

# ROBUST MIMO OFDM TRANSMIT BEAMFORMER DESIGN FOR LARGE DOPPLER SCENARIOS UNDER PARTIAL CSIT

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## ABSTRACT

Performance of OFDM (Orthogonal Frequency Division Multiplexing) systems is limited by inter carrier interference (ICI) under high Doppler scenarios such as that encountered in high speed trains like TGV. The use of multiple receive antennas is known to be a very effective way to combat ICI. In a recent publication, the authors explored the use of transmit (Tx) antennas for ICI mitigation. It considered a MIMO (multiple input multiple output) scenario with perfect channel state information at transmitter (CSIT) and iteratively designed a transmit beamformer to maximize the sum capacity across all the subcarriers in the presence of ICI. In this paper, we make the design more robust by considering only partial CSIT knowledge. The beamformer is designed by optimizing the expected weighted sum rate (EWSR) under large MIMO asymptotics regime. The convergence of the beamformer follows easily from the design.

**Index Terms**— MIMO, OFDM, high Doppler, intercarrier interference, partial CSIT

## 1. INTRODUCTION

As is well known, high Doppler encountered in HST (high speed train) environments violates the orthogonality requirement for OFDM (Orthogonal Frequency Division Multiplexing), resulting in ICI. The SINR (signal to interference plus noise ratio) analysis due to ICI can be found in ([1],[2]). Several prior publications have focused on receiver techniques to mitigate ICI. It is known that multiple receive antennas in a SIMO scenario is very effective in canceling out the ICI (for example, see [3]). In a recent publication ([4]), the authors of this paper extended the analysis to a MIMO (multiple input multiple output) scenario and iteratively designed a transmit beamformer to maximize the sum capacity across all the subcarriers in the presence of ICI. A linear model was assumed for the channel variation across each OFDM symbol as in many of the prior works ([5], [6], [3]). It was observed that the presence of ICI renders the problem similar to that encountered in a MIMO interfering broadcast channel (MIMO-IBC). Hence, authors followed the classical difference of convex ([7]) approach which they also reinterpreted as essentially

a minorization technique ([8]). This was the first time that the transmit beamformer was designed for a MIMO-OFDM case where the Doppler causes non-trivial channel variation within the duration of one OFDM symbol. However, this work assumed perfect CSIT (channel state information at transmitter) for the mean channel and the linear channel variation across the OFDM symbol. In this work, we relax this assumption to that of a partial CSIT and hence consider the optimization of Expected WSR(EWSR). Many techniques exist in the literature towards this end. For instance, the EWSMSE (Expected weighted sum mean squared error) ([9]) approach improves over Naive EWSR (NEWSR) by accounting for covariance CSIT in the interference. However, the EWSMSE approach is suboptimal and cannot even be used in the zero channel mean case (case of covariance CSIT only). Recently, [10] proposed a new approach EWSUMSE (Expected weighted sum unbiased MSE) approach which represents a better approximation of the EWSR. In this work, we follow the approach in [11] that uses large MIMO asymptotics to design a robust beamformer under partial CSIT. We come up with a suitable channel model to enable the transmitter to predict the required channel parameters while accounting for the error in the channel estimates after the prediction. Then, we propose an iterative design of transmit beamformer under partial CSIT, there by making the design more robust and practical. The design also guarantees convergence of the beamformer coefficients.

The rest of the paper is organized as follows. We first present the system model in Section 2. The large MIMO asymptotics is given in 3. This is followed by the design of the beamformer in section 4 and numerical results in section 5. Finally, conclusions are given in section 6. In the following discussions, a bold notation in small letters indicates a vector and bold notation or calligraphic notation with capital letters indicates a matrix.  $\mathbf{E}(\cdot)$  corresponds to the expectation operator and  $\log |\cdot|$  refers to log determinant.

