Relay Aided Coordinated Beamforming and Interference Neutralization

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***Abstract*—We consider the Multi-Input Single Output (MISO) Interfering Broadcast Channel (IBC), in other words the multi-user (MU) multi-cell half duplex downlink in a cellular or heterogeneous network, aided by a full duplex MIMO relay. The Degrees of Freedom (DoF) are analyzed for joint coordinated beamforming by the base stations and interference neutralization by the relay. Also Weighted Sum Rate Maximization at finite SNR is developed. The main concern however for interference neutralization is channel state information (CSI) at the relay, which does not observe the direct user links. Various solutions are explored.**

I. Introduction

In this paper, Tx may denote transmit/transmitter/ transmission and Rx may denote receive/receiver/reception. Interference is the main limiting factor in wireless transmission. Base stations (BSs) disposing of multiple antennas are able to serve multiple Mobile Terminals (MTs) simultaneously, which is called Spatial Division Multiple Access (SDMA) or Multi-User (MU) MIMO. Hence we focus here on the more challenging downlink (DL), which typically carries more traffic and requires Channel State Information at the Tx (CSIT), which is more difficult to acquire than CSI at the Rx (CSIR). In cellular systems, one can distinguish between the cell center where a single cell design is appropriate (due to high SIR) and the cell edge where a multi-cell approach is mandatory. The MU MIMO DL problem for the cell center (multi-antenna) users is called the (MIMO) Broadcast Channel (BC). For the cell edge users, the DL of multiple cells with single (multiple) users per cell corresponds to the Interference (Broadcast) Channel (IC/IBC). The recent introduction of (multi-antenna) Interference Alignment (IA) (which is joint Zero-Forcing (ZF) by Tx and Rx antennas in multi-cell systems) has shown that approaching high system capacity through agressive frequency reuse should in principle be possible. Whereas precise capacities for cellular systems remain unknown, IA allows to reach the optimal high SNR rate prelog, called Degree of Freedom (DoF) (or spatial multiplexing factor). The recent development of Massive MIMO (MaMIMO) [1] opens new possibilities for increased system capacity while at the same time simplifying system design.

Another development that has been integrated in 4G wireless systems is the use of relays. Relays have been used since a long time, for *coverage extension*. Such coverage extension can be achieved by typically single antenna (dumb) relays (also called repeaters). The relay paradigm has been shifting in recent years, to *interference management*, which is rendered possible by smart multi-antenna relays. One form of interference management is *interference neutralization* (IN), in which artificial multipath is introduced to provoke desctructive interference superposition at receivers. Relays can be single or multiple, which typically require more coordination. Relaying can be one-way (either uplink or downlink) or two-way (simultaneously UL and DL). Two-way relaying has to be implemented in a two-hop (or two-phase) fashion using half duplex (HD) nodes. The use of full duplex (FD) relays allows the implementation of single-hop instantaneous relaying. In the two-hop case, the direct link from base station (BS) to user equipment (UE) is often deemed negligible, and coverage extension is an active ingredient. IN with instantaneous relays on the other hand exploits the direct links also. Yet another aspect of relaying is the variety of communication theoretic approaches, of the form "X& F" where F stands for "Forward" and X belongs to a long list that starts with A (amplify), D (decode), C (compress), Cu (compute) etc. AF is also called non-regenerative. In DF, (part of) the signal is decoded and re-encoded. This could be done pretty much instantaneously using causal channel codes. However, most current wireless signals use OFDM or derivatives to handle frequency-selective channels. In OFDM de/re-coding requires passing between time and frequency domains which introduces an unacceptable delay for instantaneous operation. Hence instantaneous relaying not only requires a FD relay, but also AF operation in the time domain. In communication theoretic discussions, the advantage of DF over AF of denoising the signal before relaying is emphasized. However, such discussions typically overlook the limited quality CSIT which is available for either AF or DF, and which reduces the effect of the denoising advantage of DF. So far, we have assumed that all transmission or relaying is in-band, i.e. occurs in the same frequency band. Relaying schemes with out-of-band segments have also been considered. Finally, other relaying scenario variations are possible in which e.g. a small cell BS also relays for a macro BS, or in which the UEs also relay via D2D. Also, full duplex BS or dynamic TDD lead to simultaneous DL and UL users among which interference could be managed with relays.

