

# Enabling Covariance-Based Feedback in Massive MIMO: A User Classification Approach

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**Abstract**—In this paper, we propose a novel channel feedback scheme for frequency division duplexing massive multi-input multi-output systems. The concept uses the notion of user statistical separability which was hinted in several prior works in the massive antenna regime but not fully exploited so far. We here propose a hybrid statistical-instantaneous feedback scheme based on a user classification mechanism where the classification metric derives from a rate bound analysis. According to classification results, a user either operates on a statistical or instantaneous feedback mode. Our results illustrate the sum rate advantages of our scheme under a global feedback overhead constraint.

## I. INTRODUCTION

Massive multiple-input multiple-output (MIMO) is expected to be a key enabler for the next generation communication systems. It has drawn considerable interest from academia and industry for improving energy and spectral efficiency [1], [2].

However, the large scale number of antennas brings up new challenges, one of which is the acquisition of accurate instantaneous channel state information (CSI), especially downlink CSI in frequency division duplex (FDD) massive MIMO systems. In conventional MIMO FDD feedback mechanisms, users need to conduct channel estimation and feed the quantized channel back to base station (BS) [3] where quantization occurs over a certain budgeted number of bits.

Numerous works have been dedicated to reducing feedback overhead, such as trellis-extended codebook design [4], compressive sensing-based channel feedback reduction [5], channel covariance-based feedback representation [6]–[9]. In [9], a two-stage precoding structure was introduced to reduce downlink pilot and channel feedback overhead. Advanced allocation of feedback bits among users was also proposed under total overhead constraint [10]. Furthermore, the channel feedback issue was tackled by exploiting user cooperation via device-to-device communications, such as cooperative precoder feedback scheme [11]. Although several previous schemes exploit the low rank behavior of channel covariance to map the instantaneous channel into a lower dimensional vector, the pair-wise orthogonality that may exist between users' signal subspaces [9], [12] has not fully been exploited. This is because such orthogonality hinges on the degree of spatial separation between users and therefore cannot be guaranteed.

Still, we point out that, statistically speaking, some users are bound to be spatially isolated from the other users and can benefit the system by being identified and assigned a

suitable feedback mechanism. In fact, it is well known that a spatially isolated user can be served free of interference via a statistical precoder [12], [13], hence not requiring any instantaneous feedback at all.

Therefore, this motivates a hybrid statistical-instantaneous feedback method where users are classified according to their channel covariance. In this paper, some users are assigned zero bit towards instantaneous feedback and labeled as class-S users. The remaining users share the feedback bit budget evenly and are named as class-I users. The class-I users estimate their downlink instantaneous CSI and feed quantized channel back to BS. A suitable precoder design is proposed for both user classes with statistical CSI of class-S users and instantaneous feedback of class-I users. The precoder is based on signal-to-leakage-and-noise ratio (SLNR) and the closed-form precoding vectors are obtained. Moreover, the user classification criterion is derived analytically from a sum rate bound maximization argument. The proposed scheme is shown to improve significantly the overall system throughput in the presence of restricted feedback bit budget.

## II. SYSTEM AND CHANNEL MODELS

We consider a single-cell massive MIMO system where the BS is equipped with  $M$  antennas and serves  $K$  single-antenna users. We assume the BS holds channel covariance of users. When the proposed hybrid statistical-instantaneous feedback method is considered, the  $K$  users are classified into two classes based on their channel covariance, i.e.,  $K_S$  class-S and  $K_I$  class-I users. Class-S users are assigned zero bit towards instantaneous feedback, while class-I users share the feedback overhead evenly and feed their quantized instantaneous channel back to BS based on codebooks.

For any user  $k$ , a multipath channel model is considered as

$$\mathbf{h}_k \triangleq \frac{1}{\sqrt{P}} \sum_{p=1}^P \gamma_{kp} \mathbf{a}(\theta_{kp}), k = 1, \dots, K, \quad (1)$$

where  $P$  is the number of independent, identically distributed (i.i.d.) paths,  $\gamma_{kp}$  represents the complex gain of the  $p$ -th path and  $\mathbf{a}(\theta_{kp})$  is the corresponding steering vector. For simplicity, uniform linear array is considered and the steering vector is given as

$$\mathbf{a}(\theta_{kp}) \triangleq \left[ 1, e^{j2\pi \frac{d}{\lambda} \sin(\theta_{kp})}, \dots, e^{j2\pi \frac{(M-1)d}{\lambda} \sin(\theta_{kp})} \right]^T, \quad (2)$$

where  $d$  is antenna spacing,  $\lambda$  is wavelength and  $\theta_{kp}$  is the random angle of arrival (AoA) of the  $p$ -th path.

