Abstract—Coordinated vehicle control strategies aim at optimizing driving dynamics to increase traffic flow without impacting safety. These control strategies are based on the knowledge of the vehicles’ state information like position and velocity obtained through Vehicle-to-everything (V2X) communications. Literature on control strategies yet assumes perfect positions, whereas position errors are in fact present and non negligible (e.g. GPS). As a consequence, these localization errors impact the control strategies by introducing uncertainty, which must be accounted for to minimize the probability of accidents.

This paper qualifies and quantifies such uncertainty and proposes strategies to reduce it in a collision avoidance scenario. We notably relate these strategies to their impacts on traffic flow. More specifically, we model coordinated automated vehicles as a Model Predictive Control (MPC), integrate localization errors and evaluate its impact of the output to avoid accident. We then propose possibilities to mitigate accident-prone controls and quantify them on traffic flow. Our study illustrates that localization errors impact traffic flow by forcing future automated vehicles to increase gaps or reduce speed.

I. INTRODUCTION

Location-based services are a cornerstone of future Cooperative ITS (C-ITS). All traffic safety and efficiency applications defined either in ETSI ITS, SAE or ISO are based on the knowledge of positioning information. Although GNSS data is the primary source of position information, high definition maps and on-board sensors are expected to be critical complementary technologies. Position information will play an even more critical role in future automated vehicle systems due to the implementation of real-time control strategies. Vehicles do not only need to be located roughly on a street but rather exactly on a lane. It has yet been shown through recent European projects (HIGHTS1, TIMON2) that GNSS-only systems may not provide the precision required by future automated vehicle system.

Automated vehicles systems may be classified in two categories. Vehicles with cruise control capabilities are known as Adaptive Cruise Control (ACC) vehicles, whereas ACC vehicles with additionally V2X communications (ITS-G5, LTE-V2X) are known as Cooperative ACC (CACC) vehicles. ACC/CACC vehicles not only promise to increase road safety but also road capacity (increased flow). A detail survey on control strategies for automated (ACC/CACC) vehicles may be found in [1]. Signalized intersection clearance using centralized control algorithm proposed in [2] achieves a throughput increase of up to 11% for different penetration levels of CACC vehicles. Roundabout clearance strategies on automated (ACC and CACC) vehicles achieve a throughput of around 3600 veh/hr [3]. Decentralized control algorithms minimizing energy loss while clearing an intersection achieved a throughput of 9000 veh/hr [4]. Shladover et al. [5] further showed that a lane capacity increase from 2200 veh/hr to 4000 veh/hr may be achieved in a CACC-only traffic. Nevertheless, according to van Arem et al. [6] penetration of CACC vehicles below 60% will not affect flow capacity.

These described benefits of ACC/CACC vehicles on the flow capacity relates either to a decrease in inter vehicular distance or to an increase in speed. Such benefits may only be achieved if controllers of automated vehicles have perfect knowledge of the environment (e.g. state information of other vehicles). Unfortunately, state information is error prone. In particular, localization is known to have non-negligible errors, and the previously described state-of-art on coordinated control strategies (e.g. [1]) assume perfect localization. Patel et al. [7] showed that unaccounted localization errors in automated vehicle controllers potentially lead to accidents. How should control algorithms behave and account for localization errors when they are non negligible? Accordingly, under localization errors, can automated vehicles truly reach the promised flow capacity increase? How would the situation evolve in mixed traffic scenarios? These are the questions we address in this paper.

The goal of the paper is to ensure vehicles operate such that collision avoidance uncertainty is low despite the presence of localization errors. Our contributions are as follows: (i) we propose a centralized control system based on a Model-Predictive Controller (MPC) integrating a localization error model; through extensive simulations, we qualify and quantify the impact of localization errors on pure automated traffic; (ii) we propose the concept of Operational Point (OP) for MPC and observe the relationship between flow capacity and collision avoidance uncertainty. We show how the shift of OP can reduce uncertainty in collision avoidance; (iii) we evaluate the impact of localization error on mixed traffic system. In a nutshell, we show that automated vehicles will require to either reduce speed or increase inter-distance to mitigate the impact of localization errors.

The rest of this paper is organized as follows: Section II describes the proposed MPC strategy including localization errors. Section III evaluates the impact of localization errors,
suggest options to mitigate them and evaluate them on their impact on traffic flows. Towards the end of the paper we draw some conclusions in Section IV.