The extensive tutorial on relays [2] covers pretty much all aspects of relaying and exhibits a bibliography that reflects the intense research activity on this topic in recent years. Nevertheless, only a few references are provided that are relevant for the multi-antenna instantaneous relaying for interference neutralization considered here. Some relevant references are the following. [3] considers the two cell MISO IBC with as many users per cell as BS antennas, and two MIMO relays. The operation is two-hop in which the direct link is assumed to be negligible. [4] considers all half duplex units and hence operates in a two-hop fashion. Two-way AF relaying is considered in a SISO K cell interference channel with N MIMO relays. Also single antenna repeaters (dumb relays) are considered which in general just modify the direct BS-UE channels but here in fact totally determine those channels due to the bidirectional two-hop operation. In [4] all interference is managed by the smart (MIMO) relays. The IN feasibility study not only comprises system dimension considerations but also the power constraints at the relays. [5] is representative of various other bulks of works. It considers multiple antennas at all units in a relay aided interference channel, and hence considers joint IA-IN. However, it is two-phase based and uses DF. [6] considers MIMO relays in a MIMO IC but with out-of-band BS-Relay links. This is an interesting set-up when all units are constrained to be half duplex. However, additional frequency bands are needed to feed the relays. This would not impact the wireless system capacity much if the BS-Relay links would be e.g. mmWave but the BS-Relay links may be too long for such frequencies. [7] is closely related to the topic of this paper. It investigates joint ZF-IN feasibility for a square MIMO two cell IC with an instantaneous MIMO relay.

Our research target is the development of *autonomous relays* which configure themselves for relaying and esp. interference neutralization. This requires ideally no or at least minimal intrusion/modification of protocols (signaling overhead and structure). Hence single hop instantaneous (full duplex) relays are required, performing AF in the time domain.

In this paper we first briefly review the MIMO IBC and then introduce the MISO IBC plus Full Duplex MIMO Relay (IRBC), an interesting configuration that apparently has not been considered before. We study the joint ZF+IN feasibility conditions. This involves the full column rank of Khatri-Rao products. We also discuss the CSI Acquisition issue with two approaches: CSI acquisition and transfer, or direct AF filter adaptation.

II. IRBC Signal Model

A. MIMO IBC Case

We shall assume frequency-flat channels. The discrete-time index is omitted in the notations below. Consider first the general IBC setting, with C cells and K_c users in cell c, the $N_{c,k} \times 1$ received signal at user k in cell c is

$$\mathbf{y}_{c,k} = \mathbf{H}_{c,k,c}\,\mathbf{G}_{c,k}\,\mathbf{x}_{c,k} + \sum_{(j,i)=(1,1),\neq(c,k)}^{(C,K_j)} \mathbf{H}_{c,k,j}\,\mathbf{G}_{j,i}\,\mathbf{x}_{j,i} + \mathbf{v}_{c,k} \quad (1)$$

where $\mathbf{x}_{c,k}$ are the $d_{c,k} \times 1$ intended (white, unit variance) signal streams for that user, $\mathbf{H}_{c,k,j}$ is the $N_{c,k} \times M_j$ channel from BS j to user k in cell c. We assume that we are considering a noise whitened signal representation so that we get for the noise $\mathbf{v}_{c,k} \sim \mathcal{CN}(0, I_{N_{c,k}})$. The $M_c \times d_{c,k}$ matrix spatial Tx filter of beamformer (BF) is $\mathbf{G}_{c,k}$.

The single cell MU downlink or BC is obtained when $C = 1$ and the IC case corresponds to $K_c \equiv 1$, $c = 1, ..., C$. Also, the analysis simplifies significantly for the so-called symmetric (or uniform) case in which $K_c \equiv K$, $M_c \equiv M$, $N_{c,k} \equiv N$, $d_{c,k} \equiv d$ leading to the symmetric IBC configuration (M, N, C, K, d).

