A LARGE SYSTEM ANALYSIS OF WEIGHTED SUM RATE MAXIMIZATION OF SINGLE STREAM MIMO INTERFERENCE BROADCAST CHANNELS UNDER LINEAR PRECODING

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ABSTRACT

The weighted sum rate (WSR) maximizing linear precoding algorithm is studied in large correlated single stream multiple-input multiple-output (MIMO) interference broadcast channels (IBC). We consider an iterative WSR design which exploits the connection with Weighted sum Minimum Mean Squared Error (WMMSE) designs as in [1], [2], focusing on the version in [1]. We propose an large system approximation of the signal-to-interference plus noise ratio (SINR) at every iteration. The large system approximation of the SINR depends only on the slow fading terms or second order statistics of the channels. In this work, the large system approximation is used to establish a property of the Multi-users single stream MIMO communications. Simulations show that the approximations are accurate.

***Index Terms*—** random matrix theory, beamforming, weighted sum rate maximization, single stream MIMO

1. INTRODUCTION

We consider the MIMO IBC with linear precoding at the transmitter. A cellular network of C base stations (BS) is considered where each one of them possesses M antennas and serves K users multi-antenna receivers. The precoding matrix that maximizes (local optimum) the WSR for IBC is obtained from an iterative algorithm proposed by Luo et al. and Slock et al. in [1] and [2] respectively which is called the IBC WMMSE algorithm. In this contribution, we carry out a large system analysis of this latter. Herein, we extend the work based on [3] and presented in [4], that presents the deterministic equivalent expressions of the SINR of the WMMSE iterative algorithm for IBC in the case of multiple-input single-output (MISO) systems in [3], and we inspire from the works in [5] and [6] which present Massive MISO deterministic equivalents of the SINR corresponding to the sub-optimal zero-forcing (ZF) and regularized zero-forcing (RZF) precoders and , all for large M and K. Although our work will inspire from the works in [5] and [6] and will be an extension of the work in [4], however it is not straightforward and needs careful attention when dealing with single stream MIMO systems instead of MISO systems. Other works on large systems exist, e.g. [7], [8], [9], [10] and [11], where a multi cell RZF denoted iaRZF is presented in [8], this latter maximizes the sum rate as our precoder does but is not optimal for all existing scenarios, e.g. the scenario where many users are located on the cell edges, in fact, it corresponds to an optimal beamforming only in the case of identical intra-cell channel attenuation and identical inter-cell channel attenuation. Algorithms that minimize the total transmit power for large systems are presented in [9], [10] and [11], however, they are different than the WMMSE approach that maximizes the total sum rate instead of minimizing the total power. Furthermore, the deterministic limits of the SINRs corresponding to the iterative IBC WMMSE process leading to the optimal WSR are presented, which makes it possible to evaluate its performance more easily and compare with other algorithms and precoders. Simulations show that the proposed SINR approximation is close to the real performance, i.e. the performance of the IBC WMMSE algorithm. Moreover, an analysis of the SINR approximation for simple cases is conducted in order to prove some properties in the multi-user (MU) massive MIMO communications. In this work, we show that the achievable SINR of a single stream MIMO broadcast channels (BC) system scales with the number of antennas so for N-antennas users the achievable SINR equals N times the achievable SINR for MISO BC systems. We extend this result to IBC systems. Simulations show the correctness of this theoretical result. Notation: The operators $()^H, tr(.)\ and\ E[.]$ denote conjugate transpose, trace and expectation, respectively. The $M \times M$ identity matrix is denoted I_M and $\log(.)$ is the natural logarithm.

2. SYSTEM MODEL

In the following, we analyze a cellular downlink IBC single stream MIMO scenario where C cells are presented, c=1...C, each of the C cells consists of one BS associated with a number K of N-antennas receivers. We assume transmission on a single narrow-band carrier. the received signal $y_{c,k}$ at the kth user in cell c reads

$$y_{c,k} = \sum_{m=1}^{C}\sum_{l=1}^{K} H_{m,c,k} g_{m,l} s_{m,l} + n_{c,k} \tag{1}$$

