Device-Centric Cooperation in Wireless Networks

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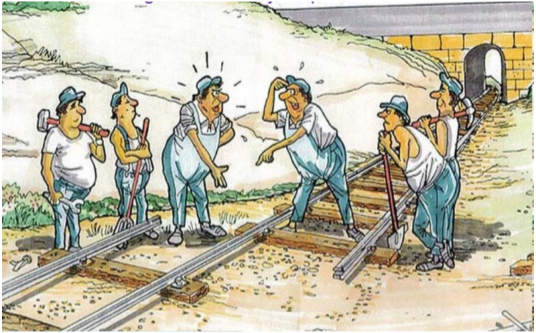
PERFUME

Coordination vs. cooperation

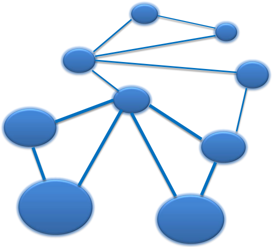
Coordination is a way to resolve complex problems among distributed agents
Can come with a notion of conflict: **coordination** → **cooperation**



Network coordination

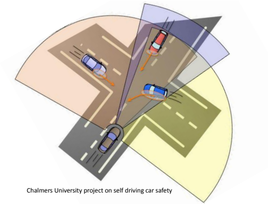
Coordination and cooperation have emerged as central concepts in many types of networks

- Autonomous robots networks
- Transportaton networks
- Sensor networks
- Processor networks
- Energy (Smart Grids) networks
- Wireless networks



Team-playing robots

- Driver-less vehicles
- Autonomous robot patrols
- Plant probes (nuclear sites,..)
- Military drones (ground, air)
- "Smart Factory" robots
- Robot sport teams "Robo-Cup"


Chalmers University project on self driving car safety

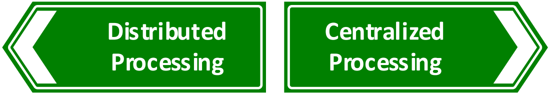
Network coordination is often, by essence, "myopic"

Outline

Outline

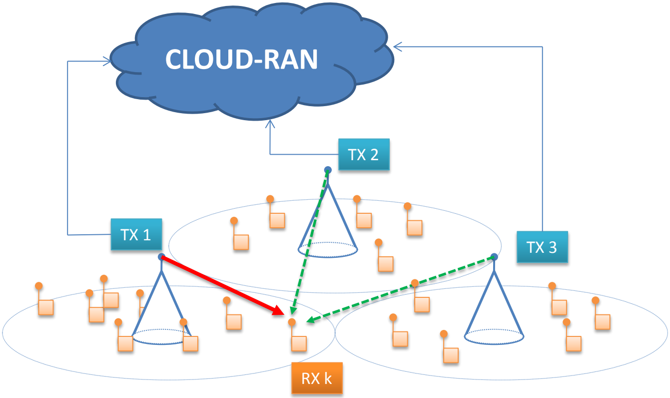
Network/Device cooperation beyond 5G

Where are we going?



Why beyond 5G may be "centralized"

- Cloud RAN is very popular, pushes for more centralization
- Centralized decision making is conceptually simple and efficient
- Coordination, coperation is easy
- Mobile service providers love it

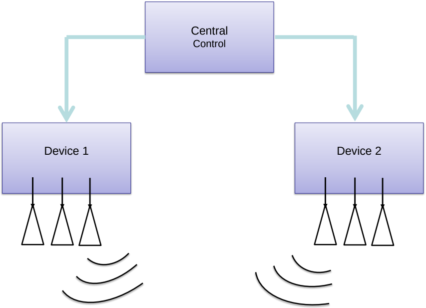


Enhancing spectral efficiency via coordination

Recent spectrum efficiency gains (or promise) from

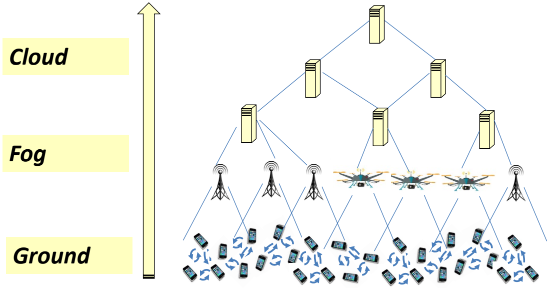
- MU-MIMO, Network MIMO (CoMP), Massive MIMO
- Dynamic cell clustering
- Beamforming
- Power control
- Channel aware scheduling
- Spectrum sharing

All made easy in centralized settings



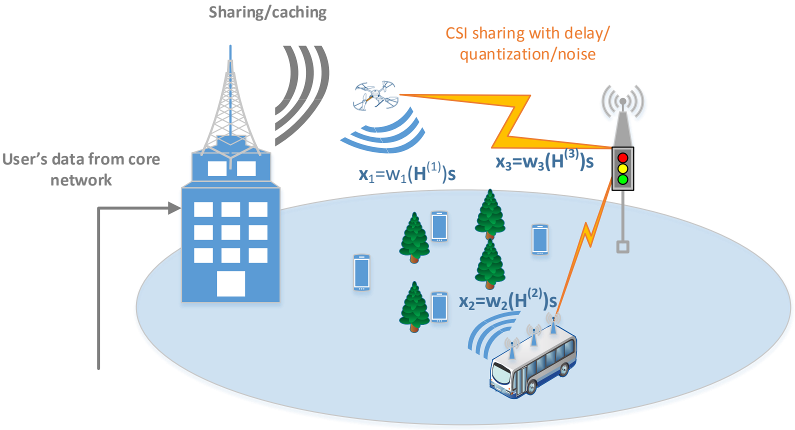
Why beyond 5G may be partly "decentralized"

- Centralization leads to expensive architectures
- Curse of dimension (IoT: billions of devices)
- Centralized processing increases latency, killer for the tactile internet.
- Wireless backhaul architectures are often heterogeneous

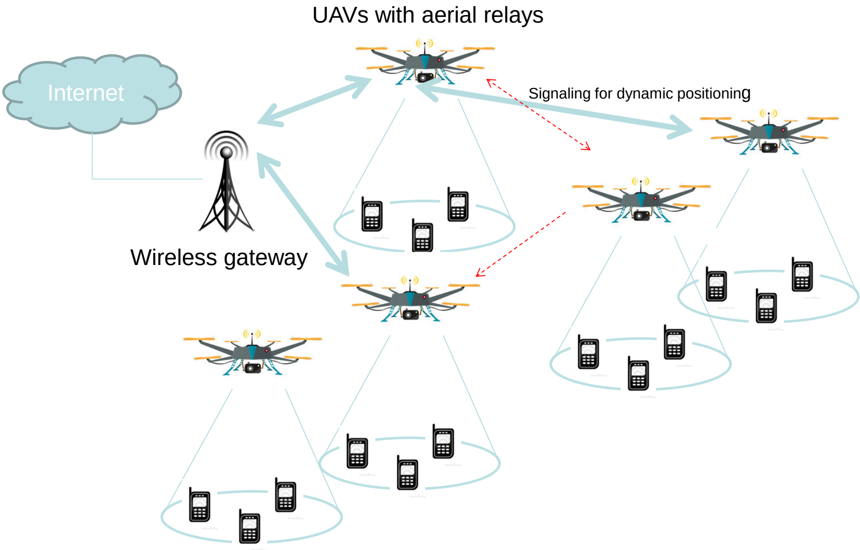


Centralization ↑ Backhaul ↑ latency ↑ timeliness ↓

Cooperation in heterogenous Wireless networks



Ultra flexible Wireless networks



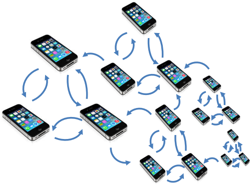
Device-centric Cooperation

Potential:

1. Many devices with substantial sensing/computing capabilities (phones, tablets, vehicles, drones, pico-BS..)
2. Huge collective intelligence
3. Local processing makes time-sensitive measurements more relevant

Challenges

1. How to model distributed information settings?
2. Is there a price of distributedness?
3. Are there robust approaches?



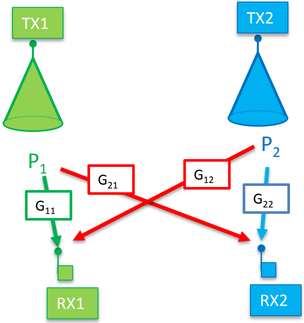
Ex 1: Power control over interference channels

- Two interfering devices, with interference channels $G_{i,j}, i = 1, 2, j = 1, 2$
- Transmit with binary power control is sum-rate optimal [Gjendemsjo et al., 2008, TWC]

$$(p_1^\star, p_2^\star) = \underset{(p_1,p_2)\in\mathcal{P}}{\operatorname{argmax}} \left[R(p_1(\{G_{i,j}\}), p_2(\{G_{i,j}\}))\right]$$

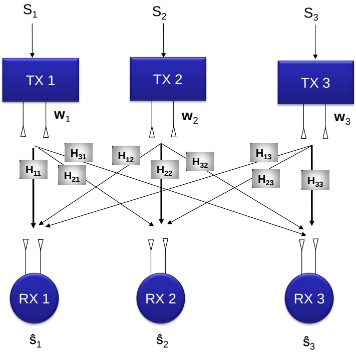
where

$$\mathcal{P} \triangleq \{(p_1, p_2) | p_j : \mathbb{R}^4 \to \{0, P_j^{\max}\}, j = 1, 2\}.$$



Hence the coordinated choice of "full power" or "stay silent" for each device requires full centralized CSI. What if not the case?

Ex 2: Interference Alignment



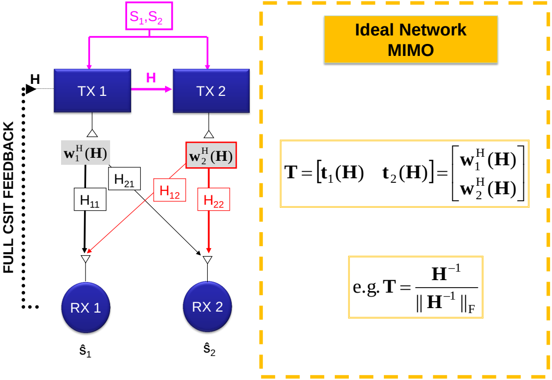
Interference Alignment Conditions with N_r=2:

$$\mathbf{H}_{12}\mathbf{w}_2 \propto \mathbf{H}_{13}\mathbf{w}_3$$
$$\mathbf{H}_{21}\mathbf{w}_1 \propto \mathbf{H}_{23}\mathbf{w}_3$$
$$\mathbf{H}_{31}\mathbf{w}_1 \propto \mathbf{H}_{32}\mathbf{w}_2$$

Alignment can be carried out in space, frequency, time domains. [Maddah-Ali et al., 2008, TIT][Cadambe and Jafar, 2008, TIT]

Realization of alignment conditions requires knowledge of all matrices $\mathbf{H}_{i,j}$ at all transmitters. What if not the case?

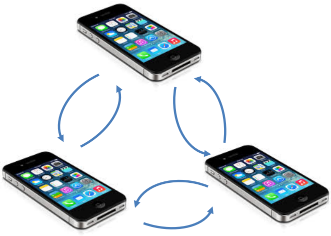
Ex 3: Network MIMO



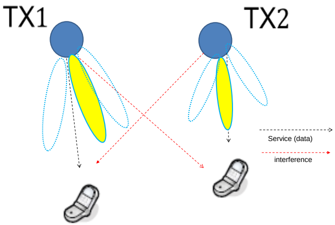
Network MIMO requires full knowledge of global **H** matrix (and data symbols) at all transmitters. What if not the case?

Ex 4: Distributed caching

- D2D can be leveraged for content sharing and caching among terminals [Golrezaei et al., 2014, TIT]
- Popular files can be cached in device memory for later use. Each device can store K files.
- N ideally close-by devices can coordinate to cache non overlapping subsets of K files, hence making the NK most popular files available in their vicinity.
- This requires full information exchange. What if not the case?



Ex 5: Coordinated beamforming/scheduling



- Each transmitter should design a beamforming vector $\mathbf{w}_i, i = 1, 2$
- The best beamformer choice strikes a optimal trade-off between matched filter (egoistic) solution and interference zero-forcing (altruistic) solution [Jorswieck et al., 2008, TSP]
- Optimal design based on knowledge of all direct and interference channel gains. what if not the case?