There are a number of cases in which the DoF of linear IA are captured completely by the proper bound [8]. The proper bound corresponds to imposing that the number of (relevant) variables in Tx/Rx filters should equal at least the number of ZF conditions. For the symmetric MIMO IC ($K = 1$), when $\min(M, N) >\geq 2d$, alignment is feasible iff $M + N \geq (C + 1)d$ (proper bound). This is a generalization of [9] which only considered the square case $M = N$. Then there is the "divisible case": if $d_{c,1} \equiv d$ and $d|N_{c,1}, \forall c$ OR $d|M_c, \forall c$ (where "$a|b$" denotes "a divides b") then alignment is feasible iff condition (11) in [8] is satisfied. This is again a bit more general than similar results by [10] where they had $d_{c,1} = 1$ which of course divides everything, or [11] where the $d_{c,1}$ needed to divide both the $N_{c,1}$ and the M_c. For the IBC, [12] finds that the proper bound $M + N \geq (CK + 1)d$ is sufficient in the symmetric IBC when either M or N is divisible by d.

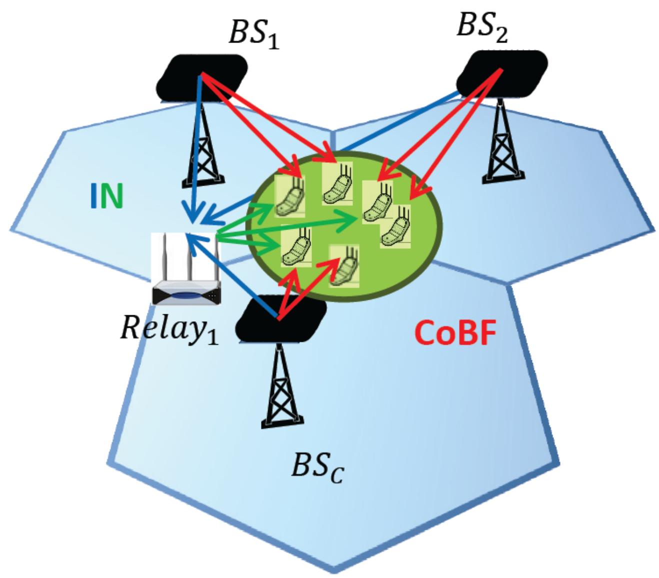

Fig. 1. MISO downlink (half duplex) IBC plus full duplex MIMO Relay. All links are in-band.

B. MISO IBC Plus MIMO FD Relay Signal Model

In what follows, we shall focus on the symmetric MISO IBC, see Fig. 1, in which case the scalar Rx signals can be written as

$$y^{'}_{c,k} = \mathbf{h}^{UB}_{c,k,c}\,\mathbf{g}_{c,k}\,x_{c,k} + \sum_{(j,i)=(1,1),\neq(c,k)}^{(C,K)} \mathbf{h}^{UB}_{c,k,j}\,\mathbf{g}_{j,i}\,x_{j,i} + \mathbf{v}_{c,k} \quad (2)$$

where we now have channel vectors $\mathbf{h}^{UB}_{c,k,j}$ from the BS cell j to user k in cell c, and BF vectors $\mathbf{g}_{c,k}$. To sustain a total of CK streams, each BS needs to have a number of antennas (sum DoF) $M \geq CK$. In other words, the number of users that can be sustained per cell is limited to $K \leq \frac{M}{C}$.

We now add a MIMO FD Relay with the hope to neutralize some interference and to be able to increase the number of users. We get the following expressions for relay Rx and Tx signals and modified UE Rx signals:

$$\begin{array}{rcl} \mathbf{y} &=& \sum_{(j,i)=(1,1)}^{(C,K)} \mathbf{H}_j^{RB}\, \mathbf{g}_{j,i}\, x_{j,i} + \mathbf{v} \\ \mathbf{x} &=& \mathbf{A}\,\mathbf{y} \\ y_{c,k} &=& y^{'}_{c,k} + \mathbf{h}_{c,k}^{UR}\,\mathbf{x} \end{array} \quad (3)$$

where $\mathbf{H}_j^{RB}$ is the $N \times M$ MIMO channel response from BS j to the relay, $\mathbf{A}$ is the $N \times N$ AF matrix, $\mathbf{v}$ is the Rx noise at the relay, and $\mathbf{h}_{c,k}^{UR}$ is the channel vector from the relay to UE k in cell c.

