Reduced-Order Zero-Forcing Beamforming vs Optimal Beamforming and Dirty Paper Coding and Massive MIMO Analysis

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***Abstract*—Optimal linear transmitter beamformers in multi-antenna multi-user systems are of the Minimum Mean Squared Error (MMSE) type (dual uplink MMSE receivers). MMSE designs make an optimal compromise between noise enhancement and interference suppression and reduce to matched filters at low SNR and zero-forcing at high SNR. We consider a realistic scenario of user channels of varying attenuation and constrain the beamformers to either zero-force or ignore each interference term. This leads to a reduced-order zero-forcing (RO-ZF) design in which the number of interference sources being zero-forced increases with SNR. We apply a simple large systems analysis (applicable to Massive MIMO) to determine the asymptotic performance of RO-ZF designs, determine the optimal ZF orders, and compare to optimal and ZF linear and Dirty Paper Coding (DPC) designs. RO-ZF designs lead to variable reductions of computational complexity and channel state information (CSI) requirements (esp. in future multi-cell extensions), both important considerations in Massive MIMO systems.**

***Keywords*—** Zero-forcing beamforming (ZF BF), massive MIMO, Dirty Paper Coding (DPC), large system analysis, complexity reduction.

I. INTRODUCTION

In this paper, Tx may denote transmit/transmitter/ transmission and Rx may denote receive/receiver/reception. Massive MIMO [1] which utilizes large number of antennas at the base station (BS) offers immense possibilities for increased system capacity. Multi-user MIMO (MU-MIMO) systems requires the global knowledge of the CSI at the Tx (CSIT) which is more difficult to acquire than CSI at the Rx. However, this leads to increased computational complexity owing to the large number of antennas. Recently, a number of research works have proposed to exploit the channel hardening in Massive MIMO (MaMIMO) to reduce global instantaneous CSIT requirements to local instantaneous CSIT plus global statistical CSIT [2]. Channel hardening occurs when the number of antennas at the BS are very high such that a fading channel behaves as if the effect of the randomness in the channel to spectral efficiency will be negligible. Extensive work on BF designs for BC (broadcast channel) or IBC (Interfering BC) with perfect or partial CSIT can be found in [3]–[7].

A significant contribution for large system analysis in MaMIMO systems appeared in [8]. It allows to compute deterministic (instead of fast fading channel dependent) expressions for various scalar quantities, facilitating the analysis and design of wireless systems. E.g. it may allow to conduct the performance analysis without computing explicit beamformers. Through large system analysis, [8] compute the optimal regularization factor in Regularized ZF (R-ZF) BF, both with perfect and partial CSIT. A little known extension appeared in [9] for weighted Sum MSE (WSMSE) based optimal beamformers, but only for the perfect CSIT MISO (Multiple-Input Single-Output) BC case. Some other extensions appeared recently in [10] where MISO IBC is considered with perfect CSIT and weighted R-ZF BF, with two optimized weight levels, for intracell or intercell interference. [11] considers the large system analysis of the MIMO IBC with optimized BF under partial CSIT. [12] studied the energy consumption dynamics in a MISO BC with users moving around according to a random walk model.

A. Contributions of this paper

In this paper:

- We introduce the concept of reduced-order ZF BF and propose a greedy approach to optimize the reduced ZF orders.
- We propose a large system analysis for optimal BF and DPC with omnidirectional but differently attenuated user channels.
- We consider a novel simple large system analysis for ZF BF or DPC transmitters with omnidirectional channel covariances.
- We illustrate with numerical evaluations the complexity-performance tradeoff that RO-ZF permits.

Notation: In the following, boldface lower-case and upper-case characters denote vectors and matrices respectively. the operators $\mathrm{E}(\cdot)$, tr$(\cdot)$, $(\cdot)^H$, $(\cdot)^T$ represents expectation, trace, conjugate transpose and transpose respectively. A circularly complex Gaussian random vector with mean μ and covariance matrix Θ is distributed as $x \sim \mathcal{CN}(\mu, \Theta)$. $\mathbf{I}_N$ represents the $N \times N$ identity matrix.