II. AUTOMATED VEHICLES COORDINATION WITH LOCALIZATION ERRORS

A. System Description

As illustrated in Fig. 1, we assume a highway scenario, where multiple vehicles with imperfect localization are following one another in a coordinated mobility. If the first vehicle has to brake on sensing an obstacle ahead, the following vehicles would need to brake as well in a coordinated way. We consider a centralized control system like a Cloud or Edge Service, located either in the Cloud or in ITS-G5 or Cellular infrastructures. All vehicles (CACC and Manually Driven Vehicles (MDV)) are connected and transmit their position, speed and acceleration estimates to the centralized controller either via Cellular or DSRC communication systems [5]. This information is used by the centralized controller to compute control inputs, which are subsequently sent back to only CACC vehicles for collision avoidance. The red ellipses in Fig. 1 depicts the CACC and MDVs’ localization errors impacting the centralized controller.

In such a scenario, we quantify the impact of localization errors on the uncertainty of collision avoidance systems for future automated vehicles (e.g. CACC). We propose a method to alter dynamics (e.g. velocity or intervehicular distance) to mitigate such uncertainty.

B. Localization Errors

Fig. 2 depicts our modeling of localization errors. In this figure, vehicle \( i \) has a perceived location \( p^*_{i} \) different from its true location \( p_{i} \) according to a localization error \( e_{i} \). The perceived and true locations of the vehicle are denoted by blue and green blocks respectively. The vehicle can be located anywhere within a circle centered at the perceived location \( p^*_{i} \) with radius equal to \( e_{i} \). As a vehicle position is usually computed in 2D, the localization error is also in 2D. But without loss of generality, we consider in this paper longitudinal motion only of vehicles on a single lane to simplify our study. Thus, we transform this 2D scenario to a 1D scenario (as shown in the bottom part of the Fig. 2). The vehicle can be located anywhere between \( p_{i,1} \) and \( p_{i,2} \).

We assume that a vehicle position refers to its front bumper and that the true occupied road length corresponds to the vehicular length \( l_{i} \) from front to rear bumpers. The potential area, where the vehicle may be located lies accordingly between \( p_{i,1} \) to \( p_{i,3} \). As we can not be certain about the occupancy between \( p_{i,1} \) to \( p_{i,3} \), in the proposed approach, the vehicle is assumed to be at \( p_{i,1} \) and the new length of the vehicle is \( l_{i,e} \). Eq. (1a), and Eq. (1b) express the above mentioned ideas mathematically.

\[
\begin{align*}
    l_{i,e} &= l_{i} + 2 \ast e_{i} \\
    p_{i,1} &= p^*_{i} + e_{i}
\end{align*}
\]  

Similarly, the perceived distance between vehicles \( d_{i,k}^{*} \) is the distance between the perceived locations of vehicles \( i \) and \( k \) and is mathematically represented in Eq. (2). Kindly refer to [7] for further details. Note that neither transmitting nor receiving vehicles are aware of their true locations.

\[
d_{i,k}^{*}(n) = p_{i,1}(n) - p_{k,1}(n) - l_{i,e} > 0
\]

C. Mixed Traffic Modeling

In this paper we consider longitudinal motion of multiple vehicles on a single lane containing CACC enabled vehicles and MDV as illustrated in Fig. 3. We assume a MDV is driven by a human and is without any control capabilities. We model a reaction time as the perception reaction time of a human driver \( (t_{\text{prt}}) \) [8], and a visibility limited to the front vehicle only. Moreover, we define \( t_{\text{prt},i} = [t_{i,i-1}, t_{i,1}] \) as the pair of perception response time of a MDV \( i \) compared to the vehicle in front and the first vehicle respectively. If all vehicles in front of vehicle \( i \), i.e.: \( i - 1, i - 2 \ldots 3, 2 \) are MDVs, then MDV \( i \) will react \( t_{i,i-1} \) seconds after vehicle \( i - 1 \) and \( t_{i,1} \) seconds after vehicle 1, where \( t_{i,1} = t_{i,i-1} + t_{i-1,i-2} + \ldots + t_{2,1} \). Thus the reaction time of a MDV is proportional to the number of other MDVs immediately ahead.

Fig. 2. Localization error concept for an error magnitude of \( e_{i} \).