Ex 6: Cell coloring/clustering

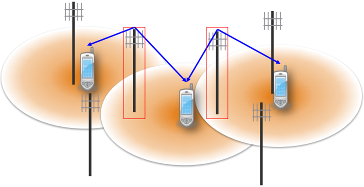


Figure: From Park, Lee, Heath, "Cooperative Base Station Coloring for Pair-wise Multi-Cell Coordination", arXiv March 2015

- Given a limited cooperation cluster size, cells can coordinate with other to design optimal clusters
- Clustering algorithms are usually centralized. But what if cells should attach to a cluster based on local CSI? (i.e. local user gains, local interference gains)
- Decentralized (heuristic) algorithm proposed in [Park et al., 2015]

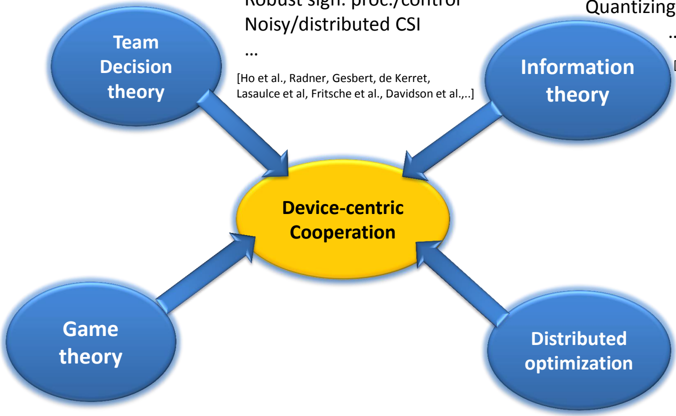
Device coordination: The many perspectives

One-shot decision
Robust sign. proc./control
Noisy/distributed CSI
...

[Ho et al., Radner, Gesbert, de Kerret,
Lasaulce et al, Fritsche et al., Davidson et al.,..]

Capacity/DoF analysis
Coordination theory
Quantizing with side info.
...

[Larousse et al,Cuff et al,
Li et al, Grover, ...]



Study of equilibria
Selfish behavior
Convergence studies
...

[Saad et al, Han et al, McKenzie et al, Lasaulce et al,
Poor et al., Rose et al., Jorswieck et al.,..]

Complexity/Convergence studies
Consensus algorithms
Delay tolerant applications
...

[Boyd et al, Inalhan et al, Colorni et al, Rabbat et al, Chen et al., Johansson et al.,
Palomar et al., Scaglione et al., Scutari et al.]

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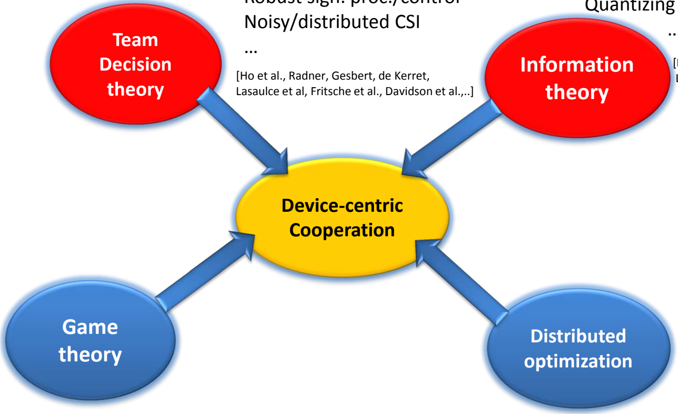

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Outline

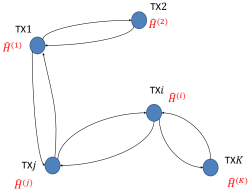
Distributed Information Models

Wireless Channel State Information (CSI) is by nature noisy and distributed

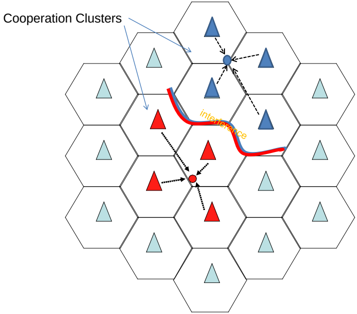
- Limited sensing and feedback
- Mobility
- Devices tend to be "myopic": They know better what is close
- CSI exchange is not free
- Devices do not need to know CSI for entire network

CSI is often transmitter dependent

→ "*Information Structure*"



Information stucture: Clustering



Approaches:

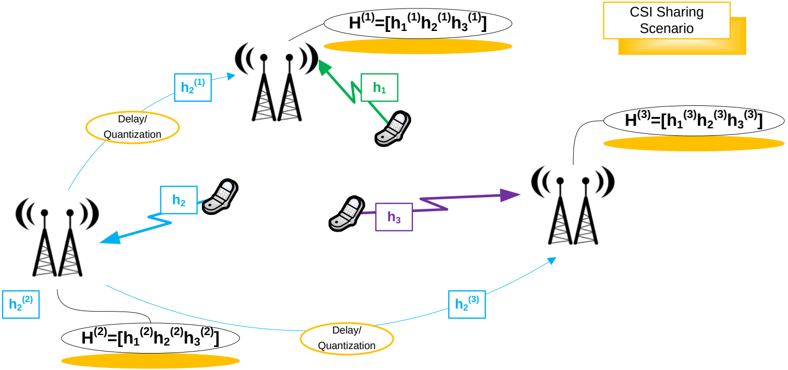
- Network-centric clustering
- User-centric clustering [Papadogiannis et al., 2008, ICC]

Limitations:

- Cluster too big: feedback sharing overhead heavy [Lozano et al., 2013, TIT]
- Cluster too small: edge-effects (inter-cluster interference) predominant

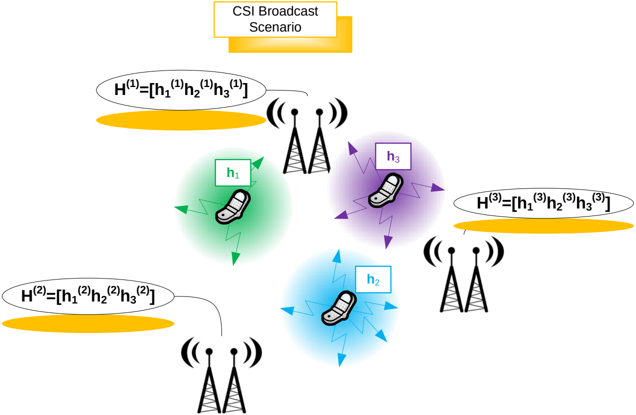
CSI information structure: LTE with limited backhaul

- Backhaul signaling introduces delays and possible quantization noise
- LTE compliant feedback: User feeds back to its home eNB only



CSI information structure: Feedback Broadcast

- CSIT can be shared directly over-the-air without backhaul links



Classical noisy CSI model (centralized)

- Every transmitter shares the same noisy channel estimate
- Imperfect (quantized, noisy, delayed,..) CSIT at TX modeled as [Wagner et al., 2012, TIT]

$$\{\hat{\mathbf{H}}\}_{i,k} = \sqrt{1-\sigma_{i,k}^2}\{\tilde{\mathbf{H}}\}_{i,k} + \sigma_{i,k}\{\mathbf{\Delta}\}_{i,k}, \qquad \forall i,k$$

where $\{\mathbf{\Delta}\}_{ik} \sim \mathcal{CN}(0,1)$

- With digital quantization $\sigma_{i,k}^2 = 2^{-B_{i,k}}$ (good approximation in the high resolution regime)
- CSIT allocation matrix $\mathbf{B}$ defined as

$$\{\mathbf{B}\}_{i,k} = B_{i,k}, \qquad \forall i,k$$

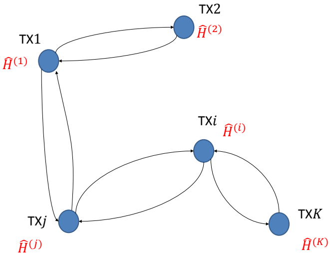
Distributed CSI Model

- CSIT is transmitter-dependent
- LOCAL CSIT at TX j modeled as

$$\{\hat{\mathbf{H}}^{(j)}\}_{i,k} = \sqrt{1-(\sigma_{i,k}^{(j)})^2}\{\mathbf{H}\}_{i,k} + \sigma_{i,k}^{(j)}\{\boldsymbol{\Delta}\}_{i,k}^{(j)}, \qquad \forall i,k$$

where $\{\boldsymbol{\Delta}\}_{i,k}^{(j)} \sim \mathcal{CN}(0,1)$

- $\sigma_{i,k}^{(j)}$ indicates quality of CSIT for channel element (i,k) at TX j



Distributed CSI structure models

Some useful particular cases:

- A CSI structure is *perfect* if $\hat{\mathbf{H}}^{(i)} = \mathbf{H}$, $\forall i$.
- A CSI structure is *centralized* if $\hat{\mathbf{H}}^{(i)} = \hat{\mathbf{H}}^{(j)}$, $\forall i, j$.
- A CSI structure is *distributed* if there exist i and j such that $\hat{\mathbf{H}}^{(i)} \neq \hat{\mathbf{H}}^{(j)}$.

Distributed CSI structure models (cont'd)

Some more particular cases:

- **Incomplete CSIT**: A CSI structure is *incomplete* if $\hat{\mathbf{H}}^{(i)}$ takes the form $\forall i\ \hat{\mathbf{H}}^{(i)} = \{\mathbf{H}_{k,l}, k \in \mathcal{S}_{\mathrm{TX}}, l \in \mathcal{S}_{\mathrm{RX}}\}$, where $\mathcal{S}_{\mathrm{TX}}$ (resp. $\mathcal{S}_{\mathrm{RX}}$) are subsets of the transmitter set (resp. receiver set).
- **Hierarchical CSIT**: A CSI structure is *hierarchical* if there exists an order of transmitter indices i_1, i_2, i_3.. such that $\hat{\mathbf{H}}^{(i_1)} \subset \hat{\mathbf{H}}^{(i_2)} \subset \hat{\mathbf{H}}^{(i_3)} \subset \ldots$
- **Master Slave**: Hierarchical where $\hat{\mathbf{H}}^{(i_1)} = [\ ]$, and $\hat{\mathbf{H}}^{(i_2)} = \mathbf{H}$ (can be extended to $K > 2$.)

Typical (practical) CSI structures

Consider the K transmitter (N antennas each) K user (single antenna) channel. Let $\boldsymbol{h}_{i,j}^{\mathrm{H}}$ be the $1 \times N$ vector channel between the jth transmitter and the ith user.

- Local CSIT with TDD reciprocity

$$(\hat{\mathbf{H}}^{(j)})^{\mathrm{H}} = \begin{bmatrix} \mathbf{0} & \boldsymbol{h}_{1,j}^{\mathrm{H}} & \mathbf{0} \\ \vdots & \vdots & \vdots \\ \mathbf{0} & \boldsymbol{h}_{K,j}^{\mathrm{H}} & \mathbf{0} \end{bmatrix}$$

- Local CSIT with LTE feedback mode

$$(\hat{\mathbf{H}}^{(j)})^{\mathrm{H}} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{h}_{j,1}^{\mathrm{H}} & \dots & \boldsymbol{h}_{j,K}^{\mathrm{H}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

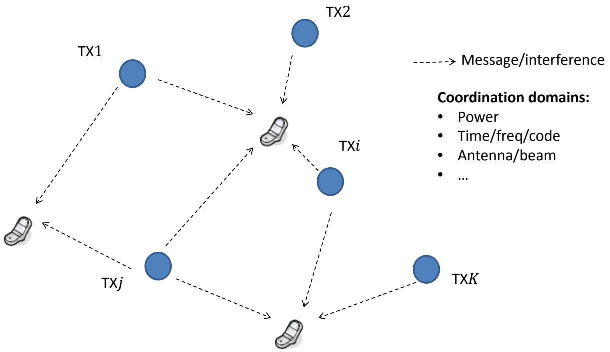
- Fully local CSIT

$$(\hat{\mathbf{H}}^{(j)})^{\mathrm{H}} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{h}_{j,j}^{\mathrm{H}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

Outline

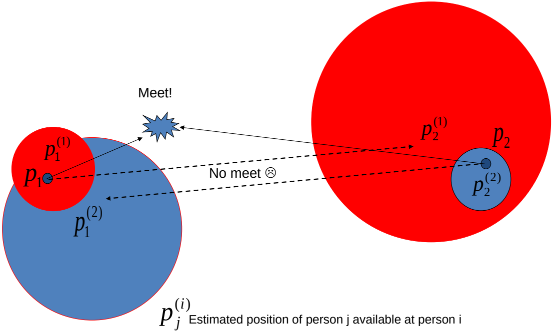
Device coordination Problem

- K nodes in a network seek to cooperate towards the maximization of a common utility
- Each node i must make best decision based on:
    - local measurement or feedback
    - finite rate signaling with neighbor nodes

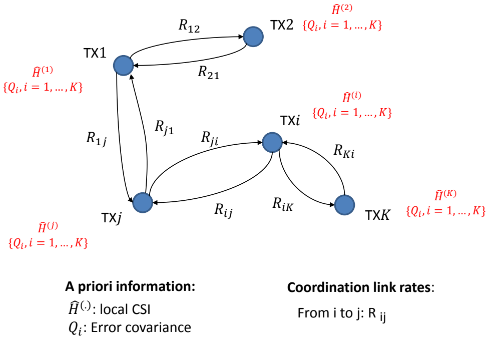


TD example: The Distributed Rendez-vous Problem

- Two visitors arrive independently in Edinburgh and seek to meet as quickly as possible.
- They have different and imprecise information about their own and each other's position.
- **Problem**: *Pick a direction to walk into*



Coordination over finite communication graphs: The big picture



- No constraint over number bits exchanged: Distributed optimization $\rightarrow$ convergence speed?
- Constraint over number of bits exchanged: What to measure? What is the most relevant information to communicate among devices?
- Decision stage (after limited communication took place): What are robust coordinated decision techniques?
- Joint communication-decision framework (challenging)

Signaling for Coordination

What is most relevant to communicate of the signaling link?