II. MULTI-USER MIMO SYSTEM MODEL

Consider a transmitter (BS) equipped with M antennas communicating with K single antenna users (MISO BC). Furthermore, under narrowband transmission, the received signal at user k can be written as,

$$y_k = \mathbf{h}_k^H \mathbf{x} + n_k, \;\; k = 1, 2, \;\; ..., \;\; K, \tag{1}$$

where $\mathbf{h}_k \in \mathcal{C}^M$ is the downlink channel between user k and BS, $\mathbf{x} \in \mathcal{C}^M$ is the transmit vector and the noise terms $n_k \in \mathcal{CN}(0, \sigma^2)$ are independent. The channel covariance matrix is defined as $\mathbf{\Theta}_k$ and thus correlated channel model can be written as, $\mathbf{h}_k = \sqrt{M}\mathbf{\Theta}_k^{1/2}\mathbf{z}_k$, where $\mathbf{z}_k$ has i.i.d complex entries of zero mean and variance $1/M$ and $\mathbf{\Theta}_k^{1/2}$ is any Hermitian square root of $\mathbf{\Theta}_k$. The correlation matrix $\mathbf{\Theta}_k$ is non-negative Hermitian and of uniformly bounded spectral norm w.r.t. M. The transmit signal $\mathbf{x}$ can be written as, $\mathbf{x} = \sum_{i=1}^{K} \mathbf{g}_i s_i$, where $\mathbf{g}_k \in \mathcal{C}^M$ represents the transmit precoder matrix for user k and s_i is the i^{th} user symbol, with $s_i \sim \mathcal{CN}(0,1)$. The transmit power constraint can be written as, $\mathrm{E}(\mathbf{x}^H\mathbf{x}) = \mathrm{tr}(\sum_{i=1}^{K} \mathbf{g}_i\mathbf{g}_i^H) \leq P$. Under optimal single user decoding, the user rate can be defined as, $R_k = \log(1+\gamma_k)$, where the signal to interference plus noise ratio (SINR), γ_k is defined as,

$$\gamma_k = \frac{|\mathbf{h}_k^H\mathbf{g}_k|^2}{\sum_{i=1, i\neq k}^{K} |\mathbf{h}_k^H\mathbf{g}_i|^2 + \sigma^2}. \tag{2}$$

The transmit SNR is defined as $\rho = \frac{P}{\sigma^2}$ and $\beta = \frac{K}{M}$. In the large system limit, we assume that $M, K \rightarrow \infty$ at a fixed ratio $\beta < 1$. Further we assume that the channel covariance matrices are represented by multiple of identity, $\mathbf{\Theta}_k = \frac{\theta_k}{M}\mathbf{I}$, with different user channel covariance matrices differentiated by the varying attenuation factor θ_k. Multiple of identity covariance structure reflects the fact that the user subspaces are randomly oriented even though we don't assume the knowledge of subspaces. Further it helps to analytically evaluate the RO-ZF BF and compare it to optimal BF. Moreover, we define the ordering of the multiple of identity for the covariance matrices as, $\theta_1 \geq \theta_2 \geq, ..., \geq \theta_K$, which means user 1 represents the strongest user and K is the weakest user.

III. Large System Analysis of Optimal BF-WSMSE

In this section, we refer to the iterative algorithm in [13] for the optimal linear transmit BF and superscript (j) refers to the iteration stage j. We simplify the large system analysis results of the optimal BF in [9] for the case of multiple of identity covariance matrices for the user channels and the result is stated below. In the following sections, we denote $(x^2)^{(j)} = (x^{(j)})^2$.