Throughout the paper, we use subscript  $(\cdot)_{I,i}$  and  $(\cdot)_{S,n}$  to denote the notations for the  $i$ -th class-I and  $n$ -th class-S users, respectively. Then, the received signals  $y_{I,i}$  and  $y_{S,n}$  are

$$y_{I,i} = p_d \mathbf{h}_{I,i}^H (\widehat{\mathbf{W}}_I \mathbf{x}_I + \widehat{\mathbf{W}}_S \mathbf{x}_S) + n_{I,i}, \quad (3)$$

$$y_{S,n} = p_d \mathbf{h}_{S,n}^H (\widehat{\mathbf{W}}_I \mathbf{x}_I + \widehat{\mathbf{W}}_S \mathbf{x}_S) + n_{S,n}, \quad (4)$$

respectively, where  $p_d$  is downlink transmit power,  $\widehat{\mathbf{W}}_I \triangleq [\widehat{\mathbf{w}}_{I,1}, \dots, \widehat{\mathbf{w}}_{I,K_I}] \in \mathbb{C}^{M \times K_I}$  and  $\widehat{\mathbf{W}}_S \triangleq [\widehat{\mathbf{w}}_{S,1}, \dots, \widehat{\mathbf{w}}_{S,K_S}] \in \mathbb{C}^{M \times K_S}$  denote downlink precoding matrices with  $\mathbb{E}\{\|\mathbf{w}\|^2\} = 1$ ,  $\mathbf{x}_I$  and  $\mathbf{x}_S$  are consisted of i.i.d. downlink data  $x_{I,i}$  and  $x_{S,n}$  with  $\mathbb{E}\{\mathbf{x}\mathbf{x}^H\} = \mathbf{I}$ , respectively, and  $n$  denotes i.i.d. additive white Gaussian noise (AWGN) with zero mean and unit variance. We take class-I users as an example and  $y_{I,i}$  is further given as

$$\begin{aligned} y_{I,i} = & \underbrace{p_d \mathbf{h}_{I,i}^H \widehat{\mathbf{w}}_{I,i} x_{I,i}}_{\text{Expected signal}} + \underbrace{p_d \sum_{j=1, j \neq i}^{K_I} \mathbf{h}_{I,i}^H \widehat{\mathbf{w}}_{I,j} x_{I,j}}_{\text{Interference from the other class-I users}} \\ & + \underbrace{p_d \sum_{n=1}^{K_S} \mathbf{h}_{I,i}^H \widehat{\mathbf{w}}_{S,n} x_{S,n}}_{\text{Interference from class-S users}} + \underbrace{n_{I,i}}_{\text{AWGN}}. \end{aligned} \quad (5)$$

Then, the signal-to-interference-plus-noise ratio (SINR) of the  $i$ -th class-I user is

$$r_{I,i} = \frac{|\mathbf{h}_{I,i}^H \widehat{\mathbf{w}}_{I,i}|^2}{\sum_{j=1, j \neq i}^{K_I} |\mathbf{h}_{I,i}^H \widehat{\mathbf{w}}_{I,j}|^2 + \sum_{n=1}^{K_S} |\mathbf{h}_{I,i}^H \widehat{\mathbf{w}}_{S,n}|^2 + p_d}. \quad (6)$$

Similarly, we can obtain SINR  $r_{S,n}$  for the  $n$ -th class-S user. Thus, the downlink ergodic sum rate of the system is

$$R_{\text{sum}} = \sum_{i=1}^{K_I} \mathbb{E}\{\log_2(1+r_{I,i})\} + \sum_{n=1}^{K_S} \mathbb{E}\{\log_2(1+r_{S,n})\}. \quad (7)$$

Note that, different from the systems under conventional feedback schemes, downlink precoding matrices  $\widehat{\mathbf{W}}_I$  and  $\widehat{\mathbf{W}}_S$  can only be designed based on the instantaneous feedback of class-I users and statistical CSI of class-S users.