- Many interesting heuristics (precoding decisions, measurements, etc.)
- Optimal signaling strategy coupled with optimum decision making W_i
- Heuristic strategies:
  1. Local decision W_i based on $\hat{\mathbf{H}}^{(i)}$ and $\mathbf{Q}_i, i = 1, .., K$, exchange quantized decisions over R_{ij} bits
     - But poorly informed nodes make bad decisions !
  2. Exchange quantized CSI $\hat{\mathbf{H}}^{(i)}$ over R_{ij} bits
     - But this ignores $\mathbf{Q}_i$!
- Optimal strategy (source coding with side-information): Create locally optimal codebooks, that are function of local CSI and neighbor CSI qualities [Li et al., 2014]

Distributed coordination

Team Decision theoretic problems:

- Several network agents wish to cooperate towards maximization of a common utility
- Each agent has its own limited view over the system state
- All need to come up with consistent actions
- Classical "robust" design does not work...
- Introduced first in economics and control [Witsenhausen68] [Ho, 1980, IEEE], recently in wireless [Zakhour and Gesbert, 2010, ITA]
- Fundamental limits rooted in Coordination Theory

Coordination Theory

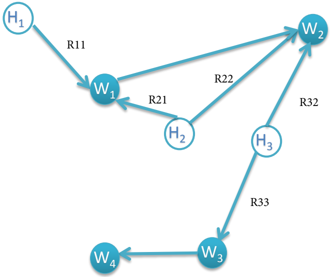


Figure: Coordination Framework[Cuff et al., 2010, TIT]

- $\mathbf{H}_1$, $\mathbf{H}_2$ and $\mathbf{H}_3$, arbitrary components of global system state, distributed according to $p_0(\mathbf{H}_1, \mathbf{H}_2, \mathbf{H}_3)$
- W_1, W_2 and W_3 are actions selected by the nodes.
- What joint distribution $p_0(\mathbf{H}_1, \mathbf{H}_2, \mathbf{H}_3)p(W_1, W_2, W_3|\mathbf{H}_1, \mathbf{H}_2, \mathbf{H}_3)$ can be achieved?
- Answer: it depends on graph topology (capacity of each edge)

Example (One Isolated Node)

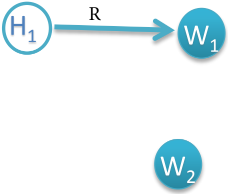


Figure: One isolated node scenario [Cuff et al., 2010, TIT]

Theorem

$$\mathcal{C}_{p_0} = \left\{ (\overbrace{R}^{rate}, \overbrace{p(W_1, W_2 | \mathbf{H}_1)}^{distribution}) \middle| R \geq I(\mathbf{H}_1; W_1 \middle| W_2) \right\}$$

Example (One Isolated Node)

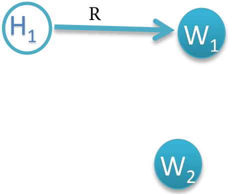


Figure: One isolated node scenario [Cuff et al., 2010, TIT]

- With Gaussian RVs, the condition becomes

$$(1 - 2^{-2R})^{-1}\rho^2_{H_1,W_1} + \rho^2_{W_1,W_2} \leq 1$$

- $R \to \infty$, $\rho^2_{H_1,W_1} + \rho^2_{W_1,W_2} \leq 1$
- $R \to 0$, $\rho^2_{H_1,W_1} = 0$ and $\rho^2_{W_1,W_2} \leq 1$

Further Results

- Results in more advanced topologies [Cuff et al., 2010, TIT]
- Polar codes used for coordination in [Chou et al., 2015, ISIT]
- Implicit coordination: Observation of action of one node by another is a non dedicated cooperation link
  - ➥ Coordination at low/no cost [Larrousse and Lasaulce, 2013, ISIT]
- Aim of this approach
  - Guidelines for network design
  - Insights for new cooperation methods

Team Decision (TD) Problems: A general formalism

$$(\boldsymbol{s}_1^\star, \dots, \boldsymbol{s}_K^\star) = \underset{\boldsymbol{s}_1,\dots,\boldsymbol{s}_K}{\operatorname{argmax}} \, \mathbb{E}_{\boldsymbol{x},\boldsymbol{x}^{(1)},\dots,\boldsymbol{x}^{(K)}} \left[f\left(\boldsymbol{x}, \boldsymbol{s}_1(\boldsymbol{x}^{(1)}), \dots, \boldsymbol{s}_K(\boldsymbol{x}^{(K)})\right)\right]$$

where

- K: Number of Decision Makers (DMs)
- $\boldsymbol{x} \in \mathbb{C}^m$: State of the world
- $\boldsymbol{x}^{(j)} \in \mathbb{C}^m$: Estimate of the state of the world $\boldsymbol{x}$ at DM j
- $\boldsymbol{s}_j : \mathbb{C}^m \to \mathcal{A}_j \subset \mathbb{C}^{d_j}$: Strategy of the j-th DM
- $\boldsymbol{s}_j(\boldsymbol{x}^{(j)}) \in \mathcal{A}_j \subset \mathbb{C}^{d_j}$: Decision at DM j for the given realization $\boldsymbol{x}^{(j)}$
- $f : \mathbb{C}^m \times \Pi_{j=1}^K \mathbb{C}^{d_j} \to \mathbb{R}$: Joint objective of the K DMs
- $p_{\boldsymbol{x},\boldsymbol{x}^{(1)},\dots,\boldsymbol{x}^{(K)}}$: Joint probability distribution of the channel and the estimates

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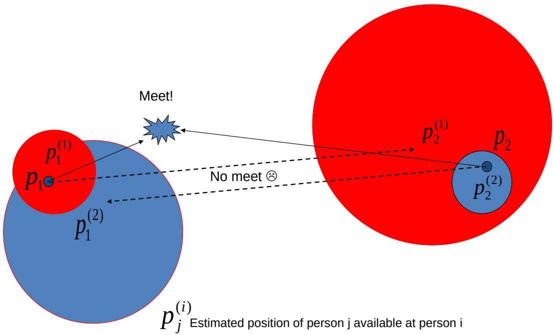


A robust solution: "Meet you at the City Hall!"

Can Team Problems be Solved with Games?

Key idea: Let autonomous transmitting devices interact to solve their interference conflicts
Players $\rightarrow$ transmitters
Actions $\rightarrow$ transmit decision (power, frequency, beam, ..)
Strategy $\rightarrow$ Utility maximization (max rate, min power, min delay,..)
Timing $\rightarrow$ simultaneous, sequential,..
Equilibrium $\rightarrow$ Nash, Stackelberg, Nash Bargaining,..

From Selfish Games to "Team Playing"

Why interference coordination can be different from a typical "game":

- Team agents (network nodes) are not conflicting players (different from players in a cooperative game)
- Agents seek maximization of the same network utility
- It is the lack of shared information which hinders cooperation, not the selfish of their interests
- Agents are not required to improve over the performance of the Nash equilibrium
- Connections to Bayesian games (see work by 1994 Nobel Prize winner John Harsanyi [Harsanyi, 1967])

A Fundamental Approach: Best Response

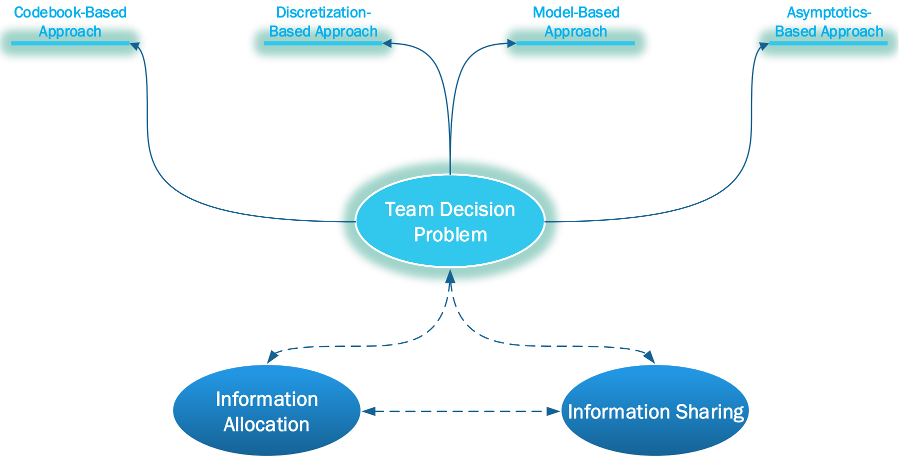
Best Response

A *Best-Response (BR)* strategy $\boldsymbol{s}_1^{\mathrm{BR}}, \dots, \boldsymbol{s}_K^{\mathrm{BR}}$ for the TD problem is a strategy such that

$$\boldsymbol{s}_j^{\mathrm{BR}} = \operatorname*{argmax}_{\boldsymbol{s}_j \in \mathcal{A}_j} \mathbb{E}_{\boldsymbol{x},\boldsymbol{x}^{(1)},\dots,\boldsymbol{x}^{(K)}|\boldsymbol{x}^{(j)}} \left[f\left(\boldsymbol{x}, \dots, \boldsymbol{s}_{j-1}^{\mathrm{BR}}(\boldsymbol{x}^{(j-1)}), \boldsymbol{s}_j(\boldsymbol{x}^{(j)}), \boldsymbol{s}_{j+1}^{\mathrm{BR}}(\boldsymbol{x}^{(j+1)}), \dots \right) \right], \quad \forall j$$

- Practical approach usually considered in the TD literature
- Still very challenging:
  - Functional optimization
  - Stochastic optimization
  - Channel space of large dimension (in most of the cases)
- In fact, Bayesian Cooperative Game with Incomplete Information [Harsanyi, 1967, Management Science]

Team Decision: Algorithm design



Model-Based Approach

Main idea: Restrict the space of possible strategies via a model

➥ Replace the strategy s_j by s_j^β with $\beta \in \mathbb{R}$ where s_j^β is a well chosen heuristic model

Example (Coordinated Beamforming [Jorswieck et al., 2008, TSP])

Beamformer in the MISO IC parameterized as

$$\boldsymbol{w}_k^\star(\lambda_k) = \frac{\lambda_k \boldsymbol{w}_k^{\mathsf{ZF}} + (1-\lambda_k)\boldsymbol{w}_k^{\mathrm{MF}}}{\|\lambda_k \boldsymbol{w}_k^{\mathsf{ZF}} + (1-\lambda_k)\boldsymbol{w}_k^{\mathrm{MF}}\|}$$

Model-based Team Decision Buying a Baguette or not?

- A french couple returns separately from work and wants baguette for dinner. their phone batteries are empty
- Personal cost for stopping at the baker is c_i .
- Each person knows its own cost c_i
- The c_i are uniformly distributed over $[0, 1]$.

Goal: maximize expectation of joint utility given by:

Person 2\Person 1	***Buy bread***	***Go home***
Buy bread	*a-c_1-c_2*	*1-c_1*
Go home	*1-c_2*	*0*

When should each person buy bread?

Optimal decision $\gamma_i^*(c_i)$ *of threshold form*

$$\boxed{\gamma_i^*(c_i) = \begin{cases} \text{Buy bread if } c_i \leq c_i^{th} \\ \text{Go home if } c_i > c_i^{th} \end{cases}}$$

Codebook-Based Approach

Main idea: Restrict the space of possible strategies to a codebook

➥ Choose s_j inside a codebook of function $\{s_j^1, \dots, s_j^m\}$

Example (Coordinated Beamforming)

- Restrict possible beamforming choices to $\mathcal{C} = \{\text{Matched Filter}, \text{Zero Forcing}\}$

Discretization-Based (1): Dimensionality Reduction

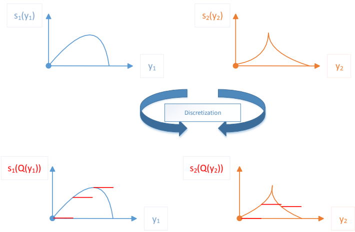
Main idea: Quantize the channel state space to reduce the dimension

➥ Replace the strategy s_j by $s_j(Q^{\mathrm{cb}})$ where

$$\begin{array}{rccc} Q^{\mathrm{cb}}: & \mathbb{C}^m & \to & \mathcal{C}^{\mathrm{cb}} \triangleq \{\boldsymbol{x}_1, \dots, \boldsymbol{x}_{n^{\mathrm{cb}}}\} \\ & \boldsymbol{x}^{(j)} & \mapsto & Q^{\mathrm{cb}}(\boldsymbol{x}^{(j)}) = \operatorname{argmin}_{\boldsymbol{x} \in \mathcal{C}^{\mathrm{cb}}} \|\boldsymbol{x} - \boldsymbol{x}^{(j)}\|^2 \end{array}$$

- Optimization subspace reduced to a space of dimension n^{cb}:

$$\begin{array}{rccc} s_j: & \mathcal{C}^{\mathrm{cb}} & \to & \mathcal{A}_j \\ & \boldsymbol{x}_i & \mapsto & s_j(\boldsymbol{x}_i) \end{array}$$