Theorem 1. *Let $\gamma_k^{(j)\,opt-WSMSE}$ be the SINR of user k* (2) *under optimal linear precoding, i.e., at the end of iteration j, $\mathbf{g}_k^{(j)} = \sqrt{\frac{P}{\psi^{(j)}}}\left(\mathbf{H}\mathbf{D}^{(j)}\mathbf{H}^H + \alpha^{(j)}\mathbf{I}\right)^{-1}\mathbf{h}_k a_k^{(j)} w_k^{(j)}$, a_k is the MMSE Rx filter, w_k is the MSE weight for user k, $\psi^{(j)}$ being the normalization constant and $\alpha^{(j)} = \frac{tr(\mathbf{D}^{(j)})}{\rho}$ with the $(k,k)^{th}$ element of the diagonal matrix $\mathbf{D}^{(j)}, d_k^{(j)} = (a_k^2)^{(j)} w_k^{(j)}$. $\mathbf{H}$ represents the channel matrix of all users, $\mathbf{H} = [\mathbf{h}_1, ..., \mathbf{h}_K]$. Then $\gamma_k^{(j)\,opt-WSMSE} - \overline{\gamma}_k^{(j)\,opt-WSMSE} \xrightarrow{M\rightarrow\infty} 0$, almost surely, where,*

$$\overline{\gamma}_k^{(j)\,opt-WSMSE} = \frac{\theta_k^2 \overline{w}_k^{(j)} (e^2)^{(j)}}{\Upsilon_k^{(j)} + \frac{\sigma^2 \overline{d}_k^{(j)} \overline{\psi}^{(j)}}{P}(1+\overline{d}_k^{(j)}\theta_k e^{(j)})^2}, \tag{3}$$

where $\overline{w}_k^{(j)}, \overline{d}_k^{(j)}, \overline{\psi}^{(j)}$ represent the deterministic equivalents for $w_k^{(j)}, d_k^{(j)}, \psi^{(j)}$ respectively, the expressions of which are given below. Further we can show that, since the logarithm is a continuous function, by applying the continuous mapping theorem [14], it follows from the almost sure convergence of $\gamma_k^{(j)\,opt-WSMSE}$ that, $R_k^{(j)} - \overline{R}_k^{(j)} \xrightarrow[a.s]{M\rightarrow\infty} 0$, where $R_k^{(j)}$ is the rate of user k, with $\overline{R}_k^{(j)} = \ln(1+\overline{\gamma}_k^{(j)\,opt-WSMSE})$.

Normalization term: A deterministic equivalent $\overline{\psi}^{(j)}$ such that $\psi^{(j)} - \overline{\psi}^{(j)} \xrightarrow{M\rightarrow\infty} 0$, almost surely, is given by

$$\overline{\psi}^{(j)} = \frac{1}{M}\sum_{k=1}^{K} \overline{w}_k^{(j)} \frac{\overline{d}_k^{(j)}\theta_k e^{(j)\prime}}{(1+\overline{d}_k^{(j)}\theta_k e^{(j)})^2}, \tag{4}$$

Using theorem 1 [8], $e^{(j)}$is given as the unique positive solution of the following equation,

$$e^{(j)} = (\sum_{i=1}^{K} \frac{\overline{d}_i^{(j)}\theta_i}{1+\overline{d}_i^{(j)}\theta_i e^{(j)}} + \alpha^{(j)})^{-1}. \tag{5}$$

$e^{(j)\prime}$, the derivative of $e^{(j)}$ w.r.t $-\alpha^{(j)}$, is obtained as,

$$e^{(j)\prime} = \frac{(e^2)^{(j)}}{1-(e^2)^{(j)}\sum_{i=1}^{K} \frac{(\overline{d}_i^2)^{(j)}\theta_i^2}{(1+\overline{d}_i^{(j)}\theta_i e^{(j)})^2}}. \tag{6}$$

Signal Power: A deterministic equivalent for the square root of the signal power, $\sqrt{\overline{P}_{S,k}^{(j)}}$ gets simplified as,

$$\sqrt{\overline{P}_{S,k}^{(j)}} = \sqrt{\frac{P}{\overline{\psi}^{(j)}}} \frac{\overline{d}_k^{(j)}\theta_k e^{(j)}}{\overline{a}_k^{(j)}(1+\overline{d}_k^{(j)}\theta_k e^{(j)})}. \tag{7}$$