### III. DOWNLINK SLNR-BASED PRECODER DESIGN

Under the proposed hybrid statistical-instantaneous feedback scheme, a SLNR-based precoding method is proposed for both class-I and class-S users. Take the  $i$ -th class-I user as an example, its SLNR is given as

$$\Gamma_{I,i} = \frac{|\mathbf{h}_{I,i}^H \widehat{\mathbf{w}}_{I,i}|^2}{\sum_{j=1, j \neq i}^{K_I} |\mathbf{h}_{I,i}^H \widehat{\mathbf{w}}_{I,j}|^2 + \sum_{n=1}^{K_S} |\mathbf{h}_{S,n}^H \widehat{\mathbf{w}}_{I,i}|^2 + \frac{1}{p_d}}. \quad (8)$$

Similarly, SLNR  $\Gamma_{S,n}$  of the  $n$ -th class-S user can be obtained. Since the instantaneous CSI of class-S users is unavailable at the BS, average SLNR  $\mathbb{E}\{\Gamma_{I,i}\}$  and  $\mathbb{E}\{\Gamma_{S,n}\}$  are exploited [14]. Moreover, based on the Mullen's inequality  $\mathbb{E}\left\{\frac{X}{Y}\right\} \geq \frac{\mathbb{E}\{X\}}{\mathbb{E}\{Y\}}$ , lower bounds of  $\mathbb{E}\{\Gamma_{I,i}\}$  and  $\mathbb{E}\{\Gamma_{S,n}\}$

are considered with the quantized channel of class-I users and channel covariance of class-S users as

$$\mathbb{E}\{\Gamma_{I,i}^{\text{LB}}\} = \frac{\widehat{\mathbf{w}}_{I,i}^H \overline{\mathbf{H}}_{I,i} \widehat{\mathbf{w}}_{I,i}}{\widehat{\mathbf{w}}_{I,i}^H \sum_{j=1, j \neq i}^{K_I} \overline{\mathbf{H}}_{I,j} \widehat{\mathbf{w}}_{I,i} + \widehat{\mathbf{w}}_{I,i}^H \sum_{n=1}^{K_S} \Phi_{S,n} \widehat{\mathbf{w}}_{I,i} + \frac{1}{p_d}},$$

$$\mathbb{E}\{\Gamma_{S,n}^{\text{LB}}\} = \frac{\widehat{\mathbf{w}}_{S,n}^H \Phi_{S,n} \widehat{\mathbf{w}}_{S,n}}{\widehat{\mathbf{w}}_{S,n}^H \sum_{q=1, q \neq n}^{K_S} \Phi_{S,q} \mathbf{h}_{S,n}^H \widehat{\mathbf{w}}_{S,n} + \widehat{\mathbf{w}}_{S,n}^H \sum_{i=1}^{K_I} \overline{\mathbf{H}}_{I,i} \widehat{\mathbf{w}}_{S,n} + \frac{1}{p_d}},$$

respectively, where  $\overline{\mathbf{H}}_{I,i} \triangleq \widehat{\mathbf{h}}_{I,i} \widehat{\mathbf{h}}_{I,i}^H$  with  $\widehat{\mathbf{h}}_{I,i}$  denoting the quantized channel of the  $i$ -th class-I user and  $\Phi_{S,n}$  is channel covariance of the  $n$ -th class-S user. To maximize  $\mathbb{E}\{\Gamma_{I,i}^{\text{LB}}\}$  and  $\mathbb{E}\{\Gamma_{S,n}^{\text{LB}}\}$ , the SLNR-based precoding vectors are

$$\widehat{\mathbf{w}}_{I,i} = \mathbf{u}_{\max} \left\{ \left( \sum_{j=1, j \neq i}^{K_I} \overline{\mathbf{H}}_{I,j} + \sum_{n=1}^{K_S} \Phi_{S,n} + \frac{\mathbf{I}_M}{p_d} \right)^{-1} \overline{\mathbf{H}}_{I,i} \right\}, \quad (9)$$

$$\widehat{\mathbf{w}}_{S,n} = \mathbf{u}_{\max} \left\{ \left( \sum_{q=1, q \neq n}^{K_S} \Phi_{S,q} + \sum_{i=1}^{K_I} \overline{\mathbf{H}}_{I,i} + \frac{\mathbf{I}_M}{p_d} \right)^{-1} \Phi_{S,n} \right\}, \quad (10)$$

where  $\mathbf{u}_{\max}(\mathbf{A})$  denotes the eigenvector corresponding to the maximal eigenvalue and  $\mathbf{I}_M$  is  $M$ -dimension identity matrix.