The BS and relay are of course subject to Tx power constraints. However, as we shall not focus on the BF design and we shall assume that the relay power constraint is generous enough to permit IN, we shall not consider the power constraints further.

III. Joint ZF-IN Feasibility Conditions

The noise-free Rx signal at a UE can be rewritten as

$$\begin{aligned} y_{c,k} = & \underbrace{(\mathbf{h}_{c,k,c}^{UB} + \mathbf{h}_{c,k}^{UR}\,\mathbf{A}\,\mathbf{H}_c^{RB})\,\mathbf{g}_{c,k}}_{\neq 0}\, x_{c,k} \\ & + \sum_{(j,i)=(1,1),\neq(c,k)}^{(C,K)} \underbrace{(\mathbf{h}_{c,k,j}^{UB} + \mathbf{h}_{c,k}^{UR}\,\mathbf{A}\,\mathbf{H}_j^{RB})\,\mathbf{g}_{j,i}}_{=0}\, x_{j,i}\,. \end{aligned} \quad (4)$$

where the conditions for joint ZF-IN on the BF vectors $\mathbf{g}_{j,i}$ and the AF matrix $\mathbf{A}$ are indicated. These conditions can perhaps be more easily interpreted in a dual UL in which we have an Interfering Multiple Access Channel (IMAC) plus Relay:

$$\mathbf{g}_{j,i}^H(\mathbf{h}_{c,k,j}^{UB\,H} + \mathbf{H}_j^{RB\,H}\,\mathbf{A}^H\,\mathbf{h}_{c,k}^{UR\,H}) = 0\,,\ \forall (j,i) \neq (c,k) \quad (5)$$

in which the BF $\mathbf{g}_{j,i}^H$ now play the role of ZF Rx. Having M antennas, the BS Rx can ZF $M-1$ interfering streams while still receiving the stream of interest. For user (j,i) let $S_{j,i}$ denote the set of $M-1$ users that will be suppressed by $\mathbf{g}_{j,i}$. Then the conditions (5) become IN conditions for the AF matrix $\mathbf{A}$ for the interfering users $(c,k) \notin \{\{(j,i)\}, S_{j,i}\}$. The number of such conditions is $KC(KC-1)-(M-1)KC = KC(KC-M)$. Note that the ZF conditions for the $\mathbf{g}_{j,i}$ and the IN conditions for $\mathbf{A}$ involve different (and hence independent) user channels $\mathbf{h}_{c,k,j}^{UB}$. Hence, even though the ZF and IN conditions are coupled, the BF can be considered as independent of $\mathbf{A}$ in the IN conditions. For the same reason also, the direct overall channel gains appearing in (4) (for $(c,k,j)=(c,k,c)$) will be non-zero w.p. 1, in spite of the conditions (5).

Introducing the $vec(.)$ operator, which stacks consecutive columns of a matrix in a supervector, with the property $vec(\mathbf{A}\,\mathbf{X}\,\mathbf{B}) = (\mathbf{B}^T \otimes \mathbf{A})\,vec(\mathbf{X})$ where $\otimes$ denotes the Kronecker product, and taking Hermitian transpose of the scalars in (5), we can rewrite the IN conditions from (5) as

$$vec^H(\mathbf{A}^H)\,(\mathbf{h}_{c,k}^{UR\,T} \otimes \mathbf{H}_j^{RB}\mathbf{g}_{j,i}) = -\mathbf{h}_{c,k,j}^{UB}\mathbf{g}_{j,i} \quad (6)$$

