers of spatial stream. As shown in this figure, the overhead registered in a 802.11ac network is inversely proportional to the size of the A-MPDU frame. Moreover, the overhead reduction is much more pronounced when there are too many spatial streams. Indeed, as the size of the A-MPDU frame increases, the 802.11ac stations are spending more time transmitting data than listening to the channel. Hence, the bandwidth is increasingly well utilized (i.e., the overhead is becoming much smaller). However, we note that upwards of a certain size of A-MPDU frame (especially in the case when the number of spatial streams is small), the overhead starts to increase again. This increase is due mostly to collision time which is getting important since the size of the A-MPDU frame is great and the number of spatial streams is small.

In Figure 14, we analyze the variation of the overhead of the 802.11ac network according to the number of spatial streams for different sizes of the A-MPDU frame. We note that for each A-MPDU frame size, there is a maximum number of spatial streams that minimizes the overhead of the 802.11ac network. If we use a lesser or greater number of spatial streams than is necessary, the overhead in both cases is not minimized. Indeed, with an insufficient number of spatial streams, the overhead is mostly caused by the collision time, resulting in a degradation of the 802.11ac throughput (as shown in Figure 10). While with an oversupply of spatial streams, the overhead is caused by the process of accessing the network, resulting in an under-utilization of the radio channel. That is why, the use of longer A-MPDU frames should improve the 802.11ac throughput (as shown in Figure 10). Hence, an optimal number of spatial streams that minimizes the 802.11ac overhead relative to a size of A-MPDU frame is the minimum number of spatial streams to be used for reducing the impact of collisions on the 802.11ac throughput. Whereas, if the number of spatial streams used exceeds the optimal number, it is necessary to increase the size of the A-MPDU frame to bring down the MAC layer overhead, thus enhancing the 802.11ac throughput.

In Figure 15, we analyze the variation of the overhead of the 802.11ac network according to the number of stations for different sizes of the A-MPDU frame. The purpose of this figure is to demonstrate, for each network size, the need to combine A-MPDU frame aggregation with spatial multiplexing to reduce the network overhead. This figure indicates that the overhead increases with the number of stations, as a result of repetitive collisions. However, the overhead is inversely proportional to A-MPDU frame size when the latter does not exceed a certain threshold. In this case, the collision time has a significant impact on the network overhead.

VI. Simulation and Model Validation

The ultimate goal of this section is to validate the accuracy of the proposed mathematical model and the theoretical results obtained. To do this, we have developed (under Matlab) a simulation program based on the Monte Carlo method [27]. Finally, to make the simulation results more credible, we have also calculated 95% confidence intervals.

In Figures 16 and 17, we respectively present a sample of numerical values of the throughput and the overhead aiming at comparing the theoretical results (i.e., those generated from the proposed mathematical model) to their corresponding simulation results (i.e., those calculated on the basis of the Monte Carlo simulation). We can clearly observe that the theoretical results match well with the simulation results.

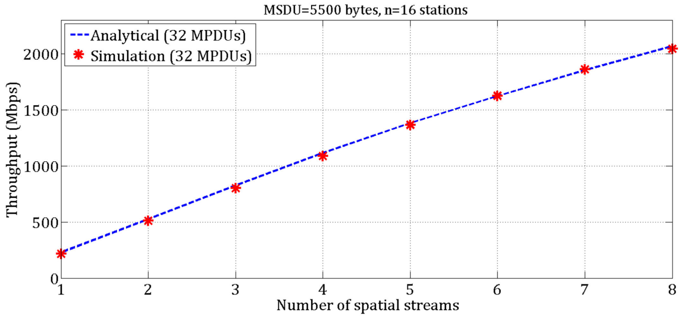

Fig. 16. Comparison between theoretical and simulation results of the 802.11ac throughput.