### IV. SYSTEM SUM RATE BOUND ANALYSIS

The system performance is highly impacted by the user classification due to the complex mutual interactions between users. Since only covariance matrices are available for user classification, we present a rate bound based on statistical information to predict the rate performance under the proposed SLNR-based precoder and any user classification solution.

#### A. Approximate Subspace and Quantized Channel Prediction

For user  $k$ , its channel covariance  $\Phi_k$  can be approximately represented in beam domain form  $\widetilde{\Phi}_k$  [15]–[17] given as

$$\widetilde{\Phi}_k = \mathbf{V} \widetilde{\Phi}_k^{\text{BD}} \mathbf{V}^H, \quad (11)$$

where  $\mathbf{V}$  is a Discrete Fourier Transform (DFT) matrix whose  $t$ -th column represents a virtual beam and is given as

$$\mathbf{v} \left( \frac{t}{M} \right) \triangleq \frac{1}{\sqrt{M}} [1, e^{j\pi(\frac{2t}{M}-1)}, \dots, e^{j\pi(M-1)(\frac{2t}{M}-1)}]^T. \quad (12)$$

The matrix  $\widetilde{\Phi}_k^{\text{BD}}$  is a diagonal matrix whose  $t$ -th diagonal element denotes the average gain of the  $t$ -th beam given as  $\mathbb{E}\left\{ \left| [h_k^{\text{BD}}]_t \right|^2 \right\}$ . When  $M \rightarrow \infty$ , each path corresponds to only one virtual beam in  $\mathbf{V}$  and the beam index  $t$  satisfies  $\left(\frac{2t}{M} - 1\right) = \frac{2d}{\lambda} \sin(\theta_{kp})$  [16]. When  $M$  is not large enough, power leakage exists. But the most power concentrates around the closest beam  $\hat{t} = \lfloor t \rfloor = \lfloor \frac{M}{2} (\sin(\theta_{kp}) + 1) \rfloor$  with half wavelength antenna spacing, where  $\lfloor t \rfloor$  denotes the closest integer to  $t$ . Thus, the approximate complex gain of the  $\hat{t}$ -th virtual beam can be obtained as  $\left| [\widetilde{h}_k^{\text{BD}}]_{\hat{t}} \right|^2 = \sum_{p \in \mathcal{B}_{k,\hat{t}}} |\gamma_{kp}|^2$ ,

where  $\mathcal{B}_{k,\hat{t}}$  denotes the set of paths whose closest beam is the  $\hat{t}$ -th beam.

The nonzero elements in  $\tilde{\Phi}_k^{\text{BD}}$  are limited since the number of multipaths is usually small in massive MIMO systems. We use  $d_{k,\min}$  and  $d_{k,\max}$  to denote the indices of the first and last nonzero elements, respectively. Taking power leakage of paths to the neighbouring beams into consideration, we can present an approximate subspace of the  $k$ -th user as

$$\mathcal{S}_k = \text{Span} \left\{ \mathbf{v} \left( \frac{x}{M} \right), x_{k,\min} \leq x \leq x_{k,\max} \right\}, \quad (13)$$

where  $x_{k,\min} = \max(d_{k,\min} - x_k, 1)$ ,  $x_{k,\max} = \min(d_{k,\max} + x_k, M)$  and  $x_k$  is a non-negative constant considered for power leakage, which can be obtained by empirical measurement.

The codebook design for spatially correlated channel usually takes channel covariance into account and codebook vectors are designed in subspaces. Thus, a codebook  $\tilde{\mathcal{C}}_k$  of size  $X$  can be predicted with the assumption that codebook vectors are isotropically distributed in subspace  $\mathcal{S}_k$ . Hence, the codebook vector  $\tilde{\mathbf{c}}_{k,u} \in \tilde{\mathcal{C}}_k$ ,  $u = 1, \dots, X$ , is constructed as

$$\tilde{\mathbf{c}}_{k,u} = \frac{1}{\sqrt{M}} [1, e^{j\pi\theta_k(u)}, \dots, e^{j\pi(M-1)\theta_k(u)}]^T, \quad (14)$$

where  $\theta_k(u) = \left( \frac{2x_{k,\min}}{M} - 1 \right) + u \frac{2(x_{k,\max} - x_{k,\min})}{MX}$ .