which need to hold for $\forall (c,k) \notin \{\{(j,i)\}, S_{j,i}\}$. There are many ways of selecting the sets $S_{j,i}$, leading to many solutions for joint ZF-IN. Each solution will correspond to a local optimum for utility optimization designs. Let us consider one specific choice for the $S_{j,i}$ in which the $M-1$ users to be ZF'd comprise in any case the $K-1$ other users in cell j and such that $S_j = \{\{(j,i)\}, S_{j,i}\}$ is independent of i. Then let $\mathbf{H}_j^{UR} = [\mathbf{h}_{c,k}^{UR\,T},\,(c,k) \notin S_j]$ which is a matrix of size $N \times (CK-M)$. Introduce also $\mathbf{G}_j = [\mathbf{g}_{j,1}\cdots\mathbf{g}_{j,K}]$ of size $M \times K$ and $\mathbf{h}_j^{UB} = [\mathbf{h}_{c,k,j}^{UB}\mathbf{G}_j,\,(c,k) \notin S_j]$, then we can rewrite (6) as

$$\begin{aligned} & vec^H(\mathbf{A}^H)\,[\mathbf{H}_1^{UR} \otimes \mathbf{H}_1^{RB}\mathbf{G}_1 \cdots \mathbf{H}_C^{UR} \otimes \mathbf{H}_C^{RB}\mathbf{G}_C] \\ & \quad = -[\mathbf{h}_1^{UB} \cdots \mathbf{h}_C^{UB}]\,. \end{aligned} \quad (7)$$

This system of equations can be solved for $vec^H(\mathbf{A}^H)$ if the matrix of coefficients has full column rank. To investigate this, we can use the following Lemma.

Lemma 1: **Full column rank conditions of Khatri-Rao product** Consider the block matrices $\mathbf{A} = [\mathbf{A}_1 \cdots \mathbf{A}_n]$, $\mathbf{A} = [\mathbf{B}_1 \cdots \mathbf{B}_n]$ with compatible column block structure, their Khatri-Rao product $\mathbf{A} \odot \mathbf{B} = [\mathbf{A}_1 \otimes \mathbf{B}_1 \cdots \mathbf{A}_n \otimes \mathbf{B}_n]$ has full column rank iff

(i) all $\mathbf{A}_i$ and $\mathbf{B}_i$ have full column rank,

(ii) at least one of $\mathbf{A}$ or $\mathbf{B}$ has full column rank. □

Proof. Sufficiency is fairly straightforward. For necessity, (i) is a result of $\text{rank}(\mathbf{A}_i \otimes \mathbf{B}_i) = \text{rank}(\mathbf{A}_i)\,\text{rank}(\mathbf{B}_i)$. (ii) for the case $n=2$, by contradiction: given that the $\mathbf{A}_i$ and $\mathbf{B}_i$ have full column rank but if now both $\mathbf{A}$ and $\mathbf{B}$ don't have full column rank, then there exist vectors $\mathbf{a}_i$, $\mathbf{b}_i$ such that $\mathbf{A}_1\,\mathbf{a}_1 = \mathbf{A}_2\,\mathbf{a}_2$ and $\mathbf{B}_1\,\mathbf{b}_1 = \mathbf{B}_2\,\mathbf{b}_2$. Then $\mathbf{A}_1\,\mathbf{a}_1\mathbf{b}_1^T\mathbf{B}_1 = \mathbf{A}_2\,\mathbf{a}_2\mathbf{b}_2^T\mathbf{B}_2$ and

$$vec(\mathbf{B}_i\mathbf{b}_i\mathbf{a}_i^T\mathbf{A}_i^T) = (\mathbf{A}_i \otimes \mathbf{B}_i)\,vec(\mathbf{b}_i\mathbf{a}_i^T) = (\mathbf{A}_i \otimes \mathbf{B}_i)\,(\mathbf{a}_i \otimes \mathbf{b}_i).$$

Hence $(\mathbf{A} \odot \mathbf{B})[(\mathbf{a}_1 \otimes \mathbf{b}_1)^T \ \ -(\mathbf{a}_2 \otimes \mathbf{b}_2)^T]^T = 0$ which means that $\mathbf{A} \odot \mathbf{B}$ would not have full column rank. ■

Applying Lemma 1 to (7) leads to the following.