Discretization-Based (2): Monte-Carlo Approximation

- Best-response optimization at DM 1: $\forall i_1 \in \{1, \dots, n^{\mathrm{cb}}\}$,

$$s_1^{\mathsf{BR}}(\boldsymbol{x}_{i_1}) = \underset{S_1 \in \mathcal{A}_1 \subset \mathbb{C}^{d_1}}{\operatorname{argmax}} \; \mathbb{E}\left[f(\boldsymbol{x}, S_1, s_2^{\mathsf{BR}}(\mathcal{Q}^{\mathrm{cb}}(\boldsymbol{x}^{(2)})), \dots, s_K^{\mathsf{BR}}(\mathcal{Q}^{\mathrm{cb}}(\boldsymbol{x}^{(K)}))) \middle| \boldsymbol{x}^{(1)} = \boldsymbol{x}_{i_1} \right]$$

- For a given $\boldsymbol{x}_i$ and given $s_2^{\mathsf{BR}}, \dots, s_K^{\mathsf{BR}}$: Standard stochastic optimization problem [Shapiro et al., 2014]
- Use Monte-Carlo approximation: $\forall i \in \{1, \dots, n^{\mathrm{cb}}\}$,

$$s_1^{\mathsf{BR}}(\boldsymbol{x}_i) = \underset{\mathbf{s}_1 \in \mathcal{A}_1}{\operatorname{argmax}} \frac{1}{n^{\mathrm{MC}}} \sum_{\ell=1}^{n^{\mathrm{MC}}} \mathrm{f}\left(\boldsymbol{x}_\ell, \mathbf{S}_1, \boldsymbol{s}_2(\mathcal{Q}(\boldsymbol{x}_\ell^{(2)})), \dots, \boldsymbol{s}_K(\mathcal{Q}(\boldsymbol{x}_\ell^{(K)}))\right)$$

where $\left(\boldsymbol{x}_\ell, \boldsymbol{x}_\ell^{(2)}, \dots, \boldsymbol{x}_\ell^{(K)}\right) \sim p_{\boldsymbol{x}, \boldsymbol{x}^{(2)}, \dots, \boldsymbol{x}^{(K)} \,|\, \boldsymbol{x}^{(1)} = \boldsymbol{x}_i}$

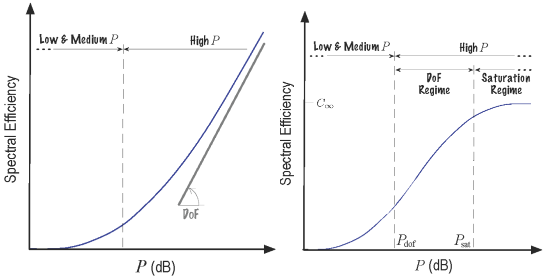
Asymptotics-Based Approach

Main idea: use asymptotic analysis to make the problem deterministic

➥ Possible to obtain new insights and transmission strategies

Example (DoF Analysis)

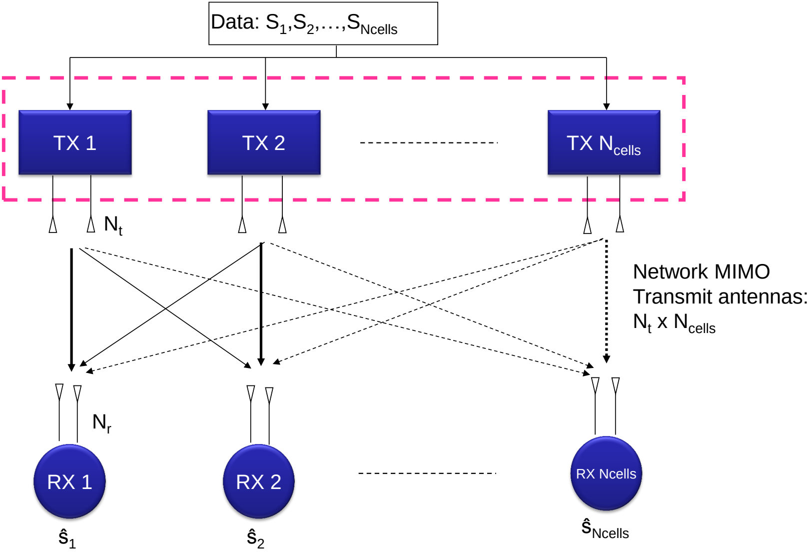
- Let the transmit SNR goes to infinity



* A. Lozano et al, "Fundamental limits of cooperation", IEEE Trans. On Information Theory, Sept. 2013.

Outline

Joint Precoding over Network MIMO



Team Decision Problem

$$(\boldsymbol{w}_1^\star, \ldots, \boldsymbol{w}_K^\star) = \underset{(p_1,\ldots,p_K)\in\mathcal{P}}{\operatorname{argmax}} \mathbb{E}[\mathrm{R}(\mathbf{H}, \boldsymbol{w}_1(\hat{\mathbf{H}}^{(1)}), \ldots, \boldsymbol{w}_K(\hat{\mathbf{H}}^{(K)}))]$$

where

$$\mathrm{R}(\mathbf{H}, \boldsymbol{w}_1(\hat{\mathbf{H}}^{(1)}), \ldots, \boldsymbol{w}_K(\hat{\mathbf{H}}^{(K)})) = \sum_{k=1}^{K} \log_2 \left| \mathbf{I}_{d_k} + \mathbf{T}_k^{\mathrm{H}} \mathbf{H}_k^{\mathrm{H}} \left(\mathbf{R}_k + \sum_{i\neq k} \mathbf{H}_i \mathbf{T}_i \mathbf{T}_i^{\mathrm{H}} \mathbf{T}_i^{\mathrm{H}} \right)^{-1} \mathbf{H}_k \mathbf{T}_k \right|$$

with

- $\mathbf{H} \in \mathbb{C}^{N_{\text{tot}} \times M_{\text{tot}}}$ the multi-user channel
- $\boldsymbol{w}_j$ the precoding function:

$$\begin{aligned} \boldsymbol{w}_j : \quad \mathbb{C}^{N_{\text{tot}} \times M_{\text{tot}}} &\rightarrow \mathbb{C}^{M_j \times d_{\text{tot}}} \\ \hat{\mathbf{H}}^{(j)} &\mapsto \boldsymbol{w}_j(\hat{\mathbf{H}}^{(j)}) \end{aligned}$$

- $\mathbf{T} \in \mathbb{C}^{M_{\text{tot}} \times d_{\text{tot}}}$ the multi-user precoder

$$\mathbf{T} = \begin{bmatrix} \mathbf{T}_1 & \ldots & \mathbf{T}_K \end{bmatrix} = \begin{bmatrix} \boldsymbol{w}_1(\hat{\mathbf{H}}^{(1)}) \\ \vdots \\ \boldsymbol{w}_K(\hat{\mathbf{H}}^{(K)}) \end{bmatrix}$$

A Key Example

- Particularly interesting because:
  - Continuous optimization with large channel state dimension
  - Strong dependency (state & TXs): see DoF results
- ➥ Many difficulties

Table: Team Decision Modeling for Joint Precoding

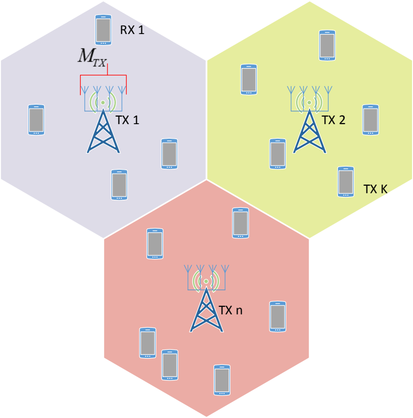
Notations for the Team Decision Problems		
State-of-the-world	$\boldsymbol{x}$	$\mathbf{H}$
Estimate at DM j	$\boldsymbol{x}^{(j)}$	$\hat{\mathbf{H}}^{(j)}$
Strategy at DM j	$\boldsymbol{s}_j$	$\boldsymbol{w}_j$
Decision space at DM j	$\mathcal{A}_j$	$\mathbb{C}^{M_j \times d_{\text{tot}}}$
Objective	f	R

Outline

A Result Based on Random Matrix Theory (RMT)

- n cooperating TXs
- Each TX has M_{TX} antennas
- K and M_{TX} grow large at the same rate

$$\beta \triangleq \lim_{M,K\to\infty} \frac{M}{K} \triangleq \lim_{M_{TX},K\to\infty} \frac{nM_{\mathrm{TX}}}{K} \geq 1$$



Distributed CSI with Correlated Noise

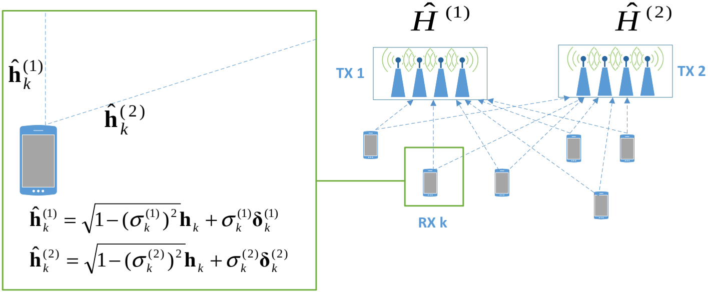
- Extend to spatial correlation in the CSI noise

$$\hat{\boldsymbol{h}}_k^{(j)} \triangleq \sqrt{1-(\sigma_k^{(j)})^2}\boldsymbol{h}_k + \sigma_k^{(j)}\boldsymbol{\delta}_k^{(j)}$$

with

$$\mathbb{E}\left[\boldsymbol{\delta}_k^{(j)}(\boldsymbol{\delta}_k^{(j')})^{\mathrm{H}}\right] = (\rho_k^{(j,j')})^2\mathbf{I}_M$$

- Extremely general model: Bridges the Gap from distributed CSIT to centralized CSIT: Can model partially centralized settings



A Practical Example

Example

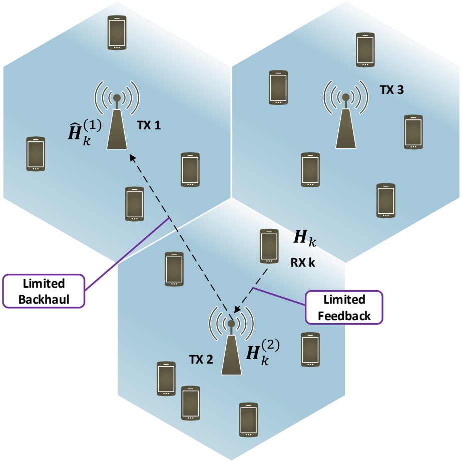
- Imperfect feedback

$$\hat{\boldsymbol{h}}_j^{(j)} = \sqrt{1-\sigma_{FB}{}^2}\boldsymbol{h}_j + \sigma_{FB}\boldsymbol{\delta}_j^{(j)}$$

- Imperfect backhaul

$$\hat{\boldsymbol{h}}_k^{(j')} = \sqrt{1-\sigma_{BH}{}^2}\hat{\boldsymbol{h}}_k^{(j)} + \sigma_{BH}\boldsymbol{\epsilon}_k^{(j,j')}$$

➥ CSI estimates error at different TXs are correlated



Model-Based Approach: Regularized ZF

- Modelization of the precoding decisions using Regularized ZF with sum power constraint P

$$\mathbf{T}_{\text{rZF}}^{(j)}\left(\gamma^{(j)}\right) \triangleq \left((\hat{\mathbf{H}}^{(j)})^{\mathrm{H}}\hat{\mathbf{H}}^{(j)} + M\gamma^{(j)}\mathbf{I}_M\right)^{-1}(\hat{\mathbf{H}}^{(j)})^{\mathrm{H}}\frac{\sqrt{P}}{\sqrt{\Psi^{(j)}}}$$

  with $\Psi^{(j)}$ the power normalization at TX j, and

$$\boldsymbol{w}_j(\hat{\mathbf{H}}^{(j)}) = \mathbf{E}_j^{\mathrm{H}}\mathbf{T}_{\text{rZF}}^{(j)}\left(\gamma^{(j)}\right)$$

  Where $\mathbf{E}_j$ is a row selection matrix

- Effective precoder is

$$\mathbf{T}^{\text{DCSI}} \triangleq \begin{bmatrix} \boldsymbol{w}_1(\hat{\mathbf{H}}^{(1)}) \\ \boldsymbol{w}_2(\hat{\mathbf{H}}^{(2)}) \\ \vdots \\ \boldsymbol{w}_n(\hat{\mathbf{H}}^{(n)}) \end{bmatrix}$$

Optimization of the Regularization Parameter

- Naive regularization

$$\gamma^{(j),\mathrm{naive}} = \mathrm{argmax}_{\gamma\in\mathbb{R}}\mathbb{E}[\mathrm{R}(\hat{\mathbf{H}}^{(j)},\ldots,\hat{\mathbf{H}}^{(j)})]$$