The class-I users choose the closest codebook vectors to their channel directions as quantized channels, such as  $\hat{\mathbf{h}}_{1,i} = \arg \max_{\mathbf{c}_u \in \tilde{\mathcal{C}}_{1,i}} |\mathbf{h}_{1,i}^H \mathbf{c}_u|^2$ . However, instantaneous CSI of class-I users is unknown to the BS during user classification stage. Hence, the BS needs to predict channel directions based on channel covariance. The idea is to take the beam  $\mathbf{v} \left( \frac{t_{1,i}^*}{M} \right)$  in matrix  $\mathbf{V}$  with the largest average gain as predicted channel direction. Then, the vector in codebook  $\tilde{\mathcal{C}}_{1,i}$  which is closest to the vector  $\mathbf{v} \left( \frac{t_{1,i}^*}{M} \right)$  is selected as feedback channel. Thus, the predicted codebook vector index is obtained as

$$\tilde{u}_{1,i}^* = \begin{cases} 1, & t_{1,i}^* = x_{1,i,\min} \\ \left\lfloor \frac{(t_{1,i}^* - x_{1,i,\min})X}{x_{1,i,\max} - x_{1,i,\min}} \right\rfloor, & x_{1,i,\min} < t_{1,i}^* \leq x_{1,i,\max}. \end{cases} \quad (15)$$

Therefore, the BS obtains the predicted instantaneous channel feedback of class-I users.

### B. Lower Bound Analysis of System Sum Rate

By substituting the predicted quantized channel and beam domain channel covariance  $\tilde{\Phi}_k$  into (9) and (10), the SLNR-based precoding vectors for the  $n$ -th class-S and the  $i$ -th class-I users are approximately presented as

$$\tilde{\mathbf{w}}_{S,n} = \mathbf{V} \mathbf{e}(\tilde{l}_{S,n}^*), \quad (16)$$

$$\tilde{\mathbf{w}}_{I,i} = \mathbf{V} \mathbf{e}(\tilde{t}_{I,i}), \quad (17)$$

respectively, where  $\mathbf{e}(x)$  is the  $x$ -th column of an identity matrix, the index  $\tilde{l}_{S,n}^*$  is obtained by  $\tilde{l}_{S,n}^* = \arg \max_{l=1,\dots,M} [\tilde{\Sigma}_{S,n}]_l$  and the  $l$ -th diagonal element is presented as  $[\tilde{\Sigma}_{S,n}]_l =$

$$\frac{[\tilde{\Phi}_{S,n}^{\text{BD}}]_l}{\sum_{q=1, q \neq n}^{K_S} [\tilde{\Phi}_{S,q}^{\text{BD}}]_l + \sum_{i=1}^{K_I} \delta(\tilde{t}_{1,i} - l) + \frac{1}{p_d}}$$

with  $\delta(\cdot)$  the Kronecker delta

function and  $\tilde{t}_{I,i} = \left\lfloor x_{I,i,\min} + \frac{x_{I,i,\max} - x_{I,i,\min}}{X} \tilde{u}_{I,i}^* \right\rfloor$ . Due to lack of space, the detailed derivation are omitted.

By substituting the approximate SLNR-based precoding given in (16) and (17) into SINR  $r_{I,i}$  and  $r_{S,n}$ , lower bounds  $r_{I,i}^{\text{LB}}$  and  $r_{S,n}^{\text{LB}}$  can be obtained. In addition, we exploit effective SINR  $E\{r_{I,i}^{\text{LB}}\}$  and  $E\{r_{S,n}^{\text{LB}}\}$  to overcome the lack of class-S users' instantaneous CSI. Hence, a lower bound of sum rate is obtained as

$$\tilde{R}_{\text{sum}}^{\text{LB}} = \sum_{i=1}^{K_I} \log(1 + E\{r_{I,i}^{\text{LB}}\}) + \sum_{n=1}^{K_S} \log(1 + E\{r_{S,n}^{\text{LB}}\}), \quad (18)$$

where the closed-form expressions of  $E\{r_{I,i}^{\text{LB}}\}$  and  $E\{r_{S,n}^{\text{LB}}\}$  are respectively given as

$$E\{r_{I,i}^{\text{LB}}\} = \frac{[\tilde{\Phi}_{I,i}^{\text{BD}}]_{\tilde{t}_{I,i}}}{\sum_{j=1, j \neq i}^{K_I} [\tilde{\Phi}_{I,i}^{\text{BD}}]_{\tilde{t}_{I,i}} + \sum_{n=1}^{K_S} [\tilde{\Phi}_{I,i}^{\text{BD}}]_{\tilde{t}_{I,i}}^* + \frac{1}{p_d}}, \quad (19)$$