Interference Power: Following [8], [9], the deterministic equivalent for the interference power can be obtained as,

$$\sum_{i=1,\neq k}^{K} \mathbf{h}_k^H \mathbf{g}_i^{(j)} \mathbf{g}_i^{(j)\,H}\mathbf{h}_k = \frac{P}{\overline{d}_k^{(j)}\overline{\psi}^{(j)}} \frac{\Upsilon_k^{(j)}}{(1+\overline{d}_k^{(j)}\theta_k e^{(j)})^2},$$
$$\text{where,}\quad \Upsilon_k^{(j)} = \frac{1}{M}\sum_{i=1,i\neq k}^{K} \overline{w}_i^{(j)} \frac{\overline{d}_i^{(j)}\theta_i e^{(j)\prime}}{(1+\overline{d}_i^{(j)}\theta_i e^{(j)})^2}. \tag{8}$$

Substituting the signal and interference powers, the deterministic equivalent of the SINR leads to (3).The deterministic equivalents for the $a_k^{(j)}, w_k^{(j)}, d_k^{(j)}$ are given by [9], $\overline{a}_k^{(j)} = \frac{\sigma}{\sqrt{\overline{P}_{S,k}^{(j-1)}}}\frac{\overline{\gamma}_k^{(j-1)}}{1+\overline{\gamma}_k^{(j-1)}}$, $\overline{w}_k^{(j)} = u_k(1+\overline{\gamma}_k^{(j-1)})$, and $\overline{d}_k^{(j)} = (\overline{a}_k^2)^{(j)}\overline{w}_k^{(j)}$.

IV. Large System Analysis of Optimal DPC

The received signal at user k with DPC [15] (which achieves the capacity region of MIMO BC) at the BS is

$$y_k = \underbrace{\mathbf{h}_k^H \, \mathbf{g}_k \, s_k}_{\text{signal}} + \underbrace{\sum_{i=k+1}^{K} \mathbf{h}_k^H \, \mathbf{g}_i \, s_i}_{\text{interf. from weaker users}} + \mathbf{n}_k. \quad (9)$$

In optimal DPC, users are ordered in decreasing strength, as in RO-ZF. The interference that a user will cause to weaker users gets canceled non-linearly at the Tx (in other words, in the Rx SINR it does not need to be considered), and the BF handles only interference to stronger users. As usual, optimal BF does something in between ZF and matched filter (MF). So there will be residual interference at the stronger users.

Let γ_k^{DPC} be the SINR of user k under optimal DPC, i.e., at the end of iteration j, $\gamma_k^{(j)\,DPC} - \overline{\gamma}_k^{(j)\,DPC} \xrightarrow{M\to\infty} 0$, almost surely, where, the expression for $\overline{\gamma}_k^{(j)\,DPC}$ is same as (3). However, the expressions for each of the scalars got modified as, $e^{(j)} = (\sum_{i=k}^{K} \frac{\overline{d}_i^{(j)}\theta_i}{1+\overline{d}_i^{(j)}\theta_i e^{(j)}} + \alpha^{(j)})^{-1}$, $\Upsilon_k^{(j)} = \frac{1}{M}\sum_{i=k+1}^{K} \overline{w}_i^{(j)} \frac{\overline{d}_i^{(j)}\theta_i e^{(j)\prime}}{(1+\overline{d}_i^{(j)}\theta_i e^{(j)})^2}$. Note that the only change compared to the optimal WSMSE BF is that each summation term get replaced from k to K or $k+1$ to K.