where the user symbols are chosen from a Gaussian codebook, i.e, $s_{m,l} \sim \mathcal{NC}(0,1)$, are linearly precoded and form the transmit signal; $g_{m,l} \in \mathbb{C}^M$ is the precoding vector of user l of cell m, $H_{m,c,k} \in \mathbb{C}^{N\times M}$ is the channel vector from the mth transmitter to the kth user of cell c, and the elements of the $N \times 1$ $n_{c,k}$ are independent complex Gaussian noise terms with zero mean and variance σ^2. Moreover, the precoders are subject to an average power constraint and the channel $H_{i,c,k}$ is correlated as $\mathbb{E}\left[H_{i,c,k}^H H_{i,c,k}\right] = \Theta_{i,c,k}$ thus

$$H_{i,c,k}^H = \sqrt{NM}\Theta_{i,c,k}^{1/2} X_{i,c,k} \Theta r, i, c, k^{1/2} \tag{2}$$

$$trG_cG_c^H \preceq P_c \; for \; c \in \mathcal{C} \tag{3}$$

where $\mathcal{C}$ is the set of all BSs, $X_{i,c,k}^H$ is an $N \times M$ matrix with i.i.d. complex entries of zero mean and variance $\frac{1}{NM}$ and the $\Theta_{i,c,k}^{1/2}$ and $\Theta_{r,i,c,k}^{1/2}$ are the Hermitian square-root of $\Theta_{i,c,k}$ and $\Theta_{r,i,c,k}$ respectively. The correlation matrix $\Theta_{i,c,k}$ at the transmitter side is non-negative Hermitian and of uniformly bounded spectral norm w.r.t. to M. The correlation matrix $\Theta_{r,i,c,k}$ at the receiver side can be taken as an identity matrix for all BSs and users. For notational convenience, we denote $\Theta c, c, k$ as $\Theta c, k$.

$G_c = [g_{c,1}, g_{c,2}, ..., g_{c,K}] \in \mathbb{C}^{M \times K}$ is the precoding matrix and P_c is the total available transmit power of cell c.

Treating interference as noise, user k of cell c will apply a linear receive filter $f_{c,k}$ to maximize the signal power (diversity) while reducing any residual interference that would not have been (sufficiently) suppressed by the precoder. Under the assumption of optimal single-user decoding and perfect Channel State Information (CSI) at the transmitters and receivers, the achievable rate of the kth user of cell c is given by

$$R_{c,k} = \log(1 + \gamma_{c,k}) \tag{4}$$

$$\gamma_{c,k} = \frac{|f_{c,k}^H h_{c,c,k} g_{c,k}|^2}{\sum\limits_{(m,l)\neq(c,k)} f_{c,k}^H H_{m,c,k} g_{m,l} g_{m,l}^H H_{m,c,k} f_{c,k} + f_{c,k}^H f_{c,k} \sigma^2} \tag{5}$$

where $\gamma_{c,k}$ is the SINR of the kth of cell c.

The precoders maximize the WSR of all users so we are facing an optimization problem which is the following

$$\begin{aligned} G^* =& arg \max_G \sum_{c=1}^{C} \sum_{k=1}^{K} u_{c,k} R_{c,k} \\ & s.t. \; trG_cG_c^H \leq P_c \; for \; c \in \mathcal{C} \end{aligned} \tag{6}$$

where G is the short notation for $\{G_c\}_{c\in\mathcal{C}}$ and where $u_{c,k} \geq 0$ is the weight of the k^{th} user of cell c. The optimization problem in (6) is hard to solve directly, since it is highly non convex in the precoding matrix G. To solve the problem in (6), we consider the linear receive filters $f_{c,k} \in \mathbb{C}$, the error variance $e_{c,k}$ after the linear receive filtering, given in (8), and we introduce additional weighting scalars $w_{c,k}$, so that the utility function (6) can be modified and an equivalent optimization problem can be formulated as in [1] and [2]

$$\begin{aligned} & \{G^*, \{f_{c,k}^*\}, \{w_{c,k}^*\}\} = \\ & arg \min_{G,\{f_{c,k}\},\{w_{c,k}\}} \sum_{(c,k)} w_{c,k} e_{c,k} - u_{c,k} \log(u_{c,k}^{-1} w_{c,k}) \\ & s.t. \; trG_cG_c \leq P_c \; for \; c \in \mathcal{C} \end{aligned} \tag{7}$$

with

$$e_{c,k} = E[(f_{c,k}^H y_{c,k} - s_{c,k})(f_{c,k}^H y_{c,k} - s_{c,k})^H]. \tag{8}$$