$$E\{r_{S,n}^{\text{LB}}\} = \frac{[\tilde{\Phi}_{S,n}^{\text{BD}}]_{\tilde{l}_{S,n}^*}}{\sum_{q=1, q \neq n}^{K_S} [\tilde{\Phi}_{S,n}^{\text{BD}}]_{\tilde{l}_{S,n}^*} + \sum_{i=1}^{K_I} [\tilde{\Phi}_{S,n}^{\text{BD}}]_{\tilde{t}_{I,i}} + \frac{1}{p_d}}. \quad (20)$$

Therefore, given user classification and global feedback overhead budget  $B^{\text{total}}$ , a lower bound of system sum rate can be predicted only based on statistical information.

### C. User Classification Algorithm for Sum Rate Maximization

The optimal classifier for the proposed feedback scheme to maximize system sum rate is computationally complex. Therefore, a sub-optimal user classifier is formulated based on the derived lower bound of sum rate and given as

$$\mathcal{K}_I^{\text{sub}}, \mathcal{K}_S^{\text{sub}} = \arg \max_{\tilde{R}_{\text{sum}}^{\text{LB}}} (\mathcal{K}_I, \mathcal{K}_S, B^{\text{total}}) \quad (21a)$$

$$\text{s.t. } K = K_I + K_S, \quad (21b)$$

$$B = \lfloor B^{\text{total}} / K_I \rfloor, \quad (21c)$$

where  $\mathcal{K}_I$  and  $\mathcal{K}_S$  denote the user sets of class-I and class-S users, respectively,  $\lfloor a \rfloor$  denotes the largest integer no more than  $a$  and the constraint (21c) denotes that class-I users share the total feedback overhead evenly and hence their codebook size is  $X = 2^B$ .

To find the solution for problem (21), a user classification algorithm is proposed in Alg. 1. First, we assume all the  $K$  users are class-I users and calculate the lower bound of sum rate  $\tilde{R}_{\text{sum}}^{\text{LB}, K}$  based on (18). Then, we select the user who can achieve the largest  $\tilde{R}_{\text{sum}}^{\text{LB}, K-1}$  as a new class-S user. Repeat this procedure until all the users have been selected as class-S users. Finally, compare all the  $K+1$  sum rate  $\tilde{R}_{\text{sum}}^{\text{LB}, f}$ ,  $f = 0, \dots, K$ , and select the maximal rate with index  $d^*$ . Thus, the optimal numbers of class-S and class-I users are  $K+1-d^*$  and  $d^*-1$ , respectively. Therefore, the optimal user set for class-S users consists of the first  $K+1-d^*$  selected class-S users and the remaining users are class-I users.

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**Algorithm 1** User Classification Algorithm with Sum Rate Bound Maximization
 

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**Input:**  $\tilde{\Phi}_k^{\text{BD}}, k = 1, \dots, K, B^{\text{total}}$ 
**Output:**  $\mathcal{K}_1^{\text{sub}}, \mathcal{K}_S^{\text{sub}}$ 

 1: **Initialize**

   Set  $f = K$  and a rate vector  $\tilde{\mathbf{r}}_{\text{sum}} = \emptyset$ 

   The set of class-I users  $\mathcal{K}_I = \{1, \dots, K\}$ 

   The set of class-S users  $\mathcal{K}_S = \emptyset$ 

   Calculate  $\tilde{R}_{\text{sum}}^{\text{LB},f}$  based on (18)

   Update  $\tilde{\mathbf{r}}_{\text{sum}} = \begin{bmatrix} \tilde{\mathbf{r}}_{\text{sum}} \\ \tilde{R}_{\text{sum}}^{\text{LB},f} \end{bmatrix}$ 

 2: **while**  $f \geq 0$  **do**

 3:   Decrease  $f$  by 1 and calculate  $B = \lfloor B^{\text{total}}/f \rfloor$ 

 4:   Find the user with index  $n_S$  as class-S user, such that

$$n_S = \arg \max_{u \in \mathcal{K}_I} \tilde{R}_{\text{sum}}^{\text{LB},f}(\mathcal{K}_S \cup \{u\}, \mathcal{K}_I \setminus \{u\}, B^{\text{total}})$$