V. Reduced Order ZF

In this section, we consider the BF to be a reduced order ZF (RO-ZF). This can be interpreted as the number of interfering channels to be zero-forced for a user k is much less than K. The RO-ZF BF $\mathbf{g}_k$ can be written as, $\mathbf{g}_k = \frac{\mathbf{P}^{\perp}_{\mathbf{H}_{I_k}} \mathbf{h}_k}{||\mathbf{P}^{\perp}_{\mathbf{H}_{I_k}} \mathbf{h}_k||}$. Here, $\mathbf{P}_H = \mathbf{H}(\mathbf{H}^H\mathbf{H})^{\#}\mathbf{H}^H$ represent the projection onto the column space of $\mathbf{H}$, $\mathbf{P}^{\perp}_{\mathbf{H}} = \mathbf{I} - \mathbf{P}_H$ is the projection onto its orthogonal complement (# represents the Moore-Penrose pseudo-inverse). For the convenience of analysis, we define the following: K_k represents the strongest interfering channel zero-forced by the BF of user k and I_k denotes the set of user indices for which the ZF is done. $\mathbf{H}_{I_k}$ represents the matrix of all the user channels in I_k. Complexity in the RO-ZF case will be about half of that of full ZF (multiplying the $M \times K$ $\mathbf{H}$ by a triangular $K \times K$ instead of a full $K \times K$, computation of the $K \times K$ inverse or triangular factor takes $O(K^3)$ operations, with a smaller factor if only a triangular factor is needed and not a full inverse).

VI. Large System Analysis for RO-ZF, Full Order ZF and ZF-DPC

In this section we consider the large system analysis for the RO-ZF scheme proposed in this paper and also the full order ZF (full order means $|I_k| = K-1, \forall k$). In this section, we split $\mathbf{g}_k = \sqrt{p_k}\mathbf{g}'_k$, where p_k is the power allocated to user k.

$$\begin{aligned} &\gamma_k^{RO-ZF} = \tfrac{P_{S,k}}{P_{I,k}+\sigma_k^2} = \frac{p_k|\mathbf{h}_k^H\mathbf{g}'_k|^2}{\sum\limits_{i=1,i\neq k}^{K} p_i|\mathbf{h}_k^H\mathbf{g}'_i|^2 + \sigma_k^2}, \\ &\mathbf{g}'_k = \frac{\mathbf{P}^{\perp}_{\mathbf{H}_{I_k}}\mathbf{h}_k}{||\mathbf{P}^{\perp}_{\mathbf{H}_{I_k}}\mathbf{h}_k||} \implies \mathbf{h}_k^H\mathbf{g}'_k = \left\|\mathbf{P}^{\perp}_{\mathbf{H}_{I_k}}\mathbf{h}_k\right\|. \end{aligned} \quad (10)$$

Further, by the law of large numbers, $P_{S,k} - \overline{P}_{S,k} \xrightarrow[a.s]{M\to\infty} 0$, where,

$$\begin{aligned} \overline{P}_{S,k} &= \mathrm{E}(|\mathbf{h}_k^H\mathbf{g}'_k|^2) = \mathrm{E}_{\mathbf{H}_{I_k}}\mathrm{E}_{\mathbf{h}_k}\mathrm{tr}(\mathbf{P}^{\perp}_{\mathbf{H}_{I_k}}\mathbf{h}_k\mathbf{h}_k^H) \\ &= \tfrac{\theta_k}{M}\mathrm{tr}(\mathbf{I}_M - \mathbf{H}_{I_k}(\mathbf{H}_{I_k}^H\mathbf{H}_{I_k})^{\#}\mathbf{H}_{I_k}^H) = \theta_k(1-\tfrac{|I_k|}{M}), \end{aligned} \quad (11)$$

where we use the property of the projection matrices that $\mathbf{P}^{\perp}_{\mathbf{H}_{I_k}}\mathbf{P}^{\perp}_{\mathbf{H}_{I_k}} = \mathbf{P}^{\perp}_{\mathbf{H}_{I_k}}$. Next, we consider the terms in $P_{I,k}$,

$$|\mathbf{h}_k^H\mathbf{g}'_i|^2 = \frac{|\mathbf{h}_k^H\mathbf{P}^{\perp}_{\mathbf{H}_{I_i}}\mathbf{h}_i|^2}{\left\|\mathbf{P}^{\perp}_{\mathbf{H}_{I_i}}\mathbf{h}_i\right\|^2}. \quad (12)$$