Theorem 1: **Interference Neutralization Feasibility** In the MISO IBC with MIMO Relay with the dimensions considered above, IN is feasible iff

$$N \geq \max(K, CK-M, C\,\min(K, CK-M))\,, \quad K \leq M\,. \quad (8)$$

This leads to the following evolution for the number of Relay antennas:

$$N = \begin{cases} 0 & ,\ 1 \leq K \leq \frac{M}{C} \\ C^2(K-\frac{M}{C}) & ,\ \frac{M}{C} \leq K \leq \frac{M}{C-1} \\ C\,K & ,\ \frac{M}{C-1} \leq K \leq M \end{cases} \quad (9)$$

where in the first regime only ZF BF is needed. □

The following are two variations on the basic scenario.

Intracell BF. In this case the BF is non-cooperative between cells and only considers the intracell users (the BF is multi-cell oblivious). In this case all intercell interference needs to be canceled by IN. Hence $N = C(CK-M)$ gets replaced by $N = C(C-1)K$.

BF-independent AF The IN eqations will not depend on the

BF $\mathbf{G}_j$ (though the BF will still depend on the AF $\mathbf{A}$) if interference is not neutralized starting from the BF inputs but starting from the BS antennas. Then the factors $\mathbf{G}_j$ disappear from the equation in (7). This leads to IN conditions: $N \geq \max(M, CK-M, C\,\min(M, CK-M))$.

The ZF and IN conditions can be solved iteratively as follows. Start e.g. with $\mathbf{A} = 0$.

(a) The BFs $\mathbf{g}_{j,i}$ can be solved by ZF the direct links in (4) w.r.t. the effective channels in (5) of the other users in S_j.

(b) The AF matrix $\mathbf{A}$ can then be determined from the equations (7).

Iterate (a) and (b) until convergence. Whereas joint ZF-IN can have many solutions, fixing the sets S_j forces convergence to one particular solution (apart from underdeterminacy issues of course if N is larger than necessary).

For the rest of this paper we shall focus on the case of $C = 2$ cells. In this case $N = 4(K - \frac{M}{2})^+$ which evolves from 0 to $2M$ as K evolves from $\frac{M}{2}$ to M. For the Intracell BF case, we get $N = 2K$, whereas the BF-independent AF case (typically) also leads to $N = 4(K - \frac{M}{2})^+$.

IV. CSI Acquisition and AF Adaptation

A. Application Scenarios

Finite MISO with (close to) fully loaded cells, $K \approx M$. In this case the previous discussion immediately applies.
Sectored Massive MIMO with sectoring that is either by fixed design or by user selection. Then per sector the system can be expected to be heavily loaded and we are possibly back to the previous scenario.
Original Massive MIMO corresponds to the finitely loaded case, leading to channel hardening. In this case Maximum Ratio Transmission leads already to weak interference and further BF and IN improvements just lead to some further interference reduction. The system design can be expected to only need few iterations in this case. For $C = 2$ cells, we have $N \geq 2K$ if $K \leq \frac{M}{2}$ (if the BFs do not do ZF).

Below, we assume TDD operation, channel reciprocity and synchronization of all units.

B. CSI Acquisition

If the CSI acquisition is addressed in the literature, it is usually discussed from a cloud processing point of view, with all channel estimates being provided to a central unit which carries out the AF calculation and then sends this to the relay somehow.

It would seem more desirable to permit the relay to perform in situ AF filter computation. The difficulty is that the Relay has no visibility on $\mathbf{H}^{UB}$, information about which has to be conveyed to it either by the BS or by the UE.

If we consider the finite MISO scenario, with $K = M$, then we have $N = 2M$ and all of $\mathbf{H}^{UR}$, $\mathbf{H}^{RB}$, $\mathbf{H}^{UB}$ are square matrices. The Relay can perform AF computation on the basis of the following training/transmission steps:

- Phase 1: UL training for the $2K = 2M$ UEs, of duration $\geq 2K$. This allows the BSs to estimate $\mathbf{H}^{UB}$ and the Relay to estimate $\mathbf{H}^{UR}$.
- Phase 2: DL training for the Relay to learn $\mathbf{H}^{RB}$, of duration $\geq 2M$.
- Phase 3: BS feeds channel estimates $\mathbf{H}^{UB}$ to the Relay, of duration $\geq 2K$.

This procedure requires very strong training coordination between the cells. Other quantities (noise variances, transmit powers, utility weights) can be exchanged also but vary more slowly and represent less signaling overhead.