 5:   Update  $\mathcal{K}_S$  and  $\mathcal{K}_I$  as

$$\mathcal{K}_S = \mathcal{K}_S \cup \{n_S\}, \mathcal{K}_I = \mathcal{K}_I \setminus \{n_S\}$$

 6:   Update  $\tilde{\mathbf{r}}_{\text{sum}} = \begin{bmatrix} \tilde{\mathbf{r}}_{\text{sum}} \\ \tilde{R}_{\text{sum}}^{\text{LB},f} \end{bmatrix}$ 

 7: **end while**

 8: Find the maximal rate with index  $d^*$  in vector  $\tilde{\mathbf{r}}_{\text{sum}}$ 

 9: The first  $K+1-d^*$  users in  $\mathcal{K}_S$  belong to  $\mathcal{K}_S^{\text{sub}}$  and  $\mathcal{K}_I^{\text{sub}}$  consists of the remaining users

 10: **Return**  $\mathcal{K}_I^{\text{sub}}, \mathcal{K}_S^{\text{sub}}$ 


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## V. SIMULATION RESULTS

In this section, the analytical result and sum rate with the proposed hybrid statistical-instantaneous feedback scheme are evaluated. SLNR precoder [18] is used for conventional feedback scheme where all the users share the global feedback bit budget evenly and feed back quantized instantaneous channel. Moreover, two types of codebook are considered:

1) *DFT-based codebook* [19]. The  $u$ -th codebook vector is defined as  $\mathbf{c}_u \triangleq \frac{1}{\sqrt{M}} [1, e^{j\pi(\frac{2u}{X}-1)}, \dots, e^{j\pi(M-1)(\frac{2u}{X}-1)}]^T$ .

2) *Skewed codebook* [20]. The codebook for the  $i$ -th class-I user is created as  $\mathcal{C}_{I,i} = \left\{ \Phi_{1,i}^{1/2} \mathbf{f}_u / \left\| \Phi_{1,i}^{1/2} \mathbf{f}_u \right\| \right\}$ , where  $\mathbf{f}_u, u = 1, \dots, X$ , is isotropically distributed on the unit-sphere. In the simulations, the AoA of the  $P$  paths for user  $k$  are uniformly distributed over  $[\bar{\theta}_k - \theta_\Delta/2, \bar{\theta}_k + \theta_\Delta/2]$  with  $\bar{\theta}_k \in [-\frac{\pi}{2}, \frac{\pi}{2}]$  denoting the mean AoA and  $\theta_\Delta$  denoting spread AoA. We set  $P = 20$  and  $\theta_\Delta = 10^\circ$ . The parameter  $x_k$  for quantized channel prediction is 10. The complex gain  $\gamma_{kp}$  is with zero mean and unit variance. Each user is assigned  $B_t = \lceil \frac{B^{\text{total}}}{K} \rceil$  bits under conventional scheme.

Fig. 1 depicts the system sum rate of the proposed feedback scheme with Monte Carlo and analytical lower bound derived in Section IV. As a comparison, the Monte Carlo result of system sum rate with perfect downlink CSI is also provided. Fig. 1 shows that the analytical lower bound can reflect the change of system sum rate although it is derived only based on channel covariance and does not rely on codebook design.

The performance comparison of conventional and the proposed feedback schemes is provided in Fig. 2. It is shown

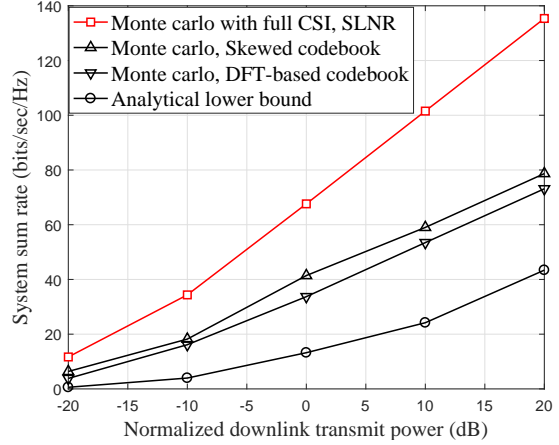


Fig. 1. Performances comparison of Monte Carlo and the analytical lower bound results with  $M = 128, K = 10$  and  $B^{\text{total}} = 40$  bits.