If $k \in I_i$, then $|\mathbf{h}_k^H\mathbf{g}'_i|^2 = 0$, else, $\mathrm{E}(|\mathbf{h}_k^H\mathbf{P}^{\perp}_{\mathbf{H}_{I_i}}\mathbf{h}_i|^2) = \mathrm{E}(\mathrm{tr}(\mathbf{P}^{\perp}_{\mathbf{H}_{I_i}}\mathbf{h}_i\mathbf{h}_i^H\mathbf{P}^{\perp}_{\mathbf{H}_{I_i}}\mathbf{h}_k\mathbf{h}_k^H)) = \frac{\theta_k\theta_i}{M^2}\mathrm{tr}(\mathbf{P}^{\perp}_{\mathbf{H}_{I_i}}) = \frac{\theta_k\theta_i}{M^2}\mathrm{tr}(\mathbf{I}_M - \mathbf{H}_{I_i}(\mathbf{H}_{I_i}^H\mathbf{H}_{I_i})^{\#}\mathbf{H}_{I_i}^H) = \frac{\theta_k\theta_i}{M}(1-\frac{|I_i|}{M})$, Finally we obtain $\mathrm{E}(|\mathbf{h}_k^H\mathbf{g}'_i|^2) = \frac{\frac{\theta_k\theta_i}{M}(1-\frac{|I_i|}{M})}{\theta_i(1-\frac{|I_i|}{M})} = \frac{\theta_k}{M}$. Further, we get the deterministic equivalent of the SINR in the large system limit as,

$$\overline{\gamma}_k^{RO-ZF} = \frac{p_k\theta_k}{\frac{1}{M}\theta_k\sum\limits_{i=1,k\notin I_i}^{K} p_i + \sigma^2}\left(1-\frac{|I_k|}{M}\right). \quad (13)$$

For the full order ZF, the interference power vanishes from the SINR terms,

$$\overline{\gamma}_k^{ZF} = \frac{p_k\theta_k}{\sigma^2}\left(1-\frac{K-1}{M}\right). \quad (14)$$

ZF-DPC combines zero-forcing and DPC technique. While DPC cancels the interference for users $i < k$, the interference of users $i > k$ are eliminated by designing the BF $\mathbf{g}_i$ such that $\mathbf{h}_k^H\mathbf{g}_i = 0$.The large system analysis for the ZF-DPC ($|I_k| = k-1$) is as follows : We define $J_k = \{1, 2, ..., k-1\}$.

$$\begin{aligned} &\gamma_k^{ZF-DPC} = \tfrac{P_{S,k}}{P_{I,k}+\sigma_k^2} = \tfrac{p_k|\mathbf{h}_k^H\mathbf{g}'_k|^2}{\sigma_k^2}, \ \text{ since, } \ P_{I,k} = 0, \\ &\mathbf{g}'_k = \tfrac{\mathbf{P}^{\perp}_{\mathbf{H}_{J_k}}\mathbf{h}_k}{||\mathbf{P}^{\perp}_{\mathbf{H}_{J_k}}\mathbf{h}_k||}, \implies \mathbf{h}_k^H\mathbf{g}'_k = \left\|\mathbf{P}^{\perp}_{\mathbf{H}_{J_k}}\mathbf{h}_k\right\|, \\ &\overline{P}_{S,k} = \mathrm{E}(|\mathbf{h}_k^H\mathbf{g}'_k|^2) = \mathrm{E}_{\mathbf{H}_{J_k}}\mathrm{E}_{\mathbf{h}_k}\mathrm{tr}(\mathbf{P}^{\perp}_{\mathbf{H}_{J_k}}\mathbf{h}_k\mathbf{h}_k^H) \\ &= \tfrac{\theta_k}{M}\mathrm{tr}(\mathbf{I}_M - \mathbf{H}_{J_k}(\mathbf{H}_{J_k}\mathbf{H}_{J_k}^H)^{\#}\mathbf{H}_{J_k}) = \theta_k(1-\tfrac{k-1}{M}). \end{aligned} \quad (15)$$

Therefore, the deterministic equivalent of the SINR becomes,

$$\overline{\gamma}_k^{ZF-DPC} = \frac{p_k\theta_k}{\sigma^2}\left(1-\frac{k-1}{M}\right). \quad (16)$$