Fig. 1. Mixed coordinated driving, with a Cloud-based centralized controller.
All human drivers cannot reach the maximum braking capacity $u_{i,\text{min}}$. In order to account for different drivers’ maximum attainable braking capacity, we randomize the braking capability of MDVs between 50% and 100% of the maximum braking capacity using a human factor ($f_h$). Assuming MDVs brake at their individual maximum attainable braking capacity after their corresponding perception response time ($t_{pr1}$) and come to a halt in $t_i^s$ seconds, the braking profile of manually driven vehicles can be given by Eq. (3):

$$u_i(n) = \begin{cases} f_h \cdot u_{i,\text{min}} & c \cdot t_{i,1} < n \leq c \cdot T_i^s \\ 0 & \text{otherwise} \end{cases} \quad \forall \ i \in Z^c$$  

where $T_i^s = t_{i,1} + t_i^s$. Values in seconds are multiplied with constant $c = 10$ and converted to instances (1 second = 10 instances). $n$ is any instant in the prediction horizon $N (n \in 1...N)$. $Z$ is the set of all CACC vehicles amongst $n_v$ vehicles, $0 \leq \text{size}(Z) \leq n_v$. Note: $Z^c$ is complement set of $Z$ which is the set of all MDV. $Z^c$ is a null set ($Z^c=\emptyset$) if there are no MDVs.

On the other hand, CACC vehicles are assumed to start implementing control action simultaneously on the reception of control inputs from the centralized controller. The frequency of received control inputs is defined by the controller’s update frequency. Thus, we can assume vehicle 1 and vehicle 4 implement their controls simultaneously (refer to Fig. 3). CACC vehicles implicitly warn following MDVs of their braking through braking lights. Accordingly, the reaction time of a MDV(i) behind a CACC vehicle will be much shorter than a MDV(k) behind another MDV i.e.: $t_{i,1} < t_{k,1}$, as indicated in Fig. 3.

As described in Section II-A, the centralized controller has a full knowledge (at instant $n=0$) about the state parameters of all MDVs and CACC vehicles, including their vehicular constraints. Our proposal is to calculate control inputs for CACC vehicles taking into account CACC vehicles and MDVs at each instant $n$ over time horizon $N (n = 1...N)$. CACC vehicles implement control inputs derived from Eq. (11), whereas MDVs implement the braking model described in Eq. (3).

### D. Centralized MPC-based Control System

We model our centralized controller as a Model Predictive Control (MPC) system. MPC is well-known method to provide sequence of actions to be followed over a finite horizon. MPC is a popular choice for trajectory planning in robotics due to their capabilities to handle system constraints and to satisfy fast convergence.

The state variable $x_i$ of a vehicle $i (i \in 1...n_v)$ is defined as the position $p_{i,1}$, velocity $v_i$ tuple in Eq. (4).

$$x_i = [p_{i,1} \ v_i]^T$$  

The relation between position, velocity, acceleration and jerks is given by Eq. (5).

$$p_{i,1} = v_i; \quad \dot{v_i} = u_i; \quad \ddot{u_i} = j_i$$  

A discrete time linear control system represented by Eq. (6) is used, where values for constants are given by Eq. (7).

$$x_i(n+1) = Ax_i(n) + Bu_i(n)$$  

$$A = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} (\Delta t)^2/2 \\ \Delta t \end{bmatrix}$$  

where $\Delta t$ is the time between two consecutive instances $n$ and $n + 1$. Vehicle and road constraints in terms of minimum and maximum values of position, velocity, acceleration are accounted for in Eq. (8a), and Eq. (8b).

$$\begin{array}{l} p_{i,1}^\text{min} \leq x_i(n) \leq p_{i,1}^\text{max} \\ u_i^\text{min} \leq u_i(n) \leq u_i^\text{max} \end{array}$$  

where $(\cdot)_{i}^\text{min}, (\cdot)_{i}^\text{max}$ corresponds to minimum and maximum value of that parameter for vehicle $i$. $u_{i}^\text{min}$ and $u_{i}^\text{max}$ stand for maximum braking and maximum acceleration capabilities. Restricting jerks ($j_i$) within certain acceptable bounds ensures smooth braking for CACC vehicles and is implemented using Eq. (9). Note: MDVs have a braking profile defined by Eq. (3) and thus jerks corresponding to MDVs can not be optimized.