## 2. SYSTEM MODEL

Consider a multiple input multiple output (MIMO) system with  $N_t$  transmit antennas and  $N_r$  receive antennas. An OFDM framework is chosen with  $N$  subcarriers and sam-

pling rate  $f_s$ . Out of the total  $N$  subcarriers, let  $N_u$  be the number of utilized subcarriers. For instance, this would account for the guard subcarriers and DC subcarrier in an OFDM system. Let  $P$  be the maximum sum power requirement across all the subcarriers and let  $P_i$  be the individual power at any subcarrier  $i$  such that  $\sum_{i=0}^{N_u-1} P_i = P$ . Consider a finite delay spread pathwise MIMO channel model as follows.

$$\mathcal{H}(\tau, t) = \underbrace{\mathcal{H}_r(\tau)\mathcal{D}(\tau, t)\mathcal{H}_t^T(\tau)}_{\text{Deterministic path wise model}} + \underbrace{\tilde{\mathcal{H}}(\tau, t)}_{\text{random part}} \quad (1)$$

where  $\mathcal{H}_r$  contains as columns the receive side path antenna array responses. Similarly,  $\mathcal{H}_t$  contains as columns the transmit side path antenna array responses.  $\mathcal{D}(\tau, t)$  is a diagonal matrix that captures the path amplitudes and the Doppler variations of the different paths and is given by  $\mathcal{D}(\tau, t) = \text{diag}(A_1 e^{j\pi f_1 t} \delta(\tau - \tau_1), \dots)$ , where  $f_i$  are the Doppler frequencies,  $A_i$  are the complex path amplitudes and  $\tau_i$  are the path delays. Note here that the time dependency (Doppler dependency) is limited to the diagonal matrix  $\mathcal{D}$ . i.e. other than the influence of the Doppler, the rest of the components are slow fading. We assume that the transmitter is capable of estimating precisely the components of the deterministic part of the channel -  $A_i$ ,  $\tau_i$  and  $f_i$ .  $\tilde{\mathcal{H}}(\tau, t)$  corresponds to the unknown random part of the channel and is the cause for partial CSIT at the transmitter. Using a precise estimate of  $\mathcal{H}_r(\tau)\mathcal{D}(\tau, t)\mathcal{H}_t^T(\tau)$  at time  $t$ , the transmitter predicts a future instance of the channel at a time offset of  $\Delta$  as  $\mathcal{H}_r(\tau)\mathcal{D}(\tau, t + \Delta)\mathcal{H}_t^T(\tau)$  assuming all components other than the Doppler for the deterministic channel component remain constant over the  $\Delta$  time duration. Thus, for the OFDM symbol for which the beamformer has to be designed, the transmitter has the channel estimate corresponding to the deterministic part of the channel.

$$\begin{aligned} \hat{\mathcal{H}}(\tau, t + \Delta) &= \mathcal{H}_r(\tau)\mathcal{D}(\tau, t + \Delta)\mathcal{H}_t^T(\tau) \\ &= \mathcal{H}(\tau, t + \Delta) - \underbrace{\tilde{\mathcal{H}}(\tau, t + \Delta)}_{\text{random error}} \end{aligned} \quad (2)$$

Now, as in [4], the time variation across the OFDM symbol of interest is approximated to be linear. Thus, let  $\mathcal{H}_0(t + \Delta, \tau)$  be the mean of the channel and  $\mathcal{H}_1(t + \Delta, \tau)$  the linear time variation. For the train velocities of interest (up to 450kmph), this is indeed the case and such assumptions have been used extensively in the prior literature ([5], [6], [3]). After FFT (Fast Fourier Transform) at the receiver, the received data at each subcarrier, therefore would be of the following form.

$$\mathbf{y}_k = \mathbf{H}_{0k} \mathbf{d}_k + \sum_{l=0, l \neq k}^{N-1} \mathbf{H}_{1l} \mathbf{d}_l \Xi_{k,l} + \nu_k \quad (3)$$

$\mathbf{H}_{0k}$  (dimension  $N_r \times N_t$ ) is the mean frequency domain channel observed at subcarrier  $k$  and is a result of  $\mathcal{H}_0(t + \Delta, \tau)$ . The second term in equation (3) represents the ICI