A. Optimization of user powers p_k

We consider here the approximation of the WSR according to the difference of convex (DC) functions approach as in [16]. Solving DC, we get the Lagrangian for the WSR,

$$\begin{aligned} &WSR(\mathbf{g},\lambda) = \lambda P + \sum_{k=1}^{K} u_k \ln\det(1 + \mathbf{g}_k^H\mathbf{B}_k\mathbf{g}_k) \\ &-\mathbf{g}_k^H(\mathbf{A}_k + \lambda\mathbf{I})\mathbf{g}_k, \ \text{ where, } \ \mathbf{B}_k = \mathbf{h}_k r_{\overline{k}}^{-1}\mathbf{h}_k^H, \\ &\mathbf{A}_k = \sum_{i\neq k, i\notin I_k}^{K} u_i\mathbf{h}_i(r_{\overline{i}}^{-1} - r_i^{-1})\mathbf{h}_i^H, \\ &r_{\overline{k}} = \sum_{i=1,i\neq k,k\notin I_i}^{K} |\mathbf{h}_k^H\mathbf{g}_i|^2 + \sigma^2, \ \ r_k = r_{\overline{k}} + |\mathbf{h}_k^H\mathbf{g}_k|^2. \end{aligned} \quad (17)$$

Here $\mathbf{g}$ represents the set of BFs $\mathbf{g}_k$. Let $\sigma_k^{(1)} = \mathbf{g}_k'^H \mathbf{B}_k \mathbf{g}_k'$ and $\sigma_k^{(2)} = \mathbf{g}_k'^H \mathbf{A}_k \mathbf{g}_k'$. For full order ZF, $\mathbf{A}_k = 0$, thus $\sigma_k^{(2)} = 0$ and (17) reduces to standard waterfilling. The advantage of formulation (17) is that it allows straightforward power adaptation: introducing stream powers $p_k \geq 0$ and substituting $\mathbf{g}_k = \mathbf{g}_k' \sqrt{p_k}$ in (17) yields

$$WSR(\mathbf{P}, \lambda) = \lambda P + \sum_{k=1}^{K} [u_k \ln(1 + p_k \sigma_k^{(1)}) - \mathrm{tr}(p_k(\sigma_k^{(2)} + \lambda))], \quad (18)$$

where $\mathbf{P}$ represents the set of powers p_k. Since this is a concave function w.r.t p_k, taking the derivative leads to the following interference leakage aware water filling (WF) (jointly for the p_k and λ)

$$p_k = \left(u_k(\sigma_k^{(2)} + \lambda)^{-1} - \sigma_k^{-(1)}\right)^+, \; \sum_k p_k = P, \quad (19)$$

where the Lagrange multiplier is adjusted to satisfy the power constraints. This can be done by bisection.

VII. Optimization of the ZF Order

In this section, we consider an alternating optimization algorithm (Algorithm 1) which computes the reduced ZF order for each user (I_k, K_k). In Algorithm 1, the text "if $u_k R_k + u_{K_k'} R_{K_k'}$ is increased" is meant to be understood "by adding the ZF from k to K_k'". In Algorithm 1, we consider the ordering for the case of, $I_k = \{K_k, K_k+1, ..., K_k + |I_k| - 1\}$ if $k < K_k$, else $I_k = \{K_k, K_k+1, ..., K_k + |I_k|\}$. $|I_k|$ represents the cardinality of the set I_k. Also, $\mathbf{H}_{I_k} = [\mathbf{h}_{K_k}, ..., \mathbf{h}_{K_k+|I_k|-1}]$ or $\mathbf{H}_{I_k} = [\mathbf{h}_{K_k}, ..., \mathbf{h}_{K_k+|I_k|}]$. Note that at finite dimension MIMO, not only the channel strengths but also the relative orientation of the channel vectors count. However, in MaMIMO with multiple of identity covariances, there is no orientation issue, only the channel strengths count. So the user ordering is simple.