$$j_i^\text{min} \leq j_i(n) \leq j_i^\text{max}$$  

Collision avoidance is achieved by ensuring the perceived distance between vehicles is always positive (see Eq. (2))

$$v_i(N) = 0$$

The cost function ($J$) is set to maximize comfort. We accordingly aim at minimizing the 2-norm of its control inputs $\|u_i(n) - u_i(n-1)\|_2$ to penalize strong deviations. By integrating all of previously defined, the optimization model for a centralized mixed vehicle braking coordination scenario can be represented as:

3The control system knows neither the true positions nor the true distance between vehicles. It computes control inputs using perceived positions and perceived distance between vehicles.
minimize \( J = \sum_{i=1}^{n_p} \sum_{n=1}^{N} \| u_i(n) - u_i(n-1) \|_2 \) \hspace{1cm} (11)
subject to
\[ Eq. \ (1a), \ (1b), \ (2), \ (3), \ (4), \ (5), \ (6), \ (7), \ (8a), \ (8b), \ (9), \ (10) \]

If perceived distance between vehicles \( d_{i+1}^*(0) \leq 0 \) the constraint set by Eq. (2) can not be fulfilled and thus the simulation is unfeasible and the algorithm is not run. In the case of a pure automated vehicle scenario (no MDVs), Eq. (3) is ignored. The centralized controller will aim at solving the convex optimization problem represented by Eq. (11). Depending on the input parameters (notably the location errors), it may fail and not return any control input, thus creating uncertainty in collision avoidance. If collisions are inevitable, this methodology will also not return any control input. Solving such a scenario is out of scope of this paper. In this paper, we rely on CVX [9] toolbox on MATLAB to solve Eq. (11).

### III. Simulations and Analysis

**A. Simulation Parameters**

We simulate a 6 vehicle \((n=6)\) braking scenario\(^6\). The location of potential collision is assumed to be the origin \((0 \text{ in 1D space})\), and vehicles are moving towards the origin. The initial (true) location of the first vehicle \((p_1(0))\) is fixed to 95.9m, considering that this is the distance at which at least one DSRC/ITS-G5 safety message would be received with 99.5% probability \([10]\). \(p_i^{\text{min}} > 0\) ensures vehicles stop before the collision. If \(i+1\) represents the vehicle following vehicle \(i\), then \(p_{i+1} > p_i\) signifies overtaking is forbidden in a single lane scenario. For all vehicles, \(u^{\text{min}}\) is set to zero, implying that vehicles cannot backup. \(u^{\text{max}} = 0\), guarantees a pure deceleration scenario, and \(j^{\text{min}}\) and \(j^{\text{max}}\) values are capped to -0.25 and 0.25 \(m/s^2\) respectively. Simulations performed in this paper don’t require \(p_i^{\text{max}}\) and \(v^{\text{max}}\). The time horizon \(N\) simulations were set to 16s (160 instants), where a second is divided into 10 instants. This is motivated by the fact that GNSS/GPS update frequency and Cooperative Awareness Message (CAM)/ Basic Safety Message (BSM) transmission frequency is 10 Hz \([11]\).

We consider localization errors to follow a zero-mean Gaussian distribution with standard deviation \(\phi\). The actual localization error \(e_i\) is drawn for each vehicle \(i\) from the Gaussian distribution. The perceived localization \(p_i^*\) is generated in 2D by adding the error \(e_i\) to the true localization value \(p_i\) in Cartesian coordinates. \(x\) component of these parameters is then chosen to convert it to a 1D scenario. \(e_i\) remains constant for each vehicle over \(N\) instances. The initial velocity \((v_{i(0)}\) ranges from 5 to 30 m/s at intervals of 5 m/s. The initial velocity for each vehicle \(v_{i}(0)\) is set to \(v_{i(0)} \pm 5\%\).

We define a simulation sample as the set of vehicle state parameters (velocity, inter-distance, location error). We simulate 100 samples per initial velocity \(v_{i(0)}\). We therefore simulate 600 samples for each \(\phi\) value. Further parameters are shown on Table I.

**B. Traffic with Automated Vehicles Only**

In this scenario, we only consider CACC vehicles and assume homogeneous capabilities, in particular \(\phi\) is the same for all CACC vehicles\(^8\). Despite the std of error \(\phi\) being constant, the magnitude of error for each vehicle is different. We vary \(\phi\) from the most precise to very imprecise situations, i.e \(\phi = [4; 2; 1; 0.5; 0.3]\). We assume that occupants of CACC vehicles may choose their preferred headway distance from 5 m up to 1.1 \(\cdot v_{i}\) m, the latter bound being taken from \([5]\). The maximum braking capacity of CACC vehicles is set to 0.6-g, which is the mean braking capacity among all vehicles \([12]\).