(inter carrier interference) caused by time variance due to Doppler.  $\mathbf{H}_{1k}$  is the frequency domain channel component corresponding to  $\mathcal{H}_1(t + \Delta, \tau)$  at subcarrier  $k$ ,  $\mathbf{d}_k = [d_k(1) \cdots d_k(N_t)]^T$  is the  $N_t \times 1$  vector of transmitted data symbols on the carrier  $k$ .  $\nu_k$  is the  $N_r \times 1$  vector of AWGN (additive white Gaussian noise) noise observed at carrier index  $k$ . The variance of  $\nu_k$  is normalized to be unity.

$$\Xi_{k,l} = \frac{1}{N} \sum_{n=0}^{N-1} \left( n - \frac{N-1}{2} \right) e^{j2\pi(k-l)\frac{n}{N}} \quad (4)$$

Now, the prediction errors that result from the unpredictable part of the channel result in errors in  $\hat{\mathcal{H}}_0(t + \Delta, \tau)$ ,  $\hat{\mathcal{H}}_1(t + \Delta, \tau)$  - the estimates of  $\mathcal{H}_0(t + \Delta, \tau)$ ,  $\mathcal{H}_1(t + \Delta, \tau)$ . Correspondingly  $\hat{\mathbf{H}}_{0k}$ ,  $\hat{\mathbf{H}}_{1k}$  (the estimates of  $\mathbf{H}_{0k}$ ,  $\mathbf{H}_{1k}$ ) are also in error. Therefore, to proceed further with the Tx (transmit) beamformer design under partial CSIT, we need a model for the errors in the estimates for  $\mathbf{H}_{0k}$ ,  $\mathbf{H}_{1k}$ .

The unpredicted part of (1) is assumed to have a separable (Kronecker) model for each path delay of the FIR channel model. A linear combination of random matrices each of which have a Kronecker correlation results in another random matrix which still retains a Kronecker correlation model. Hence, after the FFT at the receiver (a linear operation) at each subcarrier  $k$  in the frequency domain,

$$\begin{aligned} \hat{\mathbf{H}}_{0k} &= \mathbf{H}_{0k} - \mathbf{C}_r^{\frac{1}{2}} \tilde{\mathbf{H}}_{0k} \mathbf{C}_t^{\frac{1}{2}} \\ \hat{\mathbf{H}}_{1k} &= \mathbf{H}_{1k} - \beta \mathbf{C}_r^{\frac{1}{2}} \tilde{\mathbf{H}}_{1k} \mathbf{C}_t^{\frac{1}{2}} \end{aligned} \quad (5)$$

where  $\mathbf{C}_r$ ,  $\mathbf{C}_t$  are the receive and transmit side covariances for the error term. The elements of  $\tilde{\mathbf{H}}_{0k}$ ,  $\tilde{\mathbf{H}}_{1k}$  are i.i.d  $\sim \mathcal{CN}(0, 1)$ . Note that as the different channel taps are independent, the covariance matrices are subcarrier independent.  $\beta$  is a real number that signifies the extend of Doppler. As in [4], the transmit beamformer is designed to maximize the weighted sum rate (WSR). Let the transmit covariance matrix of subcarrier  $k$  be  $\mathbf{Q}_k = \mathbf{E}(\mathbf{d}_k \mathbf{d}_k^H) = \mathbf{G}_k \mathbf{G}_k^H$  where  $\mathbf{E}(\cdot)$  is the expectation operator. Thus, the WSR of this MIMO system across all the subcarriers in the presence of both ICI and AWGN noise would be given as  $\text{WSR} = \sum_{k=0}^{N-1} \log |\mathbf{I} + \mathbf{G}_k^H \mathbf{H}_{0k}^H \mathbf{R}_k^{-1} \mathbf{H}_{0k} \mathbf{G}_k|$ . where  $\mathbf{R}_k = \mathbf{I} + \sum_{l=0, l \neq k}^{N-1} |\Xi_{k,l}|^2 \mathbf{H}_{1l} \mathbf{Q}_l \mathbf{H}_{1l}^H$ . Note that this formulation can include guard subcarriers and DC subcarrier by simply forcing their respective transmit covariances to zero. In addition, in this formulation for WSR, the weights are all unity, but this is done only to simplify the notation and help focus on the main part of the work. However, as the CSIT is imperfect, to derive a Tx beamformer that is robust to the imperfections in CSIT, various optimization criterion could be considered, such as outage capacity. Here, we shall