Denote $\rho_c = \frac{P_c}{\sigma^2}$, the signal-to-noise ratio (SNR) in cell c. From (7), and after applying alternating optimization techniques, which lead to solving simple quadratic or convex functions, the precoders are obtained as the following

$$f_{c,k}^* = g_{c,k}^H H_{c,c,k}^H (\sigma^2 I_N + \sum_{m=1}^{C} \sum_{l=1}^{K} H_{m,c,k} g_{m,l} g_{m,l}^H H_{m,c,k}^H)^{-1} \tag{9}$$

$$e_{c,k}^* = (1 + \gamma_{c,k})^{-1} \tag{10}$$

$$w_{c,k}^* = u_{c,k}(e_{c,k}^*)^{-1} \tag{11}$$

$$\widetilde{g}_{c,k}^* = (\sum_{i,j} H_{c,i,j}^H f_{i,j} d_{i,j} f_{i,j}^H H_{c,i,j} + \frac{trD_c}{\rho_c} I_M)^{-1} H_{c,c,k}^H f_{c,k} w_{c,k} \tag{12}$$

where $g_{c,k}^* = \xi_c \widetilde{g}_{c,k}^*$ with $\xi_c = \sqrt{\frac{P_c}{tr\widetilde{G_c}^* \widetilde{G_c}^{*H}}}$. Also we defined $W_c = diag(w_{c,1}^*, ..., w_{c,K}^*)$, $F_c = diag(f_{c,1}^*, ..., f_{c,K}^*), D_c = F_c^H W_c F_c$. For notational convenience, we drop the superscript* in the sequel. Subsequently $f_{c,k}$ and $w_{c,k}$ are computed, which then constitute the new precoder $g_{c,k}$. This process is repeated until convergence to a local optimum.

UL/DL duality: the optimal Tx filter $g_{c,k}$ is of the form of a MMSE linear Rx for the dual UL in which $\frac{trD_c}{\rho_c}$ plays the role of Rx noise variance and $u_{c,k} w_{c,k}$ plays the role of stream variance.

3. LARGE SYSTEM ANALYSIS

In this section, performance analysis is conducted for the proposed precoder. The large-system limit is considered, where M and K go to infinity while keeping the ratio K/M finite such that $limsup_M K/M < \infty$ and $liminf_M K/M > 0$. The results should be understood in the way that, for each set of system dimension parameters M and K we provide an approximate expression for the SINR and the achieved sum rate, and the expression is tight as M and K grow large. All vectors and matrices should be understood as sequences of vectors and matrices of growing dimensions. For the precoder (12), a deterministic equivalent of the SINR is provided in the following theorem. *Theorem 2:* Let $\gamma_{c,k}$ be the SINR of the kth user of cell c with the precoder defined in (12). Then, a deterministic equivalent $\overline{\gamma}_{c,k}^{(j)}$ at iteration $j > 0$ is given by $\overline{\gamma}_{c,k}^{(j)}$ is given by

$$\overline{\gamma}_{c,k}^{(j)} = \frac{\overline{w}_{c,k}^{(j)} (\overline{m}_{c,k}^{(j)})^2}{\overline{\Upsilon}_{c,k}^{(j)} + \overline{\tilde{\Upsilon}}_{c,k}^{(j)} + \overline{d}_{c,k}^{(j)} \frac{\overline{\Psi}_c^{(j)}}{\rho_c} (1 + \overline{m}_{c,k}^{(j)})^2} \tag{13}$$

where

$$\overline{m}_{c,k}^{(j)} = \frac{1}{M} tr \overline{\Theta}_{c,k}^{(j)} V_c \tag{14}$$

$$\overline{\Psi}_c^{(j)} = \frac{1}{M} \sum_{i=1}^{K} \frac{\overline{w}_{c,i}^{(j)} e_{c,i}'}{(1 + e_{c,i})^2} \tag{15}$$

$$\overline{\Upsilon}_{c,k}^{(j)} = \frac{1}{NM} \sum_{l=1, l\neq k}^{K} \frac{1}{(1 + \overline{m}_{c,l}^{(j)})^2} e_{c,c,k,c,l}' \tag{16}$$

$$\overline{\tilde{\Upsilon}}_{c,k}^{(j)} = \frac{1}{M} \sum_{m=1, m\neq c}^{C} \frac{(1 + \overline{m}_{c,k}^{(j)})^2}{(1 + \overline{m}_{m,c,k}^{(j)})^2} \sum_{l=1}^{K} \frac{1}{(1 + \overline{m}_{m,l}^{(j)})^2} e_{m,c,k,m,l}' \tag{17}$$