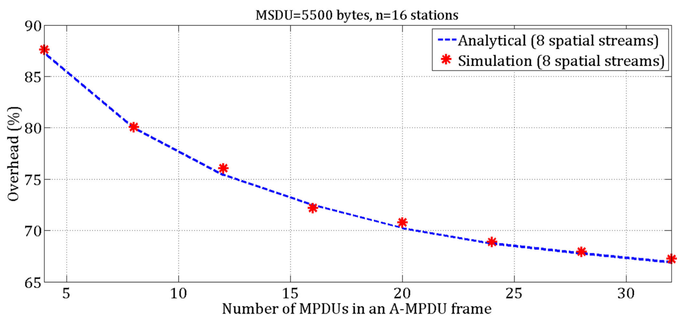

Fig. 17. Comparison between theoretical and simulation results of the 802.11ac overhead.

TABLE V
THEORETICAL THROUGHPUT, SIMULATION AND CONFIDENCE INTERVALS (GBPS)

Number of SSs	Analytical	Simulation	Confidence Intervals
1	0.2279	0.2157	[0.2093, 0.2219]
2	0.5252	0.5104	[0.5007, 0.5202]
3	0.8259	0.8002	[0.7871, 0.8142]
4	1.1150	1.0877	[1.0737, 1.1030]
5	1.3801	1.3655	[1.3497, 1.3814]
6	1.6215	1.6246	[1.6077, 1.6400]
7	1.8501	1.8604	[1.8432, 1.8760]
8	2.0659	2.0452	[2.0283, 2.0621]

TABLE VI
THEORETICAL OVERHEAD, SIMULATION AND CONFIDENCE INTERVALS (%)

Number of MPDUs	Analytical	Simulation	Confidence Intervals
4	87.2829	87.6215	[87.5983, 87.6448]
8	79.9716	80.0713	[80.0143, 80.1284]
12	75.4021	76.0541	[75.9582, 76.1499]
16	72.4810	72.1852	[72.0257, 72.3445]
20	70.2403	70.7978	[70.5955, 71.0248]
24	68.7395	68.8765	[68.6571, 69.0681]
28	67.7706	67.9441	[67.6836, 68.2037]
32	66.8928	67.2245	[66.9532, 67.4947]

To better demonstrate the matching between the theoretical and simulation results of throughput and overhead, we have provided their numerical values in Tables V and VI, respectively. We can observe again that the simulation results are well-aligned with the theoretical results.

To provide credible support to the simulation results presented in Tables V and VI, we have calculated for each simulation result its confidence interval. Since the simulation approach gives approximate results [28] (unlike mathematical modeling approach that provides exact results), the calculation of the confidence intervals is mandatory in order to determine the margin of error for the simulation results.

Finally, based on Figures 16 and 17, and on Tables V and VI, we can say that the proposed mathematical model is validated against simulation results and shows good agreement.

VII. CONCLUSIONS AND PROSPECTS

In this paper, we were interested in modeling and analyzing the performance of innovative features of the very high throughput 802.11ac standard, including: A-MPDU frame aggregation at the MAC layer and spatial multiplexing technique at the PHY layer. We have mainly highlighted the mutual dependency between the A-MPDU frame aggregation and the spatial multiplexing technique for optimizing the performance of a 802.11ac network. In addition, we have shown that optimal values of the A-MPDU frame size and the number of spatial streams should be determined in order to effectively utilize the potential of A-MPDU aggregation and spatial multiplexing. To achieve our goal, we have proposed a new mathematical model based on a Markov chain for estimating the throughput and the overhead of the 802.11ac network. The accuracy of this model has been validated by conducting Monte Carlo simulations, for which we have attached confidence intervals. The analysis of the numerical results has identified two anomalies in the functioning of the 802.11ac network with regard to A-MPDU frame aggregation and spatial multiplexing. The first one is about the decrease of the throughput as a result of increasing the size of the A-MPDU frame. The second issue is an increase of the overhead resulting from the use of multiple spatial streams. The performance analysis has also shown that each of said mechanisms is able to resolve the issue of the other. We have shown on one hand that: for each number of spatial streams, there is a maximum size of A-MPDU frame that maximizes the throughput. On the other hand, for each size of A-MPDU frame, there is a maximum number of spatial streams that minimizes the overhead. This motivates our future work, where we will model this trade-off by using multi-criteria optimisation techniques; having as objective to maximize the throughput, while minimizing the overhead of the 802.11ac network based on the size of A-MPDU frame and the number of spatial streams.

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