Only the perceived positions and perceived distance between vehicles are used to compute the CACC control inputs and evaluate collision avoidance according to Eq. (11). The true position is never known to the controller. We summarize the performance of coordinated collision avoidance subject to localization errors in Table II. As expected, for any value of \(\phi\), the number of collisions avoided using the proposed algorithm with localization errors is less than or equal to the collisions avoided when true positions are known, but see a strong drop for values of \(\phi\) greater than 1 m. Even under perfect localization conditions, the maximum number of collisions avoided is 500 out of a total of 600.

We further illustrate the impact of velocity on collision avoidance in Table III. We observe that the number of collisions avoided increases with velocity up to a certain limit. This comes from the fact that the higher velocity is, the higher

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\(^6\)The scale of this scenario is sufficient to evaluate automated vehicles, as braking is fully coordinated and shock-waves are not created.

\(^8\)This will be relaxed in future work to model different qualities of automated vehicles.
is the upper limit of the inter-vehicle distance (1.1 \cdot v_p \text{ is higher}). However, if the velocity is too high (around 30 m/s), vehicles will not be able to stop before the presumed obstacle regardless of the inter-distance (all entries for \( v_p = 30 \text{m/s} \) are 0). Collisions might still be avoided by adjusting other parameters, such as the permitted jerks used in Eq. (9) (this equation restricts how quickly vehicles can reach maximum braking), but this is out of scope of this paper.

In traffic flow theory, flows are defined using velocity and intervehicular distances. We therefore use scatter plots of simulation samples, plotted in a two-dimension velocity/inter-distance domain, with the corresponding traffic flow value in gray scale in Fig. 4-6. Large inter-distance and low speed lead to low flows, while high flows may only be reached for large velocities at low inter-distance. Our objective is to visually illustrate the impact of location errors on the distribution of avoided collisions in the figures and its impact on traffic flows.

Accordingly we need to define our methodology. On one hand, Collision Avoided (CA) samples are samples where control inputs avoided an accident despite localization errors. CA samples are represented by ‘\( \times \)’ Would the true position be known, CA samples would obviously also avoid accident. On the other hand, Probable Collision (PC) samples are samples where the controller is unable to provide control inputs. This does not strictly imply collision, but simply defines an uncertainty in the controller. PC samples are represented by ‘\( \cdot \)’. A sample might be classified as a PC sample in two cases:

(A) not feasible - due to localization errors, the perceived distance between vehicles is less than or equal to zero. In this case, the algorithm is not run as the constraint represented by Eq. (2) is not satisfied.

(B) not solvable - due to localization errors, the perceived inter-vehicular distance is greater than zero but lower than the required inter-vehicular distance to avoid collision. Accordingly, we further need to know if PC samples would lead to collision avoidance would the true position be known.

Collisions which take place (despite the information of true location) are marked as ‘\( \circ \)’. However, in real-time operation, controller would not know this and would attempt to avoid this situation at any cost. Table IV summarizes the different icons used.

We represent the collision avoidance statistics for each of the 600 samples for \( \phi = 4 \) and \( \phi = 2 \) in Fig. 4 and Fig. 5. We observe there are different regions with different concentration of CA to PC samples. PC represents uncertainty as the algorithm cannot guarantee collision free braking. Thus PC should be avoided and the objective is to make CACC vehicles operate in the region with a majority of CA samples. We illustrate such region as a convex hull, which is defined with the following parameters:

- \( \alpha \) - the target ratio of CA to PC samples; it represents the confidence in the system.
- \( \beta \) - the ratio of PC samples outside convex hull to the total PC samples; this represents the percentage of PC samples avoided by operating in the convex hull.
- margin - a truncation margin for CA to remove extreme CA samples. A low margin will provide a more compact hull.

Let the Operating Region (OR) for the controller be the area inside the convex hull. All samples inside OR form the reference database. The margin parameter is used to avoid regions with low number of CA samples and high number of PC samples from getting into the OR. The controller therefore needs to find a margin\(^9\) that creates an OR by maximizing \( \alpha \) and \( \beta \). Consider Fig. 6, and Fig. 5 depicting collision avoidance statistics for \( \phi = 2 \) and margin values of 0.15 and 0.5. \( \alpha \) values are 2.098 and 1.990 and \( \beta \) values are 0.487 and 0.415 respectively for margin values of 0.15 and 0.5 which implies the uncertainty is lower with margin value 0.15. A lower margin also makes the hull more compact, which also moves the OR away from the preferred high traffic flow configuration. This implies that the uncertainty in the system can be reduced by changing the margin, but at the cost of a reduced flow capacity.