with $\overline{\Theta}_{m,c,k} = d_{c,k}\Theta_{m,c,k}$, $\overline{m}^{(j)}_{m,c,k} = \frac{1}{M}tr\overline{\Theta}^{(j)}_{m,c,k}V_m$. Furthermore, we have

$$\overline{a}^{(j)}_{c,k} = \frac{1}{\sqrt{\overline{P}^{(j-1)}_{c,k}}}\frac{\overline{\gamma}^{(j-1)}_{c,k}}{1+\overline{\gamma}^{(j-1)}_{c,k}} \tag{18}$$

$$\sqrt{\overline{P}^{(j-1)}_{c,k}} = \frac{1}{\overline{a}^{(j-1)}_{c,k}}\sqrt{\frac{P}{\overline{\Psi}^{(j-1)}_c}}\frac{\overline{m}^{(j-1)}_{c,k}}{1+\overline{m}^{(j-1)}_{c,k}} \tag{19}$$

$$\overline{w}^{(j)}_{c,k} = (1+\overline{\gamma}^{(j-1)}_{c,k}) \tag{20}$$

$$\overline{d}^{(j)}_{c,k} = \overline{w}^{(j)}_{c,k}\overline{a}^{2,(j)}_{c,k}. \tag{21}$$

where $\overline{a}_{c,k}$ denotes the module of the linear receive filter $f_{c,k}$. Denoting

$$V_c = (T_c + \overline{\alpha}_c I_M)^{-1} \tag{22}$$

with $\overline{\alpha}^{(j)}_c = \frac{tr\overline{D}^{(j)}_c}{M\rho_c}$, three systems of coupled equations have to be solved. First, we need to introduce $e_{m,c,k}\forall\{m,c,k\} \in \{\mathcal{C},\mathcal{C},\mathcal{K}_c\}$, where $\mathcal{K}_c$ is the set of all users of cell c, which form the unique positive solutions of

$$e_{m,c,k} = \frac{1}{M}tr\overline{\Theta}_{m,c,k}V_m, \tag{23}$$

$$T_m = \frac{1}{M}\sum_{j=1}^{C}\sum_{i=1}^{K}\frac{\overline{\Theta}_{m,j,i}}{1+e_{m,j,i}}. \tag{24}$$

$e_{c,c,k}$ and $m_{c,c,k}$ denote $e_{c,k}$ and $m_{c,k}$ respectively. Secondly,we give $e'_{1,1},...,e'_{1,K},...e'_{C,1},...,e'_{C,K}$ which form the unique positive solutions of

$$e'_{c,k} = \frac{1}{M}tr\overline{\Theta}_{c,k}V_c(T'_c + I_M)V_c, \tag{25}$$

$$T'_c = \frac{1}{M}\sum_{j=1}^{C}\sum_{i=1}^{K}\frac{\overline{\Theta}_{c,j,i}e'_{j,i}}{(1+e_{c,j,i})^2}. \tag{26}$$

And finally, we provide $e'_{m,c,k,m,l}\forall\{m,c,k,l\} \in \{\mathcal{C},\mathcal{C},\mathcal{K}_c,\mathcal{K}_c\}$ which form the unique positive solutions of

$$e'_{m,c,k,m,l} = \frac{1}{M}tr\overline{\Theta}_{m,c,k}V_m(T'_{m,m,l} + \overline{\Theta}_{m,l})V_m \tag{27}$$

$$T'_{m,m,l} = \frac{1}{M}\sum_{j=1}^{C}\sum_{i=1}^{K}\frac{\overline{\Theta}_{m,j,i}e'_{m,j,i,m,l}}{(1+e_{m,j,i})^2}. \tag{28}$$