consider the expected weighted sum rate,

$$\begin{aligned} \text{EWSR} = & \sum_{k=0}^{N-1} \mathbf{E}_{(\mathbf{H}_{0k}, \mathbf{H}_{1k}) | (\hat{\mathbf{H}}_{0k}, \hat{\mathbf{H}}_{1k})} \log |\mathbf{I} + \mathbf{G}_k^H \mathbf{H}_{0k}^H \mathbf{R}_k^{-1} \mathbf{H}_{0k} \mathbf{G}_k| \\ \text{subject to } & \sum_{k=0}^{N-1} \text{tr} \{ \mathbf{G}_k \mathbf{G}_k^H \} \leq P. \end{aligned} \quad (6)$$

### 3. LARGE MIMO ASYMPTOTICS

To tackle (6), we pursue the large MIMO asymptotics and alternating optimization for multi-user systems in [11], which are based on the single-user MIMO asymptotics of [12], [13] in which both  $N_t, N_r \rightarrow \infty$  at constant ratio. This approach tends to give better approximations even when  $N_t, N_r$  are not very large. Note that

$$\begin{aligned} \log |\mathbf{I} + \mathbf{G}_k^H \mathbf{H}_{0k}^H \mathbf{R}_k^{-1} \mathbf{H}_{0k} \mathbf{G}_k| &= \log |\mathbf{I} + \mathbf{H}_{0k} \mathbf{Q}_k \mathbf{H}_{0k}^H \mathbf{R}_k^{-1}| \\ &= \log |\mathbf{R}_k| - \log |\mathbf{R}_k| \end{aligned} \quad (7)$$

where  $\mathbf{R}_k = \mathbf{R}_k + \mathbf{H}_{0k} \mathbf{Q}_k \mathbf{H}_{0k}^H$ . For the general case of Gaussian CSIT with separable covariance (which is indeed our case as is seen in (5)), we can write

$$\mathbf{H} = \bar{\mathbf{H}} + \mathbf{C}_{rx}^{\frac{1}{2}} \tilde{\mathbf{H}} \mathbf{C}_{tx}^{\frac{1}{2}} \quad (8)$$

where  $\bar{\mathbf{H}} = \mathbf{E}\mathbf{H}$ , and the elements of  $\tilde{\mathbf{H}}$  are i.i.d  $\sim \mathcal{CN}(0, 1)$ .  $\mathbf{C}_{tx}$  and  $\mathbf{C}_{rx}$  are the Tx and Rx(receive) side covariances respectively. [12], [13] lead to asymptotic expressions of the form

$$\begin{aligned} \mathbf{E}_{\mathbf{H}} \log |\mathbf{I} + \mathbf{H} \mathbf{Q} \mathbf{H}^H| = & \max_{z \geq 0, w \geq 0} \left\{ \log \left| \begin{array}{cc} \mathbf{I} + w \mathbf{C}_{rx} & \bar{\mathbf{H}} \\ -\mathbf{Q} \bar{\mathbf{H}}^H & \mathbf{I} + z \mathbf{Q} \mathbf{C}_{tx} \end{array} \right| - zw \right\} \end{aligned} \quad (9)$$

To get the terms in (6) into the format of (9), at the level of each subcarrier  $k$ , we stack the channel estimates relevant for each subcarrier  $k$ .