- Robust regularization

$$(\gamma^{(1),\star},\ldots,\gamma^{(n),\star}) = \underset{(\gamma^{(1)},\ldots,\gamma^{(n)})}{\mathrm{argmax}}\ \mathbb{E}[\mathrm{R}(\hat{\mathbf{H}}^{(1)},\ldots,\hat{\mathbf{H}}^{(n)})].$$

- Low complexity robust regularization with equal γ at all TXs

$$(\gamma^{\star},\ldots,\gamma^{\star}) = \underset{(\gamma,\ldots,\gamma)}{\mathrm{argmax}}\,\mathbb{E}[\mathrm{R}(\hat{\mathbf{H}}^{(1)},\ldots,\hat{\mathbf{H}}^{(n)})].$$

Main Result (1)

Theorem ([Li et al., 2015, Allerton])

In Joint Processing CoMP with Distributed CSI,

$$\mathrm{SINR}_k - \mathrm{SINR}_k^o \xrightarrow[K, M_{\mathrm{TX}} \to \infty]{a.s.} 0$$

with

$$\mathrm{SINR}_k^o \triangleq \frac{P\left(\frac{1}{n}\sum_{j=1}^{n}\sqrt{\frac{c_{0,k}^{(j)}}{\Gamma_{j,j}^o}}\frac{\delta^{(j)}}{1+\delta^{(j)}}\right)^2}{1+I_k^o}$$

with

$$c_{0,k}^{(j)} \triangleq 1-(\sigma_k^{(j)})^2, \quad c_{1,k}^{(j)} \triangleq (\sigma_k^{(j)})^2, \quad c_{2,k}^{(j)} \triangleq \sigma_k^{(j)}\sqrt{1-(\sigma_k^{(j)})^2}.$$

Main Result (2)

Theorem (continued)

$$I_k^o \triangleq P - P \sum_{j=1}^{n} \sum_{j'=1}^{n} \frac{\Gamma_{j,j'}^o}{\sqrt{\Gamma_{j,j}^o \Gamma_{j',j'}^o}} \left[\frac{2c_{0,k}^{(j)}}{n^2} \frac{\delta^{(j)}}{1+\delta^{(j)}} \frac{\left((\rho_k^{(j,j')})^2 c_{2,k}^{(j)} c_{2,k}^{(j')} + c_{0,k}^{(j)} c_{0,k}^{(j')} \right) \delta^{(j)} \delta^{(j')}}{n^2 \left(1+\delta^{(j)}\right) \left(1+\delta^{(j')}\right)} \right]$$

with

$$\Gamma_{j,j'}^o \triangleq \frac{\frac{1}{M} \sum_{\ell=1}^{K} \sqrt{c_{0,\ell}^{(j)} c_{0,\ell}^{(j')}} + \sqrt{c_{1,\ell}^{(j)} c_{1,\ell}^{(j')}} (\rho_\ell^{(j,j')})^2}{\frac{1+\delta^{(j)}}{\delta^{(j)}} \frac{1+\delta^{(j')}}{\delta^{(j')}} - \frac{1}{M} \sum_{\ell=1}^{K} \left(\sqrt{c_{0,\ell}^{(j)} c_{0,\ell}^{(j')}} + \sqrt{c_{1,\ell}^{(j)} c_{1,\ell}^{(j')}} (\rho_\ell^{(j,j')})^2 \right)^2}.$$

and

$$\delta^{(j)} \triangleq \frac{\beta - 1 - \gamma^{(j)}\beta + \sqrt{(\gamma^{(j)}\beta - \beta + 1)^2 + 4\gamma^{(j)}\beta^2}}{2\gamma^{(j)}\beta}.$$

Sanity Checks (1)

- Imperfect centralized CSIT:

$$\sigma_k^{(j)} = \sigma_k^{(j')} = \sigma_k, \qquad \text{(equal CSIT accuracy)}$$
$$\rho_k^{(j,j')} = 1, \qquad \text{(Full correlation)}$$
$$\gamma^{(j)} = \gamma^{(j')} = \gamma, \qquad \text{(Equal regularization)}$$

➥ Matches with [Wagner et al., 2012, TIT], [Couillet and Debbah, 2011, Theorem 14.1]

$$\mathrm{SINR}_k^{ID-DCSI,o} = \frac{(1-\sigma_k^2)\delta^2}{\Gamma^o\left(1-\sigma_k^2+(1+\delta)^2\sigma_k^2+\frac{(1+\delta)^2}{P}\right)}$$

- Also obtained with $n = 1$

Sanity Checks (2)

- Uncorrelated distributed CSIT with uniform accuracy and equal regularization:

$$\sigma_k^{(j)} = \sigma^{(j)}, \qquad \text{(Uniform CSIT)}$$
$$\rho_k^{(j,j')} = 0, \qquad \text{(Uncorrelated)}$$
$$\gamma^{(j)} = \gamma^{(j')} = \gamma, \qquad \text{(Equal regularization)}$$

➥ Matches with [de Kerret et al., 2015, ISIT]

$$\mathrm{SINR}_k^{EQ-DCSI,o} = \frac{\frac{P}{\Gamma^o}\left(\frac{1}{n}\sum_{j=1}^{n}\sqrt{c_{0,k}^{(j)}}\right)^2 \frac{\delta^2}{(1+\delta)^2}}{I_k^{EQ-DCSI,o}+1}$$

with

$$I_k^{EQ-DCSI} = P - P\sum_{j=1}^{n}\sum_{j'=1}^{n}\frac{\delta\Gamma_{j,j'}^o}{n^2(1+\delta)^2\Gamma^o}\cdot\left[2c_{0,k}^{(j)} + \delta\left(2c_{0,k}^{(j)} - c_{0,k}^{(j)}c_{0,k}^{(j')}\right)\right]$$

Cost of Distributedness

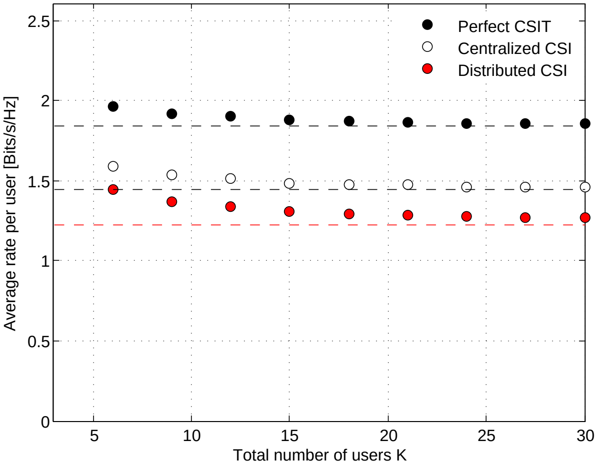


Figure: Average rate per user as a function of the number of users K with $(\sigma^{(j)})^2 = 0.1, \forall j$.

Optimization of the Regularization Parameter

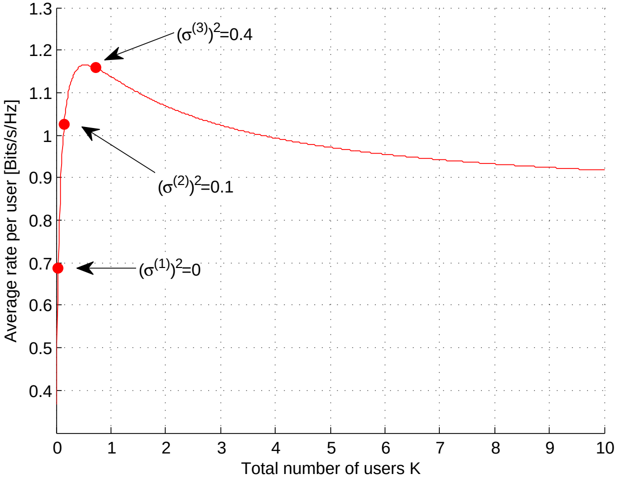
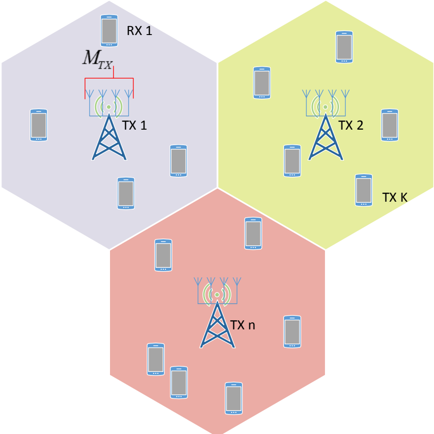


Figure: Average rate per user as a function of γ for $(\sigma^{(1)})^2 = 0, (\sigma^{(2)})^2 = 0.1, (\sigma^{(3)})^2 = 0.4$.

Simulation Settings

n	3	K	30
M	30	β	M/K=1
$(\sigma_k^{(1)})^2$	0.01	$(\sigma_k^{(2)})^2$	0.16
$(\sigma_k^{(3)})^2$	0.49	$\rho_k^{(j,j')}$	0.1
$\boldsymbol{h}_k$	$\sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \mathbf{I}_M)$	$\boldsymbol{\delta}_k^{(j)}$	$\sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \mathbf{I}_M)$



Simulations: Optimize γ

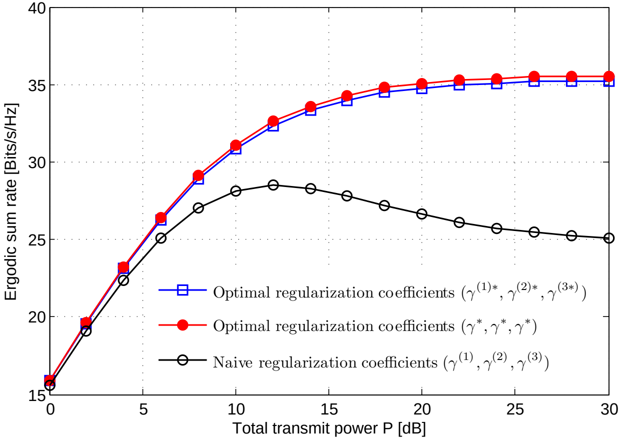


Figure: RZF, ergodic sum rate vs total transmit power P

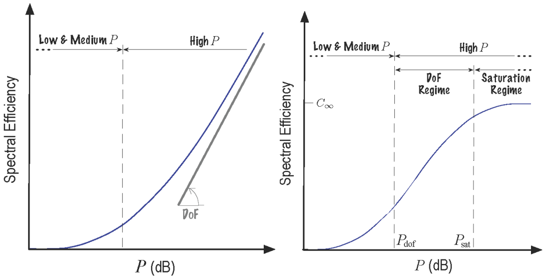
Outline

Asymtotical Analysis: DoF Approach

- First order approximation in the SNR

$$R^{\star} \approx \text{DoF} \log_2(SNR)$$

- Problem becomes deterministic: Possible to obtain analytical results to our complex TD problem

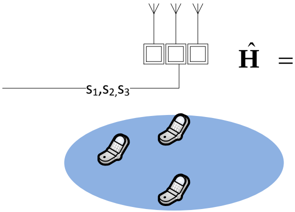


* A. Lozano et al, "Fundamental limits of cooperation", IEEE Trans. On Information Theory, Sept. 2013.

- Very successfull to obtain new innovative insights, discover new behaviours (MIMO, IA, delayed CSIT,...)

What is Known: Sum DoF with Centralized Noisy CSIT

- DoF in the K-users MIMO BC with imperfect CSIT recently confirmed [Davoodi and Jafar, 2014]

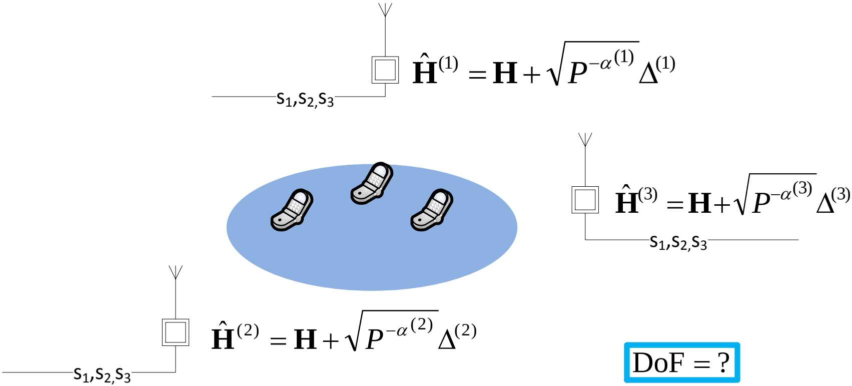


$$\hat{\mathbf{H}} = \mathbf{H} + \sqrt{P^{-\alpha}}\Delta$$

$$\boxed{\mathrm{DoF} = 1 + (K-1)\alpha}$$

- Achieved using simple ZF precoding + rate splitting

Distributed CSIT Configuration: $\alpha^{(1)}, \alpha^{(2)},..,\alpha^{(K)}$



$\boxed{\text{DoF} = ?}$

DoF under Distributed CSIT: Conventional (ZF) precoding

- DoF of Joint Precoding across K distributed TX under D-CSIT, K single-antenna users
- ZF shown to be very inefficient [de Kerret and Gesbert, 2012, TIT]:

$$\mathrm{DoF}^{\mathrm{ZF}} = 1 + (K-1)\min_{j} \alpha^{(j)}$$

- Can we do better?