For $j \geq 1$, define $\Gamma^{(j)}_c = \frac{1}{NM}H_c\overline{D}^{(j)}H^H_c + \overline{\alpha}^{(j)}_c I_M$, with $D = diag(D_1, D_2, ..., D_C)$ and $H_c = [H^H_{c,1,1},...,H^H_{c,1,K},H^H_{c,2,1},\ldots,H^H_{c,2,K},\ldots,H^H_{c,C,K}]$, the precoder at the end of iteration j is given by

$$\overline{g}^{(j)}_{c,k} = \frac{\xi^{(j)}_c}{M}(\Gamma^{(j)}_c)^{-1}H^H_{c,c,k}\overline{a}^{(j)}_{c,k}f^{(j)}_0\overline{w}^{(j)}_{c,k} \tag{29}$$

for each user k in the cell c,
where $f_{0,c,k}$ is the the normalized linear receive filter such that $f_{c,k} = a_{c,k}f_{0,c,k}$, and $\xi^{(j)}_c$ is given by

$$\xi^{(j)}_c = \sqrt{\frac{P_c}{\frac{1}{M^2}tr(\Gamma^{(j)}_c)^{-2}H_{\hat{c}}\overline{F}^{H,(j)}_c\overline{W}^{2,(j)}_c\overline{F}^{(j)}_c H^H_{\hat{c}}}} \tag{30}$$

$$= \sqrt{\frac{P_c}{\Psi^{(j)}_c}}. \tag{31}$$

where $H_{\hat{c}} = [H^H_{c,c,1},...,H^H_{c,c,K}]$ We derive the deterministic equivalents of the normalization term $\xi^{(j)}_c$, the signal power $|\overline{g}^{H,(j)}_{c,k}H^H_{c,c,k}|^2$ and the interference power $\sum_{m=1}^{C}\sum_{l\neq k\ if\ m=c}^{K}f^H_{c,k}H_{m,c,k}\overline{g}^{(j)}_{m,l}\overline{g}^{H,(j)}_{m,l}H^H_{m,c,k}f_{c,k}$ similarly to [4], [5] and [6], i.e, using the same logic and mathematical approach, but for a more complex problem. We will show that in the following.

Proof: We write $f_{c,k}$ as $f_{c,k} = a_{c,k}f_{0,c,k}$ with $a_{c,k} = \sqrt{f^H_{c,k}f_{c,k}}$ and $|f_{0,c,k}| = 1$. Let $P^{(j)}_{c,k} = |f^{H,(j)}_{c,k}H_{c,c,k}g^{(j)}_{c,k}|^2 = |H_{c,c,k}g^{(j)}_{c,k}|^2$. We have

$$g^{(j)}_{c,k} = \frac{\xi^{(j)}_c}{NM}(\Gamma^{(j)}_{c,[c,k]})^{-1}H^H_{c,c,k}f^{(j)}_{0,c,k}a^{(j)}_{c,k}w^{(j)}_{c,k} \tag{32}$$

$$-\frac{\xi^{(j)}_c}{NM}(\Gamma^{(j)}_c)^{-1}\frac{1}{NM}H^H_{c,c,k}f^{(j)}_{0,c,k}d^{(j)}_{c,k}f^{H,(j)}_{0,c,k}H_{c,c,k}(\Gamma^{(j)}_{c,[c,k]})^{-1} \tag{33}$$

$$\times H^H_{c,c,k}f^{(j)}_{0,c,k}a^{(j)}_{c,k}w^{(j)}_{c,k}; \tag{34}$$

$$g^{(j)}_{c,k} = \frac{\xi_{(j)}}{NM}(\Gamma^{-1}_{c,[c,k]}H^H_{c,c,k}f^{(j)}_{0,c,k}a^{(j)}_{c,k}w^{(j)}_{c,k} - m^{(j)}_{c,k}g^{(j)}_{c,k}); \tag{35}$$

$$g^{(j)}_{c,k} = \frac{\xi^{(j)}_c}{(1+m^{(j)}_{c,k})NM}(\Gamma^{(j)}_{c,[c,k]})^{-1}H^H_{c,c,k}f^{(j)}_{0,c,k}a^{(j)}_{c,k}w^{(j)}_{c,k} \tag{36}$$

Thus,

$$\sqrt{P^{(j)}_{c,k}} = \frac{\xi^{(j)}_c a^{(j)}_{c,k}w^{(j)}_{c,k}}{(1+m_{c,c,k})NM}|H_{c,c,k}(\Gamma_{c,[c,k]})^{-1}H^H_{c,c,k}f_{0,c,k}| \tag{37}$$

$$= \frac{\xi^{(j)}_c}{a^{(j)}_{c,k}(1+m_{c,k})}|\Theta^{1/2}_{r,c,k}X_{c,c,k}\overline{\Theta}^{1/2}_{c,k}(\Gamma_{c,[c,k]})^{-1}\overline{\Theta}^{1/2}_{c,k} \tag{38}$$