Comparing Fig. 4 and 5, we can observe the impact of location error on the convex hull (i.e. the OR) while maintaining the same value of margin. The OR is large for \( \phi = 2 \), it is significantly reduced for \( \phi = 4 \). Accordingly, localization errors de facto reduces traffic flow. The number of PC samples signifying uncertainty are more for higher values of \( \phi \).

C. Reducing Uncertainty and Impact on Traffic Flow

In the previous subsection, we illustrated the uncertainties created by location errors and creation of a convex hull as the the controller operation area with reduced uncertainty. We next outlay a method to further reduce such uncertainty within the convex hull through traffic adaptation and quantify its impact on traffic flow. The controller can identify uncertainty values in different areas within its convex hull. Conceptually speaking,

\(9\) A change in margin affects the OR and the reference database changes as well.
Fig. 4. CACC only vehicles - collision avoidance statistics for $\phi = 4$ m and a margin of 0.5

Fig. 5. CACC only vehicles - collision avoidance statistics for $\phi = 2$ m and a margin of 0.5

Fig. 6. CACC only vehicles - collision avoidance statistics for $\phi = 2$ m and a margin of 0.15

Fig. 7. Alternative Operational Point (OP) to ensure better collision avoidance

As evident, the latter two will reduce the flow, and accordingly impact the benefit of automated vehicles in future automated road transport systems. Depending on original classification of the sample and the OR, the first Keep Flow approach may not be feasible.

More specifically, the proposed flow adaptation is described as follows. Consider a sample (e.g. the triangle in Fig. 7) obtained from live traffic data. We define tile as an area bounded by CA samples within a given range of speed and intervehicular distance. If the ratio $\alpha$ in one of the three options (tiles) is bigger than that of the tile in which the sample is

- **Keep Flow** - Reduce average distance between vehicles and reduce average velocity.
- **Keep Velocity** - Maintain average velocity but reduce the distance between vehicles.
- **Keep Distance** - Maintain the same average distance between vehicles and reduce the velocity.

the approach consists of moving the operational point within its convex hull (changing velocity and intervehicular distance parameters) of a sample from a high uncertainty area to an area with less uncertainty. An automated vehicle controller has various options to do so, such as increasing jerk tolerance, reducing speed or increasing intervehicular distance. In this paper, we propose to investigate the impact of the latter two.

As illustrated in Fig. 7, a sample represented by ‘$\Delta$’ is classified in the upper circular area, which experiences a high uncertainty and will need to move to a more reliable zone. It can do so by adjusting velocity or intervehicular distance according to one of the three options, illustrated using three green arrows in Fig. 7:

- Keep Flow - Reduce average distance between vehicles and reduce average velocity.
- Keep Velocity - Maintain average velocity but reduce the distance between vehicles.
- Keep Distance - Maintain the same average distance between vehicles and reduce the velocity.
classified, the operational point must be moved.

Let us consider for example a sample with 15m average distance between vehicles and a 25m/s average velocity. Assuming a operational point tolerance of 5%, the tile is defined for this sample with distance and speed between 14 to 16m and 24 to 26m/s respectively. The $\alpha$ value for the tile in which the sample is classified is 2.6. Compared to this, three other options (potential operational points) are displayed in Fig. 6, all of which have smaller $\alpha$ values. Accordingly, the operational point must be moved. The sample is actually superimposed on top of Fig. 6 and a zoomed in image of the same has been plotted in Fig. 7. $\alpha$ values for options 1 to 3 are 7, 34, Inf respectively. All considered options have better $\alpha$ value compared to the tile in which the original sample was classified. Option 3 should be suggested which is the safest, as the $\alpha$ is the highest, but the flow would be lower. Whereas if the same flow needs to be maintained, option 1 should be suggested. No matter which option, all options are better than the actual sample. In this way, vehicles can operate in a region with less uncertainty.

For clarity, we restate important terms: OR is area inside the convex hull; Operating point is the value of the average distance between vehicles and average velocity for any sample; Tile is an area bounded by CAs with an area smaller than OR, in which an Operating Point may lie.