Principles of New Scheme (Example for $K = 3$)

Key principles:

- Layered precoding
- Layer 1: Transmit with approximate precoder
- Layer 2: Best informed TX regenerates and quantizes interference created by layer 1
- Superpose (multicast) Layer 2 on top of layer 1
- Decode and suppress interference at each user.

We distinguish:

- Arbitrary CSIT regime ($\alpha_i \in [0, 1]\,, \forall i$)
- Weak CSIT regime (α_1,α_2,..α_K) are "small"

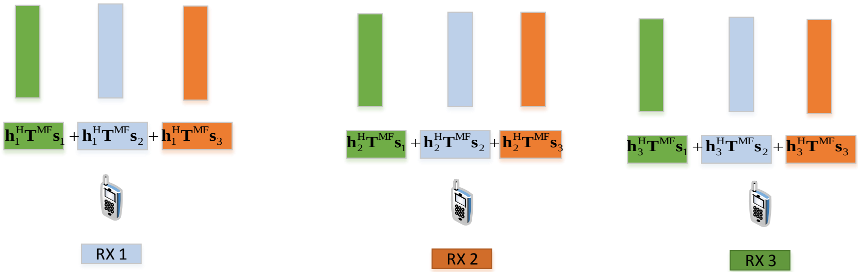
Case $K = 3$ Users: A First Simple Scheme

- Without loss of generality: TX 1 is best informed TX

$$\alpha^{(1)} \geq \alpha^{(2)} \geq \alpha^{(3)}$$

- We transmit 3 symbols per user using e.g. a distributed Matched Precoder with power $P^{(\alpha_1)}/9$

$$\mathbf{T}^{\mathrm{MF}(j)} \triangleq \frac{\hat{\mathbf{H}}^{(j)}}{\|\hat{\mathbf{H}}^{(j)}\|_{\mathrm{F}}}\sqrt{P}$$



Reconstructing the Approximate Interference

- TX 1 uses its CSIT to reconstruct the interference term:

$$\begin{aligned}(\hat{\boldsymbol{h}}_1^{(1)})^{\mathrm{H}}\mathbf{T}^{\mathrm{MF}}\boldsymbol{s}_2 &= (\boldsymbol{h}_1 + P^{-\alpha^{(1)}}\boldsymbol{\delta}_1^{(1)})^{\mathrm{H}}\mathbf{T}^{\mathrm{MF}}\boldsymbol{s}_2\\ &= \boldsymbol{h}_1^{\mathrm{H}}\mathbf{T}^{\mathrm{MF}}\boldsymbol{s}_2 + \underbrace{P^{-\alpha^{(1)}}(\boldsymbol{\delta}_1^{(1)})^{\mathrm{H}}\mathbf{T}^{\mathrm{MF}}\boldsymbol{s}_2}_{\sim P^0}\end{aligned}$$

➥ TX 1 can compute DoF-perfect estimates of the interference terms!

- Quantize the interference using $\alpha^{(1)}\log_2(P)$ bits per term if interference term scales in $P^{\alpha^{(1)}}$
- Superpose a multicast message of $(1-\alpha^{(1)})\log_2(P)$ bits, which will include quantized interference

DoF Analysis: The weak CSIT case ($\alpha^{(1)} < \frac{1}{1+K(K-1)} = 1/7$)

- The 6 quantized interference terms can be broadcast by TX 1 if

$$\underbrace{6\alpha^{(1)}\log_2(P)}_{\text{number of bits to quantize all interference terms}} \leq \underbrace{(1-\alpha^{(1)})\log_2(P)}_{\text{rate of the broadcast data symbol}} \Leftrightarrow \alpha^{(1)} \leq \frac{1}{7}$$

If the inequality is strict, we complete with fresh information bits

- DoF achieved is then

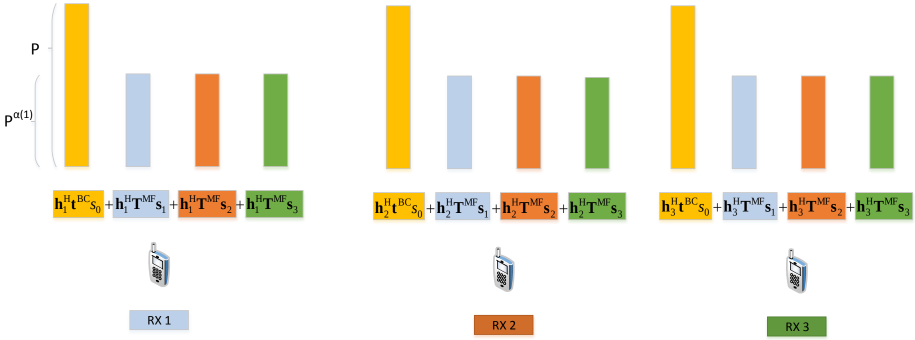
$$\text{DoF} = \underbrace{9\alpha^{(1)}}_{\text{information transmitted initially}} + \underbrace{\left(1-7\alpha^{(1)}\right)}_{\text{fresh information bits to complete the broadcast}} = 1+2\alpha^{(1)}$$

$$\text{DoF} = 1 + (K-1)\max_i \ \alpha^{(i)}$$

(instead of $\text{DoF} = 1 + (K-1)\min_i \ \alpha^{(i)}$!!)

A First Transmission Scheme in One Slide

- TX 1, 2, 3 jointly transmit K symbols to each user using a distributed Matched Precoder $\mathbf{T}^{\mathrm{MF}(j)} \in \mathbb{C}^{K\times K}$ with power $P^{\alpha^{(1)}}/K$
- TX 1 transmits the estimated quantized interference using the power $P - P^{\alpha^{(1)}}$ (equivalently from all TXs using the beamformer $\boldsymbol{t}^{\mathrm{BC}} \triangleq [1,0,0]^{\mathrm{T}}$)



Weak CSIT Regime: Improved results

- Improved scheme: TXs perform Active-Passive Zero-Forcing precoding
  - TX 2,..., TX K perform arbitrary precoding (passive)
  - TX 1 compensates with ZF precoding (active)

Theorem ([de Kerret and Gesbert, 2016, ISIT])

In the weak CSIT regime, defined by

$$\max_{j\in\{1,\dots,K\}} \alpha^{(j)} \leq \frac{1}{1+K(K-2)}$$

We have that:

$$\mathrm{DoF}^{\mathrm{DCSI}}(\boldsymbol{\alpha}) \geq 1+(K-1)\max_{j\in\{1,\dots,K\}} \alpha^{(j)}$$

Outer bound

Theorem

The Centralized Outerbound In the K-user Network MIMO channel with distributed CSIT:

$$\begin{aligned} \mathrm{DoF}^{\mathrm{DCSI}}(\boldsymbol{\alpha}) &\leq \mathrm{DoF}^{\mathrm{CCSI}}(\max_{j\in\{1,\ldots,K\}} \alpha^{(j)}) \\ &= 1 + (K-1) \max_{j\in\{1,\ldots,K\}} \alpha^{(j)} \end{aligned}$$

Key ideas:

- DoF is upperbounded by DoF achieved by full CSIT exchange
- Having multiple CSIT with α_1,α_2,..,α_K doesnt help over having just best CSIT (α_1)

$\Rightarrow$ matches the achieved DoF for weak CSIT!

Arbitrary CSIT Regime with $K = 3$

Theorem

In the 3-user Network MIMO with distributed CSIT and $\alpha^{(1)} \geq \alpha^{(2)} \geq \alpha^{(3)}$, it holds that

$$\mathrm{DoF}^{\mathrm{DCSI}}(\boldsymbol{\alpha}) \geq \begin{cases} 1 + 2\alpha^{(1)} & \text{if } \alpha^{(1)} \leq \frac{1}{4} \\ 3\frac{2\alpha^{(1)} - \alpha^{(2)} + 2\alpha^{(1)}\alpha^{(2)}}{4\alpha^{(1)} - \alpha^{(2)}} & \text{if } \alpha^{(1)} \geq \frac{1}{4}. \end{cases}$$

Optimal DoF for $K = 3$ users:

- In the weak CSIT regime
- In any CSIT regime with $\alpha^{(1)} = \alpha^{(2)}$ (regardless of what user 3 knows)

DoF for $K = 3$ Users

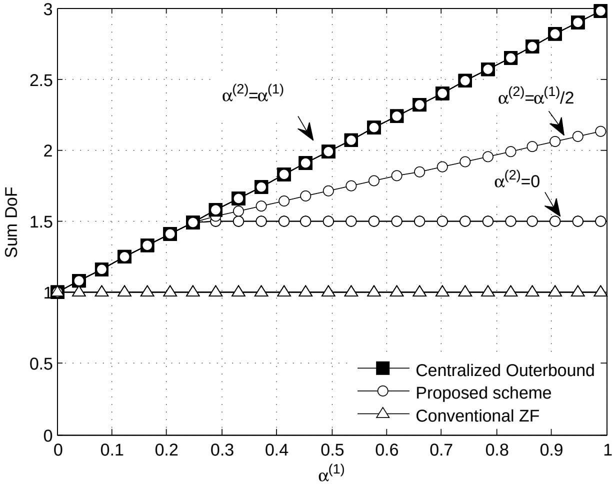


Figure: Sum DoF as a function of $\alpha^{(1)}$. User 3 has no CSIT ($\alpha^{(3)} = 0$)

Outline

Binary Power Control over Interference Channels

$$(p_1^\star, p_2^\star) = \underset{(p_1,p_2)\in\mathcal{P}}{\operatorname{argmax}} \left[\mathrm{R}(p_1(\mathbf{G}^{(1)}), p_2(\mathbf{G}^{(2)}))\right]$$

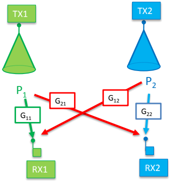
where

$$\mathrm{R}(P_1,P_2) = \log_2\left(1+\frac{G_{11}P_1}{1+G_{12}P_2}\right) + \log_2\left(1+\frac{G_{22}P_2}{1+G_{21}P_1}\right).$$

and

$$\begin{array}{cccc} p_j : & \mathbb{R}_+^4 & \rightarrow & \{P_j^{\min}, P_j^{\max}\} \\ & \mathbf{G}^{(j)} & \mapsto & p_j(\mathbf{G}^{(j)}) \end{array}$$

- Key Example because:
  - Binary optimization with (relatively) low dimensional channel state
  - Weaker depency with the channel state
  - ➥ Less difficulties



Identification of the Parameters

Table: Team Decision Modeling for Power Control

Notations for the Team Decision Problems		
State-of-the-world	$\boldsymbol{x}$	$\mathbf{G}$
Estimate at DM j	$\boldsymbol{x}^{(j)}$	$\mathbf{G}^{(j)}$
Strategy at DM j	$\boldsymbol{s}_j$	p_j
Decision space at DM j	$\mathcal{A}_j$	$\{P_j^{\min}, P_j^{\max}\}$
Objective	f	R

Outline

Discretization for Power Control –Refresher (1)–

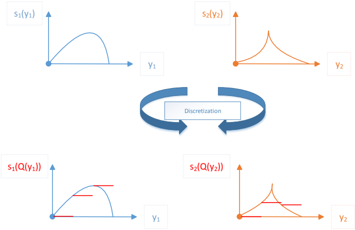
Main Idea: Quantize the channel state space to reduce the dimension

- Replace the strategy p_j by $p_j(Q^{\mathrm{cb}})$ where

$$\begin{array}{rccc} Q^{\mathrm{cb}}: & \mathbb{C}^{2\times 2} & \to & \mathcal{C}^{\mathrm{cb}} \triangleq \{\mathbf{G}_1^{\mathrm{cb}}, \ldots, \mathbf{G}_{n^{\mathrm{cb}}}^{\mathrm{cb}}\} \\ & \hat{\mathbf{G}}^{(j)} & \mapsto & \mathrm{argmin}_{\hat{\mathbf{G}}\in\mathcal{C}^{\mathrm{cb}}} \|\hat{\mathbf{G}} - \hat{\mathbf{G}}^{(j)}\|^2 \end{array}$$

- Optimization subspace reduced to a space of dimension n^{cb}

$$\begin{array}{rccc} p_j(Q^{\mathrm{cb}}): & \mathcal{C}^{\mathrm{cb}} & \to & \{P^{\mathrm{min}}, P^{\mathrm{max}}\} \\ & \mathbf{G}_i & \mapsto & p_j(\mathbf{G}_i) \end{array}$$



Discretization for Power Control –Refresher (2)–

- Best response power allocation strategy: Solve iteratively
  - At TX 1, $\forall i \in \{1, \ldots, n^{\mathrm{cb}}\}$,

$$\boldsymbol{p}_1^{\mathrm{BR}}(\mathbf{G}_i^{\mathrm{cb}}) = \operatorname*{argmax}_{P_1 \in \{P_1^{\mathrm{min}}, P_1^{\mathrm{max}}\}} \mathbb{E}\left[\mathrm{R}\left(\mathbf{G}, P_1, p_2^{\mathrm{BR}}(\mathcal{Q}(\mathbf{G}^{(2)}))\middle|\mathbf{G}^{(1)} = \mathbf{G}_i^{\mathrm{cb}}\right)\right]$$