$$\times X^H_{c,c,k}\Theta^{1/2}_{r,c,k}f^{(j)}_{0,c,k}| \tag{39}$$

$$= \frac{\xi^{(j)}_c}{a^{(j)}_{c,k}(1+m_{c,k})}|\Theta^{1/2}_{r,c,k}\frac{1}{M}tr\{\overline{\Theta}_{c,k}(\Gamma^{(j)}_c)^{-1}\}I_N\Theta^{1/2}_{r,c,k}f^{(j)}_{0,c,k}| \tag{40}$$

$$= \frac{\xi^{(j)}_c m_{c,k}}{a^{(j)}_{c,k}(1+m_{c,k})}|\Theta^{1/2}_{r,c,k}f^{(j)}_{0,c,k}| = \frac{\xi^{(j)}_c m_{c,k}}{a^{(j)}_{c,k}(1+m_{c,k})}. \tag{41}$$

Form (41) we see that if $\Theta_{r,c,c,k} = I_N$ the filters will have no effect on the signal power which justifies our choice for the channel correlation matrix at the receiver side as an identity matrix for the rest of the proof and the paper. Then,

$$\Psi^{(j)}_c = \frac{1}{(NM)^2}tr(\sum_k(\Gamma^{(j)}_c)^{-2}H^H_{c,c,k}f^{(j)}_{0,c,k}a^2_{c,k}w^2_{c,k}f^H_{0,c,k}H_{c,c,k}) \tag{42}$$

$$= \frac{1}{NM}tr(\sum_k w^{(j)}_{c,k}d^{(j)}_{c,k}z^{H,(j)}_{c,c,k}\Theta^{1/2}_{c,k}(\Gamma^{(j)}_c)^{-2}\Theta^{1/2}_{c,k}z^{(j)}_{c,c,k}) \tag{43}$$

$$= \frac{1}{NM}tr(\sum_k w^{(j)}_{c,k}z^{H,(j)}_{c,c,k}\overline{\Theta}^{1/2}_{c,k}(\Gamma^{(j)}_c)^{-2}\overline{\Theta}^{1/2}_{c,k}z^{(j)}_{c,k}) \tag{44}$$

$$\ldots \rightarrow \overline{\Psi}^{(j)}_c. \tag{45}$$

the rest of the proof is as in [3]. $z_{c,c,k} = X^H_{c,c,k}f^{(j)}_{0,c,k}$ will have i.i.d entries of zero mean and $\frac{1}{NM}$ variance if $f^{(j)}_{c,k}$ is a Matched Filter (MF) as in (46) instead of the MMSE filter in (9)

$$f^{MF}_{c,k} = g^H_{c,k}H^H_{c,c,k}(\sigma^2 I_N + H_{c,c,k}g_{c,k}g^H_{c,k}H^H_{c,c,k})^{-1} \tag{46}$$

The proof is omitted due to lack in space, it will detailed in future works. However, the optimality of the MF filters is demonstrated in Figure 1. Finally, the interference power can be given

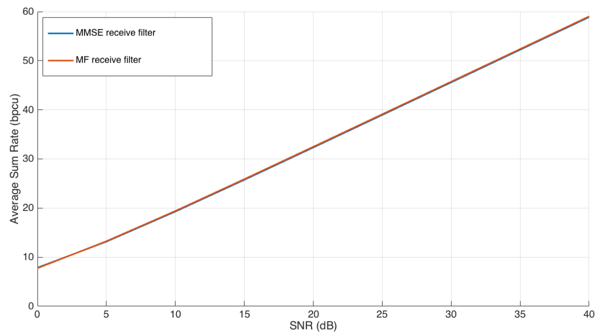

Fig. 1. Sum rate comparisons between the IBC WMMSE with MMSE filters and the IBC WMMSE with MF filters for C=1, M=10, K=4, N=2

by

$$(\frac{\overline{\xi}_c^{(j)}}{NM})^2 f_{0,c,k}^{H,(j)} H_{m,c,k} \sum_{m,l;(m,l)\neq(c,k)} H_{m,c,k}\overline{g}_{m,l}^{(j)}\overline{g}_{m,l}^{H,(j)} H_{m,c,k}^H \tag{47}$$

$$\times H_{m,c,k}^H f_{0,c,k}^{(j)} \tag{48}$$

$$= (\frac{\overline{\xi}_c^{(j)}}{1})^2 z_{m,c,k}^H \Theta_{m,c,k}^{1/2} \tag{49}$$