C. Direct AF Adaptation for Min WSMSE

Here we consider a case in which the UE convey information about $\mathbf{H}^{UB}$ to the Relay. In a utility optimization approach, we may want to consider the minimization of the Weighted Sum Mean Squared Error (WSMSE) at the UEs. We shall here not further discuss the weights, which may be adaptive also and might e.g. reflect a Weighted Sum Rate criterion. Key challenges:

- to transpose the UE level WSMSE to a cost function that can be optimized by the Relay. The Relay can form an error signal at its output but we need the error signals at the level of the UE. In Adaptive Filtering, this is called the "Filtered X" approach.
- to account for partial CSI.

In the following the discrete time index t denotes the TDD slot. We consider the channels to be slowly varying with negligible variation between consecutive slots. The BF and AF are time-varying because they are being adapted (one iteration per slot). Consider the downlink error signal in slot t

$$\mathbf{E}_U(t) = \mathbf{F}^{-1}(t)\mathbf{X}(t) - \mathbf{Y}_U(t) \text{ where} \quad (10)$$

$$\mathbf{Y}_U(t) = \underbrace{\mathbf{H}^{UB}\mathbf{G}(t)\mathbf{X}(t)}_{\mathbf{X}_U(t)} + \mathbf{H}^{UR}\mathbf{A}(t-1)\mathbf{Y}_R(t) + \mathbf{V}_U(t) \quad (11)$$

where $\mathbf{Y}_U(t)$ is a matrix of which the rows represent the $2M$ user signals, the columns represents samples collected over a subset of the downlink time span, and similarly for the other analogous quantities. $\mathbf{G}(t) = \text{blockdiag}\{\mathbf{G}_1(t), \mathbf{G}_2(t)\}$, and e.g. $\mathbf{H}^{UB}$ represents the channels from all BSs to all UEs. The matrix $\mathbf{F}(t)$ is a diagonal matrix of MMSE Rx coeffients. In the representation of (10), $\mathbf{E}_U(t)$ is a scaled version of the actual error signal $\mathbf{X}(t) - \mathbf{F}(t)\mathbf{Y}_U(t)$, the scaling of which can be compensated by a diagonal weighting matrix to be considered further. The transmitted symbols $\mathbf{X}(t)$ in 10) correspond either to training sequences or a decision directed operation.

In the next uplink phase now, the UEs transmit $\mathbf{E}_U(t)$ to the Relay, which receives

$$\check{\mathbf{Y}}_R(t) = \mathbf{H}^{UR\,H}\mathbf{E}_U(t) + \check{\mathbf{V}}_R(t) = \widehat{\mathbf{H}}^{UR\,H}\mathbf{E}_U(t) + noise \quad (12)$$

where the noise term will affect estimation variance but not bias. Now, the UE level WSMSE criterion $\text{E}||\mathbf{W}^{\frac{1}{2}}\mathbf{E}_U||^2$ for the adaptation of the AF filter $\mathbf{A}$ gets the following sample update version

$$\min_{\Delta\mathbf{A}(t)} ||\mathbf{W}^{\frac{1}{2}}((\widehat{\mathbf{H}}^{UR\,H})^H\check{\mathbf{Y}}_R(t) - \mathbf{H}^{UR}\,\Delta\mathbf{A}(t)\,\mathbf{Y}_R(t)||_F^2 \quad (13)$$

where the Relay still has stored the signal $\mathbf{Y}_R(t)$ from the preceding downlink phase. However, since we only dispose of an estimate $\widehat{\mathbf{H}}^{UR}$, we can instead consider the feasible cost function

$$\min_{\Delta\mathbf{A}(t)} \mathrm{E}_{\widetilde{\mathbf{H}}^{UR}} ||\mathbf{W}^{\frac{1}{2}}((\widehat{\mathbf{H}}^{UR\,H})^H\check{\mathbf{Y}}_R(t) - (\widehat{\mathbf{H}}^{UR} + \widetilde{\mathbf{H}}^{UR})\,\Delta\mathbf{A}(t)\,\mathbf{Y}_R(t)||_F^2 \,. \quad (14)$$