  - At TX 2, $\forall i \in \{1, \ldots, n^{\mathrm{cb}}\}$,

$$\boldsymbol{p}_2^{\mathrm{BR}}(\mathbf{G}_i^{\mathrm{cb}}) = \operatorname*{argmax}_{P_2 \in \{P_2^{\mathrm{min}}, P_2^{\mathrm{max}}\}} \mathbb{E}\left[\mathrm{R}\left(\mathbf{G}, p_1^{\mathrm{BR}}(\mathcal{Q}(\mathbf{G}^{(1)}), P_2)\middle|\mathbf{G}^{(2)} = \mathbf{G}_i^{\mathrm{cb}}\right)\right]$$

Reach optimal strategy given the strategy of the other TX

Discretization for Power Control –Refresher (3)–

- Approximation of the expectation using Monte-Carlo runs with n^{MC} samples
- At TX 1, $\forall i \in \{1, \dots, n^{\mathrm{cb}}\}$,

$$p_1^{\mathsf{BR}}(\mathbf{G}_i^{\mathrm{cb}}) = \operatorname*{argmax}_{P_1 \in \{P_1^{\min}, P_1^{\max}\}} \frac{1}{n^{\mathrm{MC}}} \sum_{i=1}^{n^{\mathrm{MC}}} \mathrm{R}\left(\mathbf{G}_i, P_1, p_2^{\mathsf{BR}}(\mathcal{Q}^{\mathrm{cb}}(\mathbf{G}_i^{(2)}))\right), \quad \forall i_1 \in \{1, \dots, n^{\mathrm{cb}}\}$$

where $(\mathbf{G}_i, \mathbf{G}_i^{(2)}) \sim f_{\mathbf{G}, \mathbf{G}^{(2)} \big| \mathbf{G}^{(1)} = \mathbf{G}_i^{\mathrm{cb}}}$.

Simulations Parameters

- Rayleigh fading with uniform pathloss
- CSIT Model

$$\mathbf{H}^{(j)} \triangleq \sqrt{1-\sigma_j^2}\mathbf{H} + \sigma_j \boldsymbol{\Delta}$$

  where $\boldsymbol{\Delta} \sim \mathcal{N}_{\mathbb{C}}(0,1)$ and $\mathbf{H} \sim \mathcal{N}_{\mathbb{C}}(0,1)$, and

$$\{\mathbf{G}^{(j)}\}_{i,k} \triangleq \left|\{\mathbf{H}^{(j)}\}_{i,k}\right|^2, \qquad \forall i,k \in \{1,\dots,K\}.$$

- Codebook:
  - Product of scalar codebooks using 10 codewords from Lloyd algorithm for each scalar.
  - Hence: $n^{\mathrm{codebook}} = 10^4 = 10000$
  - Stochastic approximation using $n^{\mathrm{MC}} = 500$

Simulation Results (1)

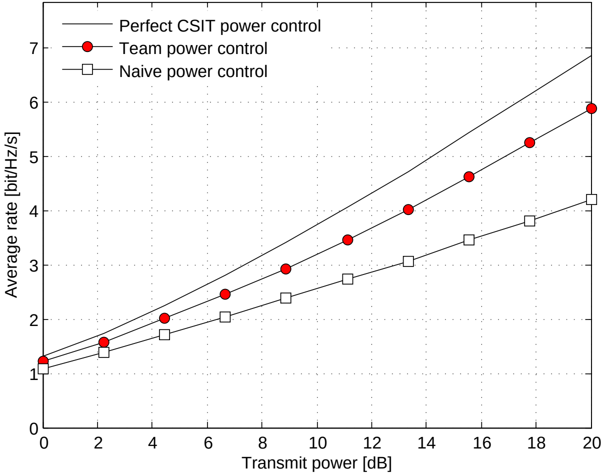


Figure: Average sum rate for $\sigma_1^2 = 1$ and $\sigma_2^2 = 0$.

Simulation Results (2)

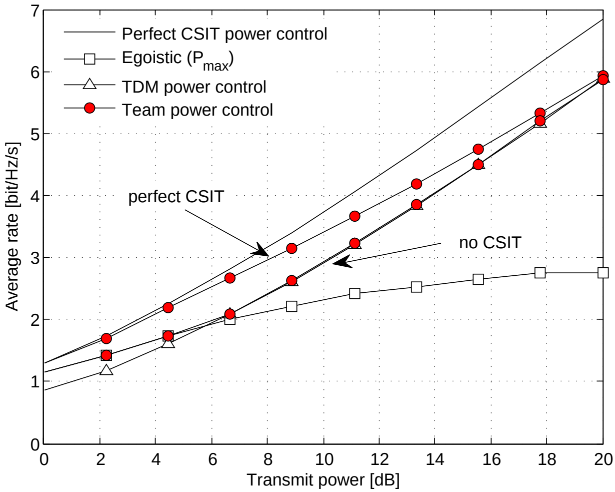
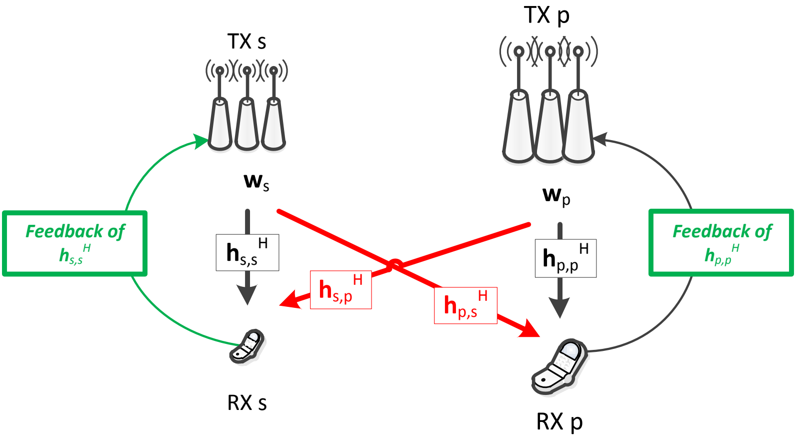


Figure: Comparison with schemes from the literature.

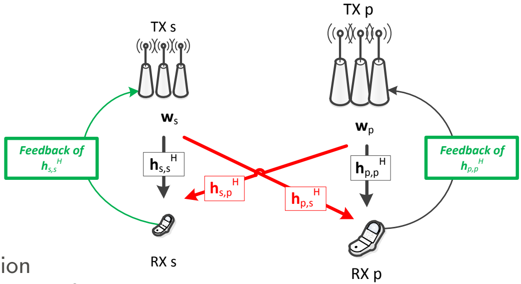
Outline

Cognitive Radio Beamforming

- Rate maximization of a secondary TX while preserving a rate constraint for the primary TX: Underlay Cognitive Radio [Haykin, 2005, JSAC]

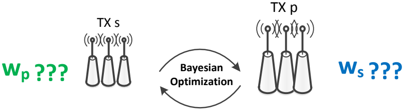


Team Decision Cognitive Radio Beamforming



- CSI configuration
  - TX s only knows $\boldsymbol{h}_{s,s}$ and multi-user statistics $\mathbf{R}_{i,j}$
  - TX p only knows $\boldsymbol{h}_{p,p}$ and multi-user statistics $\mathbf{R}_{i,j}$

➥ Coordination using only the statistics



Functional Optimization Problem

Optimization Problem

$$
\begin{aligned}
(\boldsymbol{w}_p^\star, \boldsymbol{w}_s^\star) = \underset{(\boldsymbol{w}_p, \boldsymbol{w}_s)}{\operatorname{argmax}} \, & \mathbb{E}\left[\mathrm{R}_s(\mathbf{w}_p(\mathbf{h}_{p,p}), \mathbf{w}_s(\mathbf{h}_{s,s}))\right] \\
\text{s. to} \quad & \mathbb{E}\left[\mathrm{R}_p(\mathbf{w}_p(\mathbf{h}_{p,p}), \mathbf{w}_s(\mathbf{h}_{s,s}))\right] \geq \tau > 0, \\
& 0 \leq \left\|\mathbf{w}_p(\mathbf{h}_{p,p})\right\|^2 \leq P_p^{\max} \\
& 0 \leq \left\|\mathbf{w}_s(\mathbf{h}_{s,s})\right\|^2 \leq P_s^{\max}
\end{aligned} \tag{R}
$$

where for $j \in \{s, p\}$,

$$
\begin{aligned}
\boldsymbol{w}_j : \quad \mathbb{C}^{M_j} & \rightarrow \mathbb{C}^{M_j} \\
\boldsymbol{h}_{j,j} & \mapsto \boldsymbol{w}_j(\boldsymbol{h}_{j,j})
\end{aligned}
$$

Identification of the Parameters

Table: Team Decision Modeling for Cognitive Radio

Notations for the Team Decision Problems		
State-of-the-world	$\boldsymbol{x}$	$\{\boldsymbol{h}_{s,s}, \boldsymbol{h}_{s,p}, \boldsymbol{h}_{p,s}, \boldsymbol{h}_{p,p}\}$
Estimate at DM j	$\boldsymbol{x}^{(j)}$	$\boldsymbol{h}_{j,j}$
Strategy at DM j	$\boldsymbol{s}_j$	$\boldsymbol{w}_j$
Decision space at DM j	$\mathcal{A}_j$	$\mathbb{C}^{M_j}$
Objective	f	R_s s.t. (R)

Outline

Key Idea: Codebook of Functions [Filippou et al., 2016, TWC]

Functional optimization difficult $\Rightarrow$ Parametrization of the decision space using a codebook of functions

- Here: 2 functions (strategies) labelled s and p
- Choose these strategies from efficient heuristics

Optimization Problem

$$
\begin{aligned}
(\boldsymbol{w}_p^\star, \boldsymbol{w}_s^\star) = \underset{(\boldsymbol{w}_p, \boldsymbol{w}_s)}{\operatorname{argmax}}\, & \mathbb{E}\left[\mathrm{R}_s(\mathbf{w}_p(\mathbf{h}_{p,p}), \mathbf{w}_s(\mathbf{h}_{s,s}))\right] \\
\text{s. to } \; & \mathbb{E}\left[\mathrm{R}_p(\mathbf{w}_p(\mathbf{h}_{p,p}), \mathbf{w}_s(\mathbf{h}_{s,s}))\right] \geq \tau > 0, \\
& 0 \leq \left\|\mathbf{w}_p(\mathbf{h}_{p,p})\right\|^2 \leq P_p^{\max} \\
& 0 \leq \left\|\mathbf{w}_s(\mathbf{h}_{s,s})\right\|^2 \leq P_s^{\max} \\
& (\boldsymbol{w}_p^\star, \boldsymbol{w}_s^\star) \in \left\{ (\boldsymbol{w}_p^{\mathrm{cb},1}, \boldsymbol{w}_s^{\mathrm{cb},1}), \ldots, (\boldsymbol{w}_p^{\mathrm{cb},n^{\mathrm{cb}}}, \boldsymbol{w}_s^{\mathrm{cb},n^{\mathrm{cb}}}) \right\}
\end{aligned} \tag{R}
$$

where for $j \in \{s, p\}$,

$$
\begin{aligned}
\boldsymbol{w}_j^{\mathrm{cb},i} : \quad \mathbb{C}^{M_j} & \rightarrow \mathbb{C}^{M_j} \\
\boldsymbol{h}_{j,j} & \mapsto \boldsymbol{w}_j^{\mathrm{cb}}(\boldsymbol{h}_{j,j})
\end{aligned}
$$

Strategy p

- TX p
  - TX p uses matched precoding
$$\boldsymbol{u}_p^{\mathrm{MF}} \triangleq \frac{\boldsymbol{h}_{p,p}}{\|\boldsymbol{h}_{p,p}\|}$$
  - TX p transmits with full power $\bar{P}_p = P_p^{\max}$
- TX s:
  - TX s transmits using the statistical ZF precoder
$$\boldsymbol{u}_s^{\mathrm{sZF}} \triangleq \operatorname*{argmin}_{\boldsymbol{u}} \boldsymbol{u}^{\mathrm{H}} \mathbf{R}_{p,s} \boldsymbol{u}$$
  - TX s controls its average transmit power $\bar{P}_s$ to fulfill the rate constraint (R)

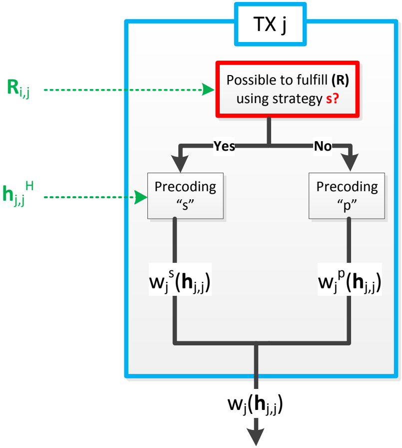
Strategy s

- TX s
  - TX s uses matched precoding
$$\boldsymbol{u}_s^{\mathrm{MF}} \triangleq \frac{\boldsymbol{h}_{s,s}}{\|\boldsymbol{h}_{s,s}\|}$$
  - TX s transmits with full power $\bar{P}_s = P_s^{\max}$
- TX p:
  - TX p transmits using the statistical ZF precoder
$$\boldsymbol{u}_p^{\mathrm{sZF}} \triangleq \operatorname*{argmin}_{\boldsymbol{u}} \boldsymbol{u}^{\mathrm{H}} \mathbf{R}_{s,p} \boldsymbol{u}$$
  - TX p controls its average transmit power $\bar{P}_p$ to fulfill the rate constraint (R)