$$\times \sum_{m,l;(m,l)\neq(c,k)} H_{m,c,k}\overline{g}_{m,l}^{(j)}\overline{g}_{m,l}^{H,(j)} H_{m,c,k}^H \Theta_{m,c,k}^{1/2} z_{m,c,k} \tag{50}$$

$$= \frac{\xi_c^{2,(j)}}{d_{c,k}^{(j)}} z_{m,c,k}^H \overline{\Theta}_{m,c,k}^{1/2} \sum_{m,l;(m,l)\neq(c,k)} \overline{w}_{m,l}^{(j)} (\overline{\Gamma}_m^{(j)})^{-1} \tag{51}$$

$$\times \overline{\Theta}_{m,l}^{1/2} z_{m,l} z_{m,l}^H \overline{\Theta}_{m,l}^{1/2} (\overline{\Gamma}_m^{(j)})^{-1} \overline{\Theta}_{m,c,k}^{1/2} z_{m,c,k} \tag{52}$$

$$\ldots \to \frac{\xi_c^{2,(j)}}{\overline{d}_{c,k}^{(j)}} \frac{\overline{\Upsilon}_{c,k}^{(j)} + \overline{\Upsilon}_{c,k}^{(j)}}{(1+\overline{m}_{c,c,k}^{(j)})^2}. \tag{53}$$

as in [3]. The filters are MF filters as denoted previously. which completes the proof.

3.1. Applications of the deterministic equivalent of the SINR

In this subsection, the deterministic equivalent of the SINR of section 3.1 is used in order to prove a property of the MU communications that can't be established or hardly established without the use of large system analysis. We prove that for a BC system, the achievable SINR for a system with N-antennas receivers (NRx) and where only a single stream (SS) is allowed equals N times the SINR achieved in the case of single-antenna receivers (MISO) for identical channel covariance matrices and equal weights u_k. Thus,

$$\gamma_{BC,SS,NRx} = N \times \gamma_{BC,MISO} \tag{54}$$

Proof: For a BC system with K users where all channel covariances matrices Θ_k are identical, the equations (25) and (27) can be written as:

$$e^{',(j)} = \frac{e_i^{',(j)}}{\overline{d}_i} = \frac{1}{\Xi^{(j)}} \frac{e_1^{(j)}}{1-c^{(j)}e_2^{(j)}}, e_k^{',(j)} = \frac{e_{i,k}^{',(j)}}{\overline{d}_i} = \frac{\overline{d}_k^{(j)}}{\Xi^{(j)}} \frac{e_2^{(j)}}{1-c^{(j)}e_2^{(j)}}, \tag{55}$$

where $c^{(j)} = \frac{\Delta^{(j)}}{\Xi^{(j)}}$,

$$\Delta^{(j)} = \frac{1}{M}\sum_{k=1}^{K}(\frac{1}{\overline{d}_k^{(j)}} + e^{(j)})^{-2}, \Xi^{(j)} = \frac{1}{M}\sum_{k=1}^{K}(\frac{1}{\overline{d}_k^{(j)}} + e^{(j)})^{-1}, \tag{56}$$

$$e^{(j)} = \frac{e_k^{(j)}}{\overline{d}_k^{(j)}} = \frac{1}{M} tr\Theta V, e_1^{(j)} = \Xi^{(j)} \frac{1}{M} tr\Theta V^2, \tag{57}$$

and

$$e_2^{(j)} = \Xi^{(j)} \frac{1}{M} tr\Theta^2 V^2. \tag{58}$$

Furthermore, the equations (15-17) can be written as:

$$\overline{\Psi}^{(j)} = e^{',(j)}\Omega^{(j)}, \overline{\Upsilon}_k^{(j)} = e_k^{',(j)}\Omega_k^{(j)} \tag{59}$$

where

$$\Omega^{(j)} = \sum_{i=1}^{K} \frac{\overline{w}_i^{(j)}}{\overline{d}_i^{(j)}} (\frac{1}{\overline{d}_i^{(j)}} + e^{(j)})^{-2}, \; \Omega_k^{(j)} = \sum_{i=1, i\neq k}^{K} \frac{\overline{w}_i^{(j)}}{\overline{d}_i^{(j)}} (\frac{1}{\overline{d}_i^{(j)}} + e^{(j)})^{-2} \tag{60}$$