$$\begin{aligned} \mathbf{H}_k &= [\mathbf{H}_{10} \Xi_{k,0} \cdots \mathbf{H}_{1,k-1} \Xi_{k,k-1} \quad \mathbf{H}_{0k} \quad \mathbf{H}_{1,k+1} \Xi_{k,k+1} \cdots] \\ &= \bar{\mathbf{H}}_k + \mathbf{C}_r^{\frac{1}{2}} \tilde{\mathbf{H}}_k \mathbf{C}_{tx,k}^{\frac{1}{2}} \\ \mathbf{C}_{tx,k} &= \text{diag}\{\gamma_{k,0} \cdots \gamma_{k,k-1} \quad 1 \quad \gamma_{k,k+1} \cdots \gamma_{k,N-1}\} \otimes \mathbf{C}_t \end{aligned} \quad (10)$$

where the elements of  $\tilde{\mathbf{H}}_k$  are i.i.d  $\sim \mathcal{CN}(0, 1)$  and  $\bar{\mathbf{H}}_k$  refers to the mean part of  $\mathbf{H}_k$ .  $\gamma_{k,l} = \beta^2 |\Xi_{k,l}|^2$  and  $\otimes$  refers to the Kronecker product. Let  $\mathbf{Q}$  be a block diagonal matrix with the each diagonal block being  $\mathbf{Q}_k$ .  $\mathbf{Q}_k$  is similar to  $\mathbf{Q}$  but with the  $k^{\text{th}}$  block diagonal set to all zeros. Then,

$$\mathbf{R}_k = \mathbf{I} + \mathbf{H}_k \mathbf{Q} \mathbf{H}_k^H, \quad \mathbf{R}_k = \mathbf{I} + \mathbf{H}_k \mathbf{Q}_k \mathbf{H}_k^H. \quad (11)$$

Equation (6) now becomes (under large MIMO asymptotics),

$$\begin{aligned} \text{EWSR} = & \sum_{k=0}^{N-1} \left( \max_{z_k, w_k \geq 0} \{ \log |\mathbf{S}_k(\mathbf{Q}, z_k, w_k)| - z_k w_k \} - \right. \\ & \left. \max_{z_k, w_k \geq 0} \{ \log |\mathbf{S}_k(\mathbf{Q}_k, z_k, w_k)| - z_k w_k \} \right) \end{aligned} \quad (12)$$

where

$$\mathbf{S}_k(\mathbf{Q}, z_k, w_k) = \begin{bmatrix} \mathbf{I} + w_k \mathbf{C}_r & \bar{\mathbf{H}}_k \\ -\mathbf{Q} \bar{\mathbf{H}}_k^H & \mathbf{I} + z_k \mathbf{Q} \mathbf{C}_{tx,k} \end{bmatrix} \quad (13)$$

Further, by the rules of determinant for block matrices ([14])

$$\begin{aligned} \log |\mathbf{S}_k(\mathbf{Q}, z_k, w_k)| &= \log |\mathbf{I} + w_k \mathbf{C}_r| + \log |\mathbf{I} + \mathbf{Q} \mathbf{T}_k(z_k, w_k)| \end{aligned} \quad (14)$$

where  $\mathbf{T}_k(z_k, w_k) = z_k \mathbf{C}_{tx,k} + \bar{\mathbf{H}}_k^H (\mathbf{I} + w_k \mathbf{C}_r)^{-1} \bar{\mathbf{H}}_k$  can be seen as some kind of generalized Tx side channel covariance matrix.

### 4. BEAMFORMER DESIGN

The overall optimization involves several iterations of alternating optimization over  $\mathbf{Q}_k, z_k, w_k, z_k, w_k$ . To determine the  $\mathbf{Q}_k$ , we observe the following.

$$\begin{aligned} \log |\mathbf{I} + \mathbf{Q} \mathbf{T}_k(z_k, w_k)| &= \log |\mathbf{I} + \sum_{l=1}^N \mathcal{I}_l \mathbf{Q}_l \mathcal{I}_l^H \mathbf{T}_k(z_k, w_k)| \\ &= \log |\bar{\mathbf{R}}_{k,\bar{k}}| + \log |\mathbf{I} + \mathbf{Q}_k \mathcal{I}_k^H \mathbf{T}_k(z_k, w_k) \bar{\mathbf{R}}_{k,\bar{k}}^{-1} \mathcal{I}_k| \end{aligned} \quad (15)$$