Assuming that this cost function is overdetermined for $\Delta\mathbf{A}(t)$, the optimizer allows us to update the AF filter as $\mathbf{A}(t) = \mathbf{A}(t\text{–}1)+\Delta\mathbf{A}(t)$. Alternative Recursive Least-Squares versions can be considered also, or gradient methods. This approach requires the following training/transmission steps:

- Phase 1: UL training for the $2K = 2M$ UEs, of duration $\geq 2K$. This allows the BSs to estimate $\mathbf{H}^{UB}$ (or the equivalent overall UL channel comprising the Relay AF also) and the Relay to estimate $\mathbf{H}^{UR}$.
- Phase 2: DL training for the Relay to learn $\mathbf{H}^{RB}$, of duration $\geq 2M$.
- Phase 3: UEs feed error signals to the Relay, of variable duration.

V. Conclusions

The use of a MIMO FD Relay allows Interference Neutralization and hence allows to serve more users than what the BS beamformers could accommodate normally. However, the adaptation of IN is cumbersome and merits further work. Various extensions of the configuration treated here can be considered, including multiple antennas at the UEs, multiple relays (which corresponds to block diagononal AF $\mathbf{A}$ of which the blocks will need to be adapted separately however), and frequency-selective channels.

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References

[1] E.G. Larsson, O. Edfors, F. Tufvesson, and T.L. Marzetta, "Massive MIMO for Next Generation Wireless Systems," *IEEE Comm's Mag.*, Feb. 2014.

[2] G. Liu, F.R. Yu, H. Ji, V.C.M. Leung, and X. Li, "In-Band Full-Duplex Relaying: A Survey, Research Issues and Challenges," *IEEE COM Surveys & Tutorials*, 2nd Qtr 2015.

[3] N. Lee and Jr. R.W. Heath, "Degrees of Freedom for the Two-Cell Two-Hop MIMO Interference Channel: Interference-Free Relay Transmission and Spectrally Efficient Relaying Protocol," *IEEE Trans. Info. Theory*, May 2013.

[4] J. Zhang, Z.K.M. Ho, E.A. Jorswieck, and M. Haardt, "Interference Neutralization for Non-Regenerative Two-Way Relaying Networks - Part I: Feasibility," *IEEE Trans. Sig. Proc.*, Sept. 2015.

[5] Y. Shu, Q. Wang, J. Zhang, and T. Sun, "Relay-aided interference alignment and neutralization for 3-cellular interference channels," in *Proc. IEEE/CIC ICCC Symp. Wireless Comm's Systems*, 2014.

[6] D. Wu, C. Yang, T. Liu, and Z. Xiong, "Feasibility Conditions for Interference Neutralization in Relay-aided Interference Channel," *IEEE Trans. Sig. Proc.*, Mar. 2014.

[7] Q. Wang, Y. Dong, J. Zhao, N. Li, J. Qian, and B. Liu, "Instantaneous Relaying: Feasibility Conditions for Interference Neutralization," *IEEE Communications Letters*, Aug. 2015.

[8] L. Ruan, V.K.N. Lau, and M.Z. Win, "The Feasibility Conditions for Interference Alignment in MIMO Networks," *IEEE Trans. Signal Processing*, Apr. 2013.

[9] G. Bresler, D. Cartwright, and D.N.C. Tse, "Feasibility of Interference Alignment for the MIMO Interference Channel: the Symmetric Square Case," in *Proc. Information Theory Workshop (ITW)*, 2011.

[10] C.M. Yetis, Tiangao Gou, S.A. Jafar, and A.H. Kayran, "On feasibility of interference alignment in mimo interference networks," *Signal Processing, IEEE Transactions on*, vol. 58, no. 9, pp. 4771 –4782, 2010.

[11] M. Razaviyayn, L. Gennady, and Z. Luo, "On the Degrees of Freedom Achievable Through Interference Alignment in a MIMO Interference Channel," *IEEE Trans. Signal Processing*, Feb. 2012.

[12] T. Liu and C. Yang, "Genie Chain and Degrees of Freedom of Symmetric MIMO Interference Broadcast Channels," [Online]. Available: http://arxiv.org/abs/1309.6727, Sep. 2013.