Thus, the SINR in (13) will be equivalent to:

$$\gamma_k^{(j)} = e^{(j)}\overline{w}_k^{(j)}\Xi^{(j)} \frac{\overline{d}_k^{(j)} e^{(j)}[1-c^{(j)}e_2^{(j)}]}{e_2^{(j)}\Omega_k^{(j)} + \frac{e_1^{(j)}}{\rho}\Omega^{(j)}(1+\overline{d}_k^{(j)})e^{(j)}} \tag{61}$$

However, our case of interest is when the weights u_k are identical. In that case, $\forall k$ $\Theta_k = \Theta$, $u_k = u$, $w_k^{(j)} = w$, $d_k^{(j)} = d$, $\Omega_k^{(j)} = N \times \Omega^{(j)}$, $c^{(j)} = \beta = \frac{K}{M}$. Let e be the unique positive solution of (37), then we can show that

$$e = \beta(1+e)e_2 + \frac{\beta}{\rho}(1+e)^2 e_1 \tag{62}$$

Using the relation (62) and the fact that the interference is diminished after the convergence of the WMMSE so $\Omega_k^{(j)} \to 0$, we get

$$\gamma_k^{(j)} = N \times e \tag{63}$$

or for single-antenna receivers:

$$\gamma_k^{(j)} = e \tag{64}$$

which completes the proof. We can extend the result in (54) to IBC systems.

3.2. Numerical Results

In this section, we will prove using numerical simulations the double findings of this paper. We prove the correctness of the deterministic equivalent of the SINR of MIMO single stream system as well as the validity of (54). We have seen in the previous section, that the SINR scales with N. Thus, the weighted sum rate function of SNR curve in the case of NRx must be parallel to the one obtained in the case of MISO. Figure 2 shows the simulation of WMMSE precoder for $C = 1, K = 15, M =$

30 and its approximation for the both cases of $N = 1$ and $N = 2$. For the simulations of the WMMSE algorithm, we have used 200 channel realizations. It can be observed that for i.i.d channels the approximation is accurate and that our asymptotic sum rate follows the simulated one; which validates our asymptotic approach. Although the sum rate expression for the approximation approach (13) seems to be complex, however we need to calculate it only once per a given SNR, while we need to run the IBC WMMSE simulations as many times as the number of channel realizations, i.e. 200 times. Moreover, we can observe that the curves of $N = 1$ and $N = 2$ are parallel which validates our proposition (54). Similarly, Figure 3 for $C = 1, K = 15, M = 30$ for the both cases of $N = 1$ and $N = 2$ validates our results for IBC systems.

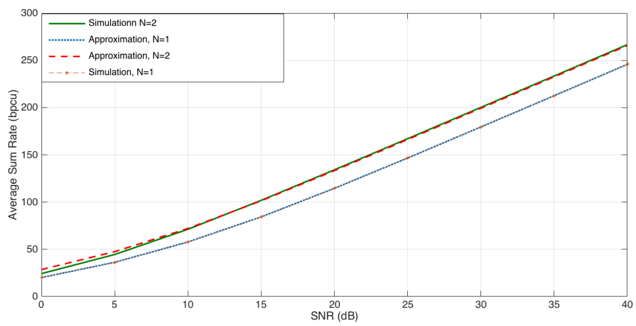

Fig. 2. Sum rate comparisons between the IBC WMMSE and our proposed approximation for C=1,K=20,M=30,N={1,2}

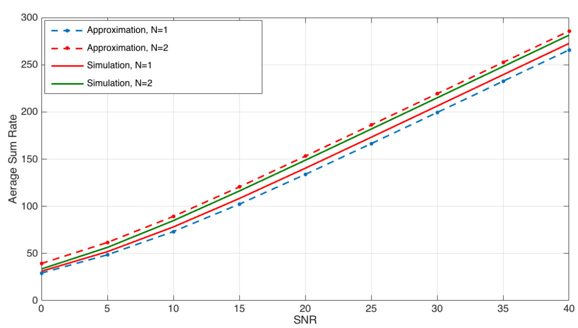

Fig. 3. Sum rate comparisons between the IBC WMMSE and our proposed approximation for C=2,K=10,M=30,N={1,2}

4. CONCLUSION

In this paper, we presented a consistent framework to study the optimal WMMSE precoding scheme based on the theory of large-dimensional random matrices. The tools from Random Matrix Theory (RMT) allowed us to study the behavior of single stream MIMO systems and to compare them to the behavior of MISO systems.

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