$\bar{\mathbf{R}}_{k,\bar{k}} = \mathbf{I} + \sum_{l \neq k} \mathcal{I}_l \mathbf{Q}_l \mathcal{I}_l^H \mathbf{T}_k(z_k, w_k)$  and  $\mathcal{I}_k$  is a single column block matrix having an identity matrix on  $k^{\text{th}}$  block and zeros elsewhere. Pre-multiplying a matrix with  $\mathcal{I}_k^H$  and post-multiplying it with  $\mathcal{I}_k$  results in selection of  $k^{\text{th}}$  diagonal block of that matrix. Note that  $\mathbf{Q}_k$  refers to the  $k^{\text{th}}$  diagonal block of  $\mathbf{Q}$ . On the same lines as [7], split  $\text{EWSR} = \text{EWSR}_k + \text{EWSR}_{\bar{k}}$ . The derivative of  $\text{EWSR}_{\bar{k}}$  with respect to  $\mathbf{Q}_k$  is given by,

$$\begin{aligned} \mathbf{B}_k &= -\frac{\partial \text{EWSR}_{\bar{k}}}{\partial \mathbf{Q}_k} = \\ & \mathcal{I}_k^H \left( \sum_{l \neq k} [\mathbf{T}_l(z_l, w_l) \bar{\mathbf{R}}_l^{-1} - \mathbf{T}_l(z_l, w_l) \bar{\mathbf{R}}_l^{-1}] \right) \mathcal{I}_k \quad (16) \\ \bar{\mathbf{R}}_l &= \mathbf{I} + \mathbf{Q} \mathbf{T}_l(z_l, w_l) \\ \bar{\mathbf{R}}_{\bar{l}} &= \mathbf{I} + \mathbf{Q}_{\bar{l}} \mathbf{T}_{\bar{l}}(z_{\bar{l}}, w_{\bar{l}}). \end{aligned}$$

Thus, the beamforming directions are obtained as the solution for the generalized eigenmatrix condition,

$$\mathbf{A}_k \mathbf{G}_k = (\mathbf{B}_k + \mu_k \mathbf{I}) \mathbf{G}_k \boldsymbol{\Sigma}. \quad (17)$$

**Table 1.** Overall Algorithm for beamformer design

Initialize $\mathbf{Q}$ , $P_k$ , $w_k$ , $z_k$ , $z_{\bar{k}}$ , $w_{\bar{k}}$ for used subcarriers
Compute $\bar{\mathbf{H}}_k$ for all used subcarriers
Initialize $\mathbf{T}_k(z_k, w_k)$ , $\mathbf{T}_k(z_{\bar{k}}, w_{\bar{k}})$ for used subcarriers
Repeat until convergence
For every used subcarrier $k$
Maximize alternatively $w_k$ , $z_k$ , $z_{\bar{k}}$ , $w_{\bar{k}}$ (see (18))
Compute $\mathbf{T}_k(z_k, w_k)$ , $\mathbf{T}_k(z_{\bar{k}}, w_{\bar{k}})$ for used subcarriers
For every used subcarrier $k$
Update $\mathbf{Q}_k$ based on (17)
For every used subcarrier $k$
Update power allocation $P_k$ , see from [4]

where  $\mathbf{A}_k = \mathcal{I}_k^H \mathbf{T}_k(z_k, w_k) \mathbf{R}_{k,\bar{k}}^{-1} \mathcal{I}_k$ .  $\mu_k$  is the Lagrangian corresponding to the power constraint  $P_k$  at subcarrier  $k$  at the current stage of the iteration and  $\mathbf{\Sigma}$  is a diagonal matrix with non-negative real entries. For further details of power allocation across subcarriers and the interference aware waterfilling, see [4]. Given  $\mathbf{Q}$ , the optimum values of  $z_k, w_k$  is obtained as,

$$w_k = \text{tr}\{\mathbf{Q}\mathbf{C}_{tx}(\mathbf{I} + \mathbf{Q}\mathbf{T}_k(z_k, w_k))^{-1}\}$$