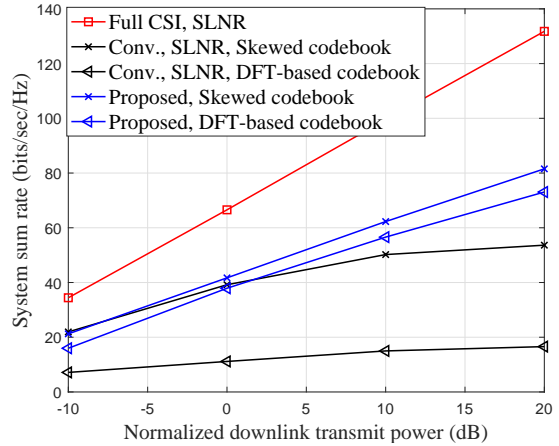


Fig. 2. System sum rate comparison versus downlink transmit power when  $M = 128, K = 10$  and  $B^{\text{total}} = 40$  bits.

that the proposed feedback scheme outperforms the conventional one, especially when DFT-based codebook is used. The skewed codebook outperforms DFT-based codebook due to the consideration of channel statistics. Moreover, the conventional scheme can only obtain marginal performance gain in high transmit power regime, while the performance of the proposed feedback scheme keeps rising with transmit power increasing.

It is shown from Fig. 3 that the performance of conventional scheme with DFT-based codebook deteriorates sharply with  $K$  increasing, and the performance with skewed codebook is also restricted. However, the performance of the proposed feedback scheme keeps increasing. When  $K = 20$ , the system sum rate under the proposed feedback scheme is 20 times larger than the conventional one with DFT-based codebook and 1.8 times larger with skewed codebook.

Fig. 4 shows that the proposed feedback scheme achieves much better performance than the conventional one, especially when total feedback bits are very limited, i.e., 10 feedback bits for 20 users. With  $B^{\text{total}}$  increasing, the performance of the

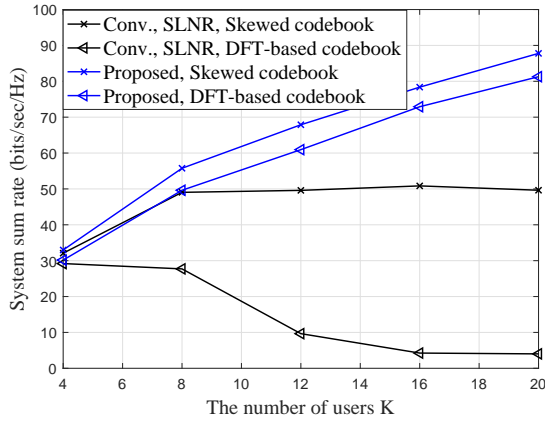


Fig. 3. System sum rate versus the number of users when  $M = 128$ ,  $p_d = 10$  dB and  $B^{\text{total}} = 40$  bits.

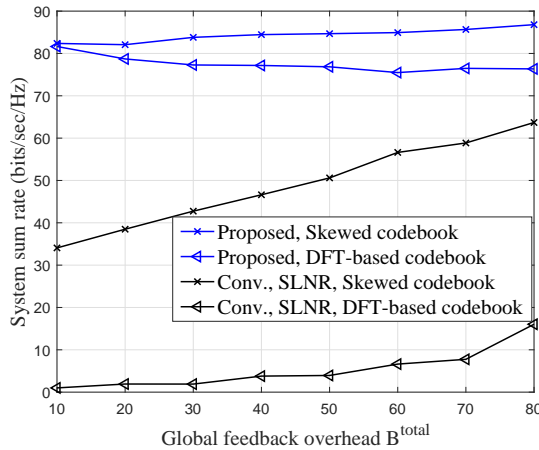


Fig. 4. System sum rate versus global feedback overhead when  $M = 128$ ,  $K = 20$  and  $p_d = 10$  dB.

proposed scheme with skewed codebook keep rising and the performance with DFT-based codebook slightly decreases.

## VI. CONCLUSIONS

In this paper, a hybrid statistical-instantaneous feedback scheme was proposed, where only class-I users share the feedback bit budget evenly and feed the quantized instantaneous channel back to BS. We proposed a SLNR-based precoding method capable of handling the mixed statistical CSI of class-S users and quantized channel of class-I users. Then, an analytical lower bound of sum rate was derived and a user classification algorithm was proposed with the purpose to maximize sum rate. Finally, simulations verify the performance gain compared to the conventional feedback schemes, especially under limited feedback bit budget.

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