Algorithm 1 Reduced Zero-Forcing Order Determination

Given: $K, M, \sigma^2, \theta_i, \forall i$, with ordering $\theta_1 \geq \theta_2 \geq ... \geq \theta_K$.
Initialization: Start with $K_k = k+1, \forall k = 1, ..., K-1$ and for user K, $|I_K| = 0$.

for $k = 1 : K$
  $K_k' = K_k$.
  while $(K_k' > 1)$
    $K_k' = K_k' - 1$.
    if$(K_k' \neq k)$
      if $u_k R_k + u_{K_k'} R_{K_k'}$ is increased
        $K_k = K_k'$, else exit while loop
    else end if
  end while
  $K_k' = K_k$.
  while $(K_k' < K)$
    $K_k' = K_k' + 1$.
    if$(K_k' \neq k)$
      if $u_k R_k + u_{K_k'} R_{K_k'}$ is increased
        $K_k = K_k'$, else exit while loop
    else end if
  end while
end for
Continue until convergence of $I_k, \forall k$.

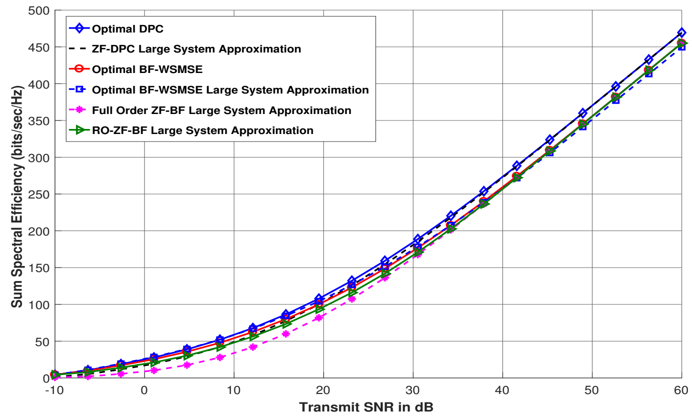

Fig. 1. Sum rate comparison for $M = 64, K = 30$.

VIII. Simulation Results

In this section we illustrate the simulation results to validate our theoretical results. We compare the sum rate performance of RO-ZF BF scheme (which has the least complexity) to the optimal BF-WSMSE [13], optimal DPC and to the large system approximations of optimal BF-WSMSE, full order ZF and the ZF DPC. For the SNR ranges of interest, it can be seen that RO-ZF performs close to the optimal schemes with much lower complexity. Figure 2 illustrates the sum rate difference of the various BF designs from the optimal DPC.

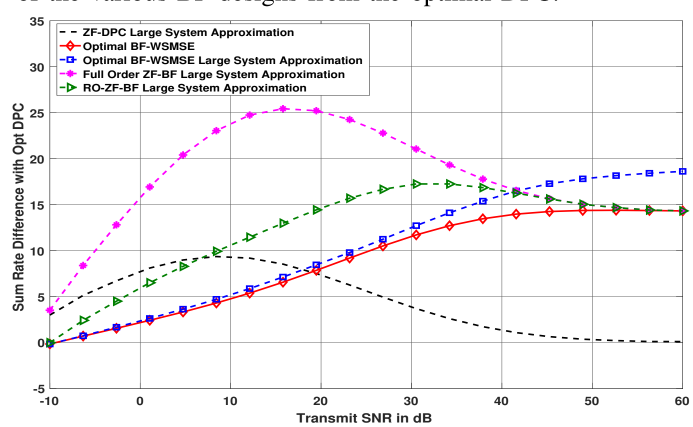

Fig. 2. Sub-optimality compared to Optimal DPC for $M = 64, K = 30$.

IX. Conclusion

In this paper, we investigate the performance-complexity tradeoffs for the reduced order ZF BF. We propose the large system analysis for the RO-ZF BF, optimal BF, optimal DPC, ZF-DPC and full order ZF for the case of omnidirectional but differently attenuated user channels. Simulation results indicate that our RO-ZF BF scheme has a performance very close to the optimal BFs such as WSMSE and DPC, but with much lesser complexity compared to the full order ZF. We also propose an alternating optimization algorithm which computes the optimal ZF order for each user.

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