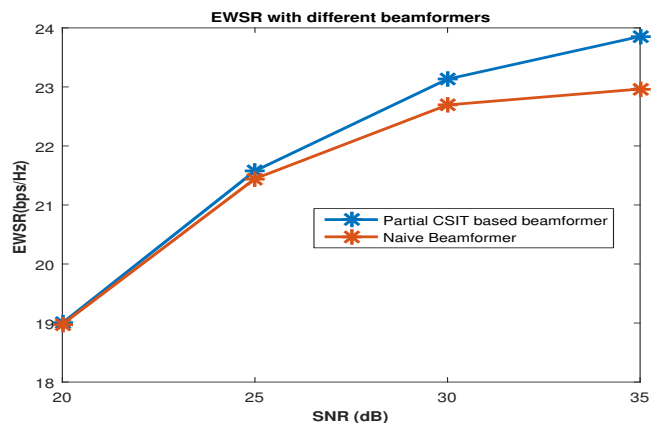
$$z_k = \text{tr}\left\{\mathbf{C}_r(\mathbf{I} + w_k\mathbf{C}_r + \bar{\mathbf{H}}_k(\mathbf{I} + z_k\mathbf{Q}\mathbf{C}_{tx,k})^{-1}\mathbf{Q}\bar{\mathbf{H}}_k^H)^{-1}\right\}. \quad (18)$$

Due to the interdependency between  $w_k$  and  $z_k$ , they have to be iterated among themselves until convergence. The equations for  $z_{\bar{k}}, w_{\bar{k}}$  are similar except for  $\mathbf{Q}$  being replaced by  $\mathbf{Q}_{\bar{k}}$ . The overall steps are summarized in Table 1. As always, there are multiple ways of performing the alternating optimization and this is just one possible approach. Note also that the algorithm only involves steps of minorization (for the updates of  $\mathbf{Q}$ , see also [4]) and steps in alternative minimization, so the convergence is guaranteed. It is also illustrative to observe that in the extreme case of  $\mathbf{C}_r$  and  $\mathbf{C}_t$  being all zeros (implying perfect CSIT), equation (9) is satisfied with  $z = 0, w = 0$  and the algorithm reduces to that given in [4].

## 5. SIMULATIONS

An LTE OFDM system operating at unlicensed 2.4GHz band is considered with 15KHz of channel spacing and 128 subcarriers. For every Tx-Rx pair, FIR Rayleigh fading channels are generated independently with the power delay profile (PDP) as [0 -5 -5] in dB. A Doppler frequency corresponding to 450kmph is assumed. The receive and transmit variance of the un-estimated part of the channel are chosen to be identity matrices reflecting a worst case scenario of no covariance knowledge about the un-estimated part. The total power in the un-estimated part is assumed to be 6dB lower than the es-

timated portion. In the simulation results presented, all subcarriers are assumed to be used. The scale factor  $\beta$  in equation (5) is taken as 0.0033 corresponding to a Doppler variation of 450kmph. For every subcarrier  $k$  parameters  $z_k, w_k, z_{\bar{k}}, w_{\bar{k}}$  are initialized to 0. Figure 1 shows the EWSR computed across 500 different channel realizations with the proposed beamformer for  $N_t = 6, N_r = 3$ . Also shown is the performance with a naive beamformer that does not take into account the unknown error part (partial CSIT) and computes the beamformer using the mean predicted channel. As expected, the gains from explicit use of the partial CSIT information become more pronounced at higher SNR.



**Fig. 1.** EWSR comparison for  $N_t = 6, N_r = 3$

## 6. CONCLUSION

In this paper, we present a method to solve the waterfilling problem for an OFDM system, in the presence of ICI and with only partial CSIT. We formulate a system model with Gaussian mean and covariance CSIT and come up with the EWSR objective be optimized. Large MIMO asymptotics that tend to work even when the antenna dimension are not large is used to re-formulate the problem based on the knowledge of the channel mean and covariance. Once this is done, the classical difference of convex ([7]) approach is used for the beamformer design. As the steps involved are minorization (the difference of convex approach is better interpreted as a minorization) and alternating minimization, the design is guaranteed to converge.

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