# High Doppler MIMO OFDM Capacity Maximizing Spatial Transceivers Exploiting Excess Cyclic Prefix

Kalyana Gopala, Dirk Slock
EURECOM, Sophia-Antipolis, France
Email: gopala,slock@eurecom.fr

***Abstract*—Performance of OFDM systems is limited by inter carrier interference (ICI) under high Doppler scenarios such as that encountered in high speed trains such as TGV. Several publications have addressed the receiver design for SISO (single input single output) and SIMO (single input multiple output) to combat ICI. Notably, the use of multiple receive antennas is a known to be a very effective way to combat ICI. In a recent publication, the authors explored the use of transmit (Tx) antennas for ICI mitigation. It considered a MIMO (multiple input multiple output) scenario and iteratively designed a transmit beamformer to maximize the sum capacity across all the subcarriers in the presence of ICI. Another important tool in the mitigation of ICI is the exploitation of excess CP (cyclic prefix). With an appropriate window function, the excess CP at the receiver may be exploited to reduce the ICI. In this work, we extend the transmit beamformer design to include the excess CP exploitation at the receiver. We jointly optimize both the receive window coefficients and the transmit beamformer using alternating minimization. The optimal window is determined using gradient descent algorithm. The convergence of the iterative approach is proved and the theory is validated via numerical simulations.**

***Keywords*—MIMO, ICI, Excess Cyclic Prefix, OFDM, Beamforming***

## I. INTRODUCTION

As is well known, high Doppler encountered in HST (high speed train) environments violates the orthogonality requirement for OFDM (Orthogonal Frequency Division Multiplexing), resulting in ICI. The SINR (signal to interference plus noise ratio) analysis due to ICI can be found in ([1],[2]). Several prior publications have focused on receiver techniques to mitigate ICI. It is known that multiple receive antennas in a SIMO scenario is very effective in cancelling out the ICI (for example, see [3]). In a recent publication ([4]), the authors of this paper extended the analysis to a MIMO (multiple input multiple output) scenario and iteratively designed a transmit beamformer to maximize the sum capacity across all the subcarriers in the presence of ICI. Another important tool in the mitigation of ICI is the exploitation of excess CP (cyclic prefix). With an appropriate window function, the excess CP at the receiver may be exploited to reduce the ICI. This is particularly relevant for HST scenarios where due to the close proximity of the Base station towers to the railway tracks, the delay spread expected is very minimal. The significance of using the excess CP and the Nyquist criterion can be found in [5]. For a SISO scenario, optimal window coefficients were derived to minimize the combined ICI and noise power in [6]. [7] derived the window coefficients to alleviate the impact of phase noise on SISO OFDM systems. More recently, a raised cosine window was used in [8] in an OFDMA uplink scenario with varying carrier frequency offsets (CFO) across different users to reduce the extent of spread of ICI across the subcarriers. This was in turn utilized to aid in inverting an ICI interference matrix.

In this paper, we focus on a MIMO scenario and derive the channel capacity in the presence of ICI caused by channel variation. Our interest lies in analyzing the impact of excess of antennas in combating ICI. Specifically, for LTE systems, due to the large subcarrier spacing, we consider a linear channel variation as was done in ([6],[7],[9], [10]). We consider a frequency selective scenario and iteratively arrive at the optimal beamformer for every subcarrier. The transmit beamformer design takes into account the excess CP exploitation at the receiver. In fact, we jointly optimize both the receive window and the transmit beamformer using a cyclic minimization approach. The algorithm can also easily account for the presence of the guard and DC subcarriers to model a realistic transmission scenario. The convergence of the proposed methodology is also shown. The main contributions of this paper are as follows.

- We design an ICI-aware beamformer for a MIMO OFDM system under FIR (finite impulse response) multipath channel that also takes into account exploitation of excess CP by the receiver.
- We iteratively arrive at the optimal window coefficients to exploit the excess CP using the gradient descent method.
- The convergence of the entire beamformer design is proved and then shown numerically via simulations.

The rest of the paper is organized as follows. We first present the system model in Section II. This is followed by the design of the beamformer in section III. Simulation results are presented for different scenarios in section IV. Finally, conclusions are given in section V. In the following discussions, a bold notation in small letters indicates a vector and bold notation with capital letters indicates a matrix.