### D. Traffic with mixed Traffic

For the sake of completeness, mixed vehicle simulations were also performed. In the presence of MDVs, CACC vehicles will have to behave like MDVs, i.e.: observe larger inter vehicular distances. Thus we assume the time headway observed by all vehicles is between vehicles is the recommended distance of 1.8 s ± 20% [13]. Moreover the presence of human factor $t_h$ randomizes the maximum achievable braking capacity of MDVs where as for CACC vehicles it is fixed to mean braking capacity of 0.6 g [12]. Moreover, we assume MDVs use GNSS based localization techniques, with a std of error 4 m, whereas CACC vehicles use advanced localization techniques and compute localization with a std of error of 30 cm. Rest of the parameters remain the same. The perception response time of a manually driven vehicle $t_{prt}$ is drawn from a normal distribution $\mathcal{N}(1.33,(0.27)^2)$ [8] and is capped between 0.8 s and 1.8 s. Other parameters are kept the same.

Simulation results for mixed vehicle scenario is summarized in Table V. The number of collisions avoided with localization error (values in second column) is always lower than the number of collisions that could have been avoided had true position been known (values in third column). We use values in the third column as baseline for comparison to evaluate the performance. We observe that as the number of CACC vehicles increase, the number of collisions avoided increase. This can be attributed to two main reasons: 1. CACC vehicles are assumed to have better localization capability and lower standard deviation of error 2. CACC vehicles can adjust its controls based on the state of neighboring vehicles and thus aid in avoiding accidents. Results corresponding to two CACC and four CACC vehicles (out of six) has been plotted in Fig. 8, 9 respectively. In this mixed vehicle scenario, CACC vehicles would imitate MDVs and thus vehicles will have similar inter

<table>
<thead>
<tr>
<th>Number of CACC vehicles $(naa)$</th>
<th>Collisions avoided with localization errors using proposed approach</th>
<th>Collisions avoided with true position information</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>270</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>329</td>
</tr>
<tr>
<td>3</td>
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<td>4</td>
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<td>430</td>
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<td>5</td>
<td>449</td>
<td>458</td>
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</tbody>
</table>

Fig. 8. Mixed vehicle analysis with 2 CACC vehicles out of 6: collision avoidance statistics

Fig. 9. Mixed vehicle analysis with 4 CACC vehicles out of 6: collision avoidance statistics
vehicular distances and thus the distribution of samples is dense. A convex hull if drawn would be so tight that the methodology described earlier will not be successful. If we compare plots of CACC only traffic and of Mixed vehicle traffic (e.g.: Fig. 5 and Fig. 9) using the flow bar on the right, we observe that flow capacity in general of CACC only traffic can be much larger compared to the flow capacity of a mixed traffic scenario.

IV. Conclusions

Advanced centralized vehicle control techniques aim at improving flow capacity using parameters like location, velocity, etc of vehicles. In circumstances where vehicles need to brake and come to a halt, they are expected to be able to avoid collisions. But the presence of localization errors creates uncertainty in collision avoidance. Vehicles must operate such that collision avoidance uncertainty is low. In this paper we have presented a method that proposes different options with alternate parameters where vehicles should operate such that collision avoidance uncertainty is reduced in an environment with varied localization errors.

We first create a database which relates localization errors, uncertainty in collision avoidance and flow capacity for a wide range of vehicle state parameters like average velocity, intervehicular distances, etc. For a given traffic scenario uncertainty in collision avoidance is computed by referencing the sample data with the database. The proposed method is then implemented to evaluate uncertainty values of three options and the one with the least uncertainty should be chosen.

We observe that higher the localization error more is the uncertainty. The best way to reduce uncertainty whilst maintaining the same flow would be to reduce localization error, which unfortunately is not always feasible. Other ways to reduce uncertainty include reducing velocity or increasing intervehicular distances or both, but these usually impact the flow capacity.

Future work will involve creating a model where localization error changes over time for every vehicle to simulate an even more realistic scenario. Moreover simulations need to be carried out on a much larger scale to generate a reference database with different parameters for proposed method to be effectively implemented.

V. Acknowledgement

Raj Haresh Patel is a recipient of a PhD Grant from the Graduate School of the University Pierre Marie Curie (UPMC), Paris. EURECOM acknowledges the support of its industrial members, namely BMW Group, IABG, Monaco Telecom, Orange, SAP, ST Microelectronics and Symantec.

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