Some Intuition

- TX p can reduce its power only if TX s can anticipate it: Coordination required to guarantee (R)
- Strategy s
  - Large objective
  - Rate constraint might be unfeasible
- Strategy p
  - Low objective
  - Rate constraint guaranteed

Statistical Coordination Algorithm [Filippou et al., 2016, TWC]



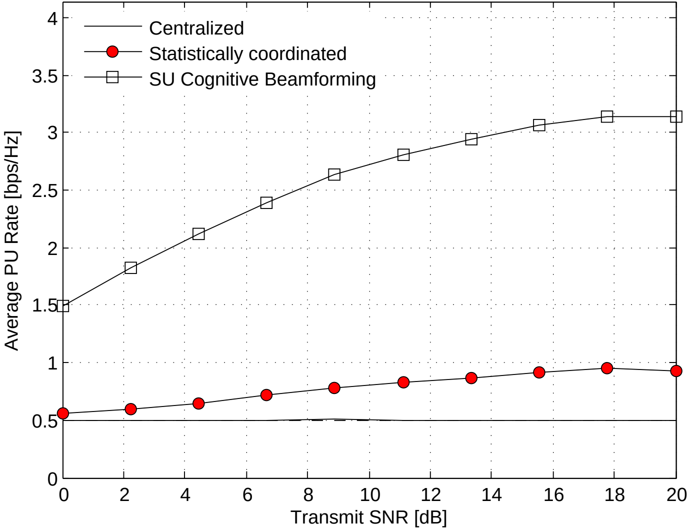
Simulations Parameters

- $M_s = M_p = 3$ antennas per-TX
- Correlation matrices

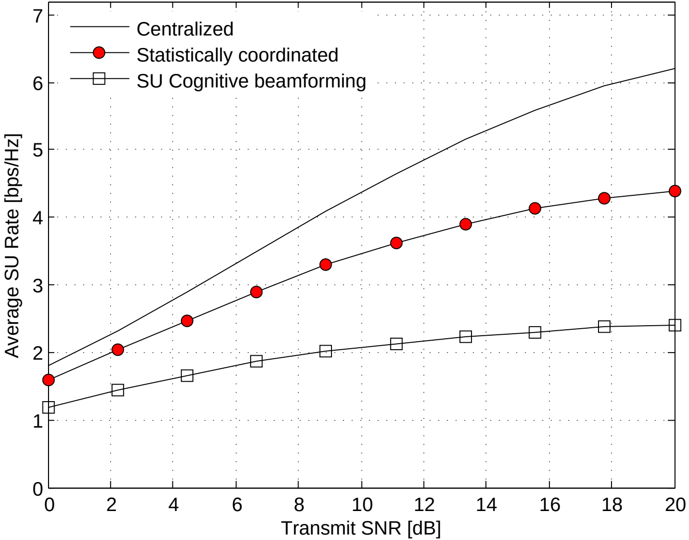
$$\mathbf{R}_{p,p} = \mathbf{R}_{s,s} = \mathbf{I}_3, \quad \mathbf{R}_{p,s} = \mathbf{R}_{s,p} = \begin{bmatrix} 1 & \rho & \rho^2 \\ \rho & 1 & \rho \\ \rho^2 & \rho & 1 \end{bmatrix}$$

- Use in the following $\rho = 0.5$ and $\tau = 0.5$bps/Hz

Ergodic rate of the PU



Ergodic rate of the SU

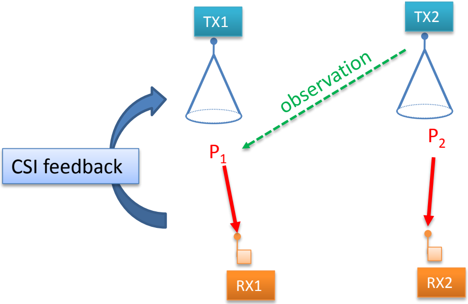


Outline

Coded Power Control

- TX 1 with non-causal CSI knowledge
- TX 2 observes transmit power P_1:

➥ implicit coordination



Modelization

- random state $H = \{G_{1,1}, G_{2,1}, G_{1,2}, G_{2,2}\}$ with fixed law $\rho(H)$ over $\mathcal{H}$
- TX i chooses its power levels P_i in $\mathcal{P}_i$
- TX 2 observes Z with fixed law $\Gamma(z|P_1)$.
- Strategy of Agent 1: $(w_{1,i})_{1\leq i\leq T}$ with:

$$w_{1,i} : \underbrace{\mathcal{H}^T}_{\text{CSI}} \times \underbrace{\mathcal{P}_1^{i-1}}_{\text{past actions}} \times \emptyset \rightarrow \mathcal{P}_1$$

- Strategy of Agent 2: $(w_{2,i})_{1\leq i\leq T}$ with:

$$w_{2,i} : \underbrace{\mathcal{H}^{i-1} \times \mathcal{Z}^{i-1} \times \mathcal{P}_2^{i-1}}_{\text{past}} \rightarrow \mathcal{P}_2$$

Auxiliary notion

Definition (Implementability)

$P_{H_i,P_{1,i},P_{2,i},Z_i}$: joint distribution induced by $(w_{1,i}, w_{2,i})_{i\geq 1}$ at stage i.
The distribution $\mathcal{Q}(h, p_1, p_2)$ is implementable if there exists a pair of strategies $(w_{1,i}, w_{2,i})_{i\geq 1}$ such that for all (h, p_1, p_2),

$$\frac{1}{T}\sum_{i=1}^{T}\sum_{y} P_{H_i,P_{1,i},P_{2,i},Y_i}(h, p_1, p_2, z) \to \mathcal{Q}(h, p_1, p_2)$$

as $T \to \infty$.

Feasible utilities

A certain utility value $\underline{f}$ is reachable if and only if there exists an implementable distribution $\mathcal{Q}$ such that $\underline{f} = \mathbb{E}_{\mathcal{Q}}[f]$.

Theorem ([Larrousse and Lasaulce, 2013])

Let $\overline{Q} \in \Delta(\mathcal{H} \times \mathcal{P}_1 \times \mathcal{P}_2)$ *with* $\sum_{(p_1,p_2)} \overline{Q}(h, p_1, p_2) = \rho(h)$. *The distribution* $\overline{Q}$ *is implementable if there exists* $Q \in \Delta(\mathcal{H} \times \mathcal{P}_1 \times \mathcal{P}_2 \times \mathcal{Z})$ *which verifies:*

$$I_Q(H; P_2) \leq I_Q(P_1; Z|H, P_2)$$

where the arguments of the mutual information $I_Q(.)$ *are defined from* Q *and* $Q(h, p_1, p_2, y) = \overline{Q}(h, p_1, p_2)\Gamma(z|p_1)$.

Remark: This theorem also characterizes expected payoff.

Convex Optimization Problem

$$
\begin{aligned}
\text{maximize } & \mathbb{E}_{\underline{q}}[f] = \sum_{\ell=1}^{L} q_\ell f_\ell \\
\text{subject to } & I_{\underline{q}}(H; P_2) \leq I_{\underline{q}}(P_1; Z|H, P_2) \\
& q_\ell \geq 0 \\
& \sum_{\ell=1}^{L} q_\ell = 1 \\
& \sum_{\ell \in \mathcal{L}_H(h)} q_\ell = \rho(h), \qquad \forall\, h, \\
& \frac{\sum_{\ell \in \mathcal{L}_{P_1,Z}(p_1,z)} q_\ell}{\sum_{\ell \in \mathcal{L}_{P_1}(p_1)} q_\ell} = \Gamma(z|p_1), \qquad \forall\, (p_1, z)
\end{aligned}
$$

Application: MAC Power Control

- 2-user MAC with binary power control
- 2 possible states: $\mathcal{H} = \{\{g_{\min}, g_{\max}\}, \{g_{\max}, g_{\min}\}\}$

	$P_{\min}$	$P_{\max}$
$P_{\min}$	0	20
$P_{\max}$	1	$\simeq 10$

$x_0 = (g_{\min}, g_{\max})$

	$P_{\min}$	$P_{\max}$
$P_{\min}$	0	1
$P_{\max}$	20	$\simeq 10$

$x_0 = (g_{\max}, g_{\min})$

Figure: Payoff table [Larrousse and Lasaulce, 2013]

Source Coding [Larrousse and Lasaulce, 2013]

- Source coding:

$$
\begin{array}{rccc}
f_{\mathrm{S}}: & \mathcal{H} & \rightarrow & \{m_0, m1\} \\
 & h & \mapsto & i
\end{array}
$$

TABLE V

PROPOSED SOURCE CODING AND DECODING FOR $p = \frac{1}{2}$.

x_0^3	Index $i = f_{\mathrm{S}}(x_0^3)$	$g_{\mathrm{S}}(i)$
(0,0,0)	m_0	(1,1,1)
(0,0,1)	m_0	(1,1,1)
(0,1,0)	m_0	(1,1,1)
(0,1,1)	m_0	(1,1,1)
(1,0,0)	m_0	(1,1,1)
(1,0,1)	m_0	(1,1,1)
(1,1,0)	m_1	(0,0,1)
(1,1,1)	m_1	(0,0,1)

Channel Coding

- Channel coding:

$$\begin{array}{cccc} f_{\mathrm{C}}: & \{m_0, m_1\} & \to & \mathcal{P}_1 \\ & i & \mapsto & \boldsymbol{p}_1 = [p_1(1), p_1(2), p_1(3)] \end{array}$$

- at block b, find optimal $\boldsymbol{p}_1^{\text{opt}} = [p_1(1)^{\text{opt}}, p_1(2)^{\text{opt}}, p_1(3)^{\text{opt}}]$ and second optimal $\boldsymbol{p}_1^{\text{opt}''} = [p_1(1)^{\text{opt}''}, p_1(2)^{\text{opt}''}, p_1(3)^{\text{opt}''}]$
  - If $i = m_0$, send with $\boldsymbol{p}_1^{\text{opt}}$
  - If $i = m_1$, send with $\boldsymbol{p}_1^{\text{opt}''}$

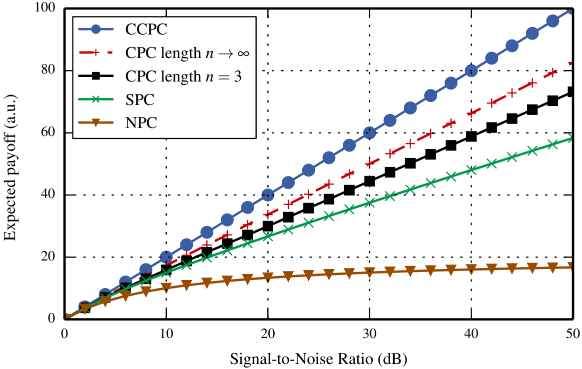
Channel Coding [Larrousse and Lasaulce, 2013]

TABLE VI

PROPOSED CHANNEL CODING FOR $p = \frac{1}{2}$.

$x_0^3(b)$	$x_2^3(b)$	i_{b+1}	$x_1^3(b)$
(0,0,0)	(1,1,1)	m_0	(0,0,0)
		m_1	(0,0,1)
(0,0,1)	(1,1,1)	m_0	(0,0,1)
		m_1	(0,0,0)
(0,1,0)	(1,1,1)	m_0	(0,1,0)
		m_1	(0,0,0)
(0,1,1)	(1,1,1)	m_0	(0,1,1)
		m_1	(0,0,1)
(1,0,0)	(1,1,1)	m_0	(1,0,0)
		m_1	(0,0,0)
(1,0,1)	(1,1,1)	m_0	(1,0,1)
		m_1	(0,0,1)
(1,1,0)	(0,0,1)	m_0	(1,1,0)
		m_1	(1,1,1)
(1,1,1)	(0,0,1)	m_0	(1,1,1)
		m_1	(1,1,0)

Simulations [Larrousse and Lasaulce, 2013]



Outline

Device centric coordination

- Relying on local communications and decentralized computations
- Decentralized cooperation can aim at the good of the network
- Challenge: Develop robust one-shot schemes that cope with arbitrary information structures
- Heuristics can be obtained by decoupling the communication from decision problems
- Open problems
  - Joint optimization of communications and decision is very challenging
  - Low complexity methods?
  - Information theoretic aspects (capacity under decentralized information settings..) ?
  - Coordination-aware feedback designs (hierarchical,...)

Other impact

- Coordination theory leads to new insights: Impact over network design?
- Implicit coordination: Coordination for free?
- Bridge the gap from implicit coordination to distributed optimization?
- Interactions with distributed optimization

Big thanks to

Artist4G

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