## II. SYSTEM MODEL

Consider a multiple input multiple output (MIMO) system with $N_t$ transmit antennas and $N_r$ receive antennas. An OFDM framework is chosen with $N$ subcarriers and sampling rate $f_s$. Out of the total $N$ subcarriers, let $N_u$ be the number of utilized subcarriers. For instance, this would account for the guard subcarriers and DC subcarrier in an OFDM system. We consider a time-varying Rician fading FIR channel of length $L$. Thus for every combination of Tx(transmit) and Rx(receive)

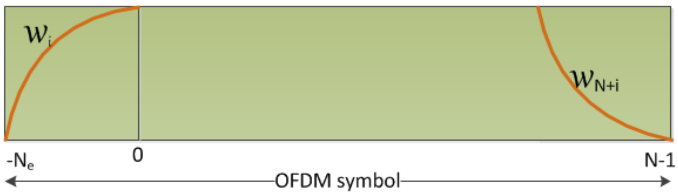


Fig. 1. Illustration of Excess CP windowing in an OFDM symbol

antenna, the time domain channel at sample $n$ of an OFDM symbol may be represented as

$$\mathbf{h}(n) = \mathbf{h}_0 + \mathbf{h}'(n) \tag{1}$$

where $\mathbf{h}_0$ is of dimension $L \times 1$ and represents the average channel across the OFDM symbol. $\mathbf{h}'$ is also of dimension $L \times 1$ and captures the time variation, has average value zero, and is orthogonal to $\mathbf{h}_0$. It is easy to see that with this formulation, the ICI contribution comes entirely from $\mathbf{h}'(n)$ (see [3] for example).

The length of the CP is considered to be greater than the channel delay spread by $N_e$ samples. The total length of the OFDM symbol including the excess CP length is taken as $N_s = N + N_e$. It is also assumed that the receiver would take advantage of this excess CP through windowing. As shown in Figure 1, let $w_i$ be the window weights. In order to satisfy the Nyquist criterion ([5]),

$$w_i + w_{N+i} = 1, \qquad i \in \{-N_e \cdots - 1\} \tag{2}$$

To continue the analysis, we can approximate $\mathbf{h}'(n)$ by a polynomial function. For an LTE-like OFDM system, we choose a linear model due to the significant subcarrier spacing compared to the Doppler frequency being considered. A similar assumption is made in previous works exploiting the excess CP as well ([6],[7],[9], [10]). Thus, for the duration of an OFDM symbol including the excess CP, (1) may be rewritten in terms of orthogonal basis functions for every tx-rx antenna pair as,

$$\begin{bmatrix} \mathbf{h}^T(-N_e) \\ \vdots \\ \mathbf{h}^T(N-1) \end{bmatrix} = \begin{bmatrix} 1 & (-N_e - \frac{N_s-1}{2}) \\ \vdots & \vdots \\ 1 & (N-1-\frac{N_s-1}{2}) \end{bmatrix} \begin{bmatrix} \mathbf{h}_0^T \\ \mathbf{h}_1^T \end{bmatrix} \tag{3}$$

where $\mathbf{h}_1$ is a constant across the OFDM symbol and captures the time variation per sample.

In what follows, without loss of generality, we take the CP length as the same as that of the excess CP length. The receiver output across all the receive antennas and subcarriers after the windowing and $N$-point FFT may be expressed as column vector of length $N_r N$ ($N_r$ received elements for each subcarrier). This may be expressed as,

$$\begin{aligned} \mathbf{y} =& \mathbf{F}_{N,N_r}\mathbf{T}^T_{cp,N_r}\mathbf{D}_{w,N_r}\{\mathbf{T}_{cp,N_r}\breve{\mathbf{H}}_0 + \\ & \mathbf{D}_{b,N_r}\mathbf{T}_{cp,N_r}\breve{\mathbf{H}}_1\}\mathbf{F}^{-1}_{N,N_t}\mathbf{s} + \mathbf{F}_{N,N_r}\mathbf{T}^T_{cp,N_r}\mathbf{D}_{w,N_r}\breve{\mathbf{v}} \end{aligned} \tag{4}$$

$\breve{\mathbf{H}}_0$ and $\breve{\mathbf{H}}_1$ are time domain block circulant channel matrices of dimension $N_r N \times N_t N$. Each block in $\breve{\mathbf{H}}_0$ or $\breve{\mathbf{H}}_1$ is of dimension $N_r \times N_t$ and there are $N \times N$ such blocks in these matrices. $\breve{\mathbf{v}}$ is the AWGN noise observed at the receiver and is normalized to have unit variance. $\mathbf{s}$ is the concatenated transmit data vector across all the transmit antennas and subcarriers and is of dimension $NN_t$. $\mathbf{F}_{N,N_t} = \mathbf{F}_N \otimes \mathbf{I}_{\mathbf{N_t}}$, where $\mathbf{F}_N$ is the DFT (discrete Fourier transform) matrix.

$$\begin{aligned} \mathbf{D}_w &= \mathtt{diag}(w_{-N_e} \dots w_{-1} \; 1 \dots 1 \; w_1 \dots w_{N_e}) \\ \mathbf{D}_b &= \mathtt{diag}\left(-(N_e - \frac{N_s-1}{2}) \dots (N-1-\frac{N_s-1}{2})\right) \\ \mathbf{T}_{cp} &= \begin{bmatrix} \mathbf{0}_{N_e \times N - N_e} & \mathbf{I}_{N_e} \\ \mathbf{I}_N & \end{bmatrix} \end{aligned}$$

Further, $\mathbf{D}_{w,N_r} = \mathbf{D}_w \otimes \mathbf{I}_{N_r}$, $\mathbf{D}_{b,N_r} = \mathbf{D}_b \otimes \mathbf{I}_{N_r}$ and $\mathbf{T}_{cp,N_r} = \mathbf{T}_{cp} \otimes \mathbf{I}_{N_r}$. Note that we can directly verify that the Nyquist criterion given in equation (2) results in $\mathbf{T}^T_{cp}\mathbf{D}_w\mathbf{T}_{cp} = \mathbf{I}_N$. The dimension of $\mathbf{D}_w$ and $\mathbf{D}_b$ are $(N+N_e) \times (N+N_e)$. Hence, assuming that the window parameters satisfy the Nyquist condition, equation (4) may be rewritten as

$$\begin{aligned} \mathbf{y} &= \tilde{\mathbf{H}}_0\mathbf{s} + \mathbf{F}_{N,N_r}\mathbf{T}^T_{cp,N_r}\mathbf{D}_{w,N_r}\mathbf{D}_{b,N_r}\mathbf{T}_{cp,N_r}\breve{\mathbf{H}}_1\mathbf{F}^{-1}_{N,N_t}\mathbf{s} \\ &\quad + \mathbf{F}_{N,N_r}\mathbf{T}^T_{cp,N_r}\mathbf{D}_{w,N_r}\breve{\mathbf{v}} \\ &= \mathbf{H}_0\mathbf{s} + \breve{\Xi}\mathbf{H}_1\mathbf{s} + \mathbf{v} \end{aligned} \tag{5}$$

where $\tilde{\mathbf{H}}_0 = \mathbf{F}_{N,N_r}\breve{\mathbf{H}}_0\mathbf{F}^{-1}_{N,N_t}$ is a block diagonal matrix corresponding to the time-invariant part . $\mathbf{H}_1 = \mathbf{F}_{N,N_r}\breve{\mathbf{H}}_1\mathbf{F}^{-1}_{N,N_t}$ is a block diagonal matrix corresponding to the time varying part of the channel. $\breve{\Xi} = \mathbf{F}_{N,N_r}\mathbf{T}^T_{cp,N_r}\mathbf{D}_{w,N_r}\mathbf{D}_{b,N_r}\mathbf{T}_{cp,N_r}\mathbf{F}^{-1}_{N,N_r}$, where $\Xi$ is a block circulant matrix. It can be easily observed using the properties of the Kronecker product that $\breve{\Xi} = \Xi \otimes \mathbf{I}_{N_r}$, where $\Xi = \mathbf{F}_N\mathbf{T}^T_{cp}\mathbf{D}_w\mathbf{D}_b\mathbf{T}_{cp}\mathbf{F}^{-1}_N$ is a circulant matrix of dimension $N \times N$. $\mathbf{v} = \mathbf{F}_{N,N_r}\mathbf{T}^T_{cp,N_r}\mathbf{D}_{w,N_r}\breve{\mathbf{v}}$ is the frequency domain correlated noise samples seen across the subcarriers and receive antennas. It must be noted that the correlation of the noise is across the subcarriers and is a result of the windowing operation. For any given subcarrier, the noise across the various receive antennas is uncorrelated, as expected.

Thus, at any subcarrier $k$, the received data may be written as

$$\begin{aligned} \mathbf{y}_k &= \{\tilde{\mathbf{H}}_{0k} + \mathbf{H}_{1k}\Xi_{k,k}\}\mathbf{d}_k + \sum_{l=0,l\neq k}^{N-1} \mathbf{H}_{1l}\mathbf{d}_l\Xi_{k,l} + \mathbf{v}_k \\ &= \underbrace{\mathbf{H}_{0k}\mathbf{d}_k}_{\text{Signal term}} + \underbrace{\sum_{l=0,l\neq k}^{N-1} \mathbf{H}_{1l}\mathbf{d}_l\Xi_{k,l} + \mathbf{v}_k}_{\text{ICI and noise terms}} \end{aligned} \tag{6}$$

$\mathbf{H}_{0k}$ (dimension $N_r \times N_t$) is the mean frequency domain channel observed at subcarrier $k$. The second term in equation (6) represents the ICI (inter carrier interference) caused by time variation due to Doppler. $\mathbf{d}_k = [\mathbf{s}(kN_t+1) \cdots \mathbf{s}(kN_t+N_t-1)]^T$ is the $N_t \times 1$ vector of transmitted data symbols on the carrier $k$. $\Xi_{k,l}$ refers to the ($k$,$l$) element of the matrix $\Xi$. $\mathbf{v}_k$ is the $N_r \times 1$ vector of AWGN (additive white Gaussian noise) noise observed at carrier index k, with the following variance.

$$\begin{aligned} \mathbf{R}_{\mathbf{v}_k} &= (e_k^T\mathbf{F}_N\mathbf{T}^T_{cp}\mathbf{D}_w\mathbf{D}^H_w\mathbf{T}_{cp}\mathbf{F}^H_N e_k) \otimes \mathbf{I}_{N_r} \\ &= (e_k^T\mathbf{F}_N\mathbf{T}^T_{cp}\mathbf{D}_w\mathbf{D}^H_w\mathbf{T}_{cp}\mathbf{F}^H_N e_k)\mathbf{I}_{N_r} \end{aligned} \tag{7}$$

where $e_k$ is a column vector with 1 at the $k^{th}$ element.

Let $P$ be the maximum sum power requirement across all the subcarriers and let $P_i$ be the individual power at any subcarrier $i$ such that $\sum_{i=0}^{N-1} P_i = P$. Let the transmit covariance matrix of subcarrier $k$ be $\mathbf{Q}_k = \mathtt{E}(\mathbf{d}_k\mathbf{d}_k^H)$ where $\mathtt{E}(\cdot)$ is the expectation operator. Thus, the capacity of this MIMO system across all the subcarriers in the presence of both ICI and AWGN noise would be given as follows.

$$\mathbf{C} = \sum_{k=0}^{N-1} \log|\mathbf{I} + \mathbf{H}_{0k}\mathbf{Q}_k\mathbf{H}_{0k}^H\bar{\mathbf{R}}_k^{-1}| \tag{8}$$

where $\bar{\mathbf{R}}_k = \mathbf{R}_{\mathbf{v}_k} + \sum_{l=0,l\neq k}^{N-1} |\Xi_{k,l}|^2\mathbf{H}_{1l}\mathbf{Q}_l\mathbf{H}_{1l}^H$. Note that this formulation can include guard subcarriers and DC subcarrier by simply forcing their respective transmit covariances to zero. We are interested in determining the optimal $\mathbf{Q}_k$ and the window weights $w_i$ such that the capacity of the link is maximised under a power constraint

$$\begin{aligned}\mathtt{f}_0 : \max_{\mathbf{Q}_k, w_{-N_e},\ldots,w_{-1}} \mathbf{C} &= \max_{\mathbf{Q}_k} \sum_{k=0}^{N-1} \log|\mathbf{I} + \mathbf{H}_{0k}\mathbf{Q}_k\mathbf{H}_{0k}^H\bar{\mathbf{R}}_k^{-1}| \\ \mathtt{subject\ \ to} \quad &\sum_{k=0}^{N-1} \mathtt{tr}\{\mathbf{Q}_k\} \leq P.\end{aligned} \tag{9}$$

## III. Beamformer Design

The objective function $\mathtt{f}_0$ in (9) is non-convex in the covariance matrix $\mathbf{Q}_i$ and hence we follow an iterative approach as in [4]. In addition, to solve the joint problem of optimizing the window design we employ the alternating (cyclic) minimization approach to alternately optimize the precoder design and window design. At the beginning of the iteration for the subcarrier $i$, let $P_i$ be the power constraint, $\bar{\mathbf{Q}}_i$ be the current values of the precoder and $w_i$ be the window values. The steps involved in optimization are

- Update the value of $\mathbf{Q}_i$ for every used subcarrier $i$ using the majorization technique ([11]). This is given in subsection III-A.
- Update of power allocation across all the subcarriers. This is given in subsection III-B.
- Update of window parameters. This is given in section III-C.

### A. Covariance matrix update

Our iterative optimization algorithm operates one subcarrier at a time. With focus on subcarrier $i$, on the same lines as [12], the objective function $\mathtt{f}_0$ may be rewritten as

$$\begin{aligned}&\max_{\mathbf{Q}_i} \sum_{k=0}^{N-1} \log|\mathbf{I} + \mathbf{H}_{0k}\mathbf{Q}_k\mathbf{H}_{0k}^H\bar{\mathbf{R}}_k^{-1}| \\ &= \max_{\mathbf{Q}_i}\{\log|\mathbf{I} + \mathbf{H}_{0i}\mathbf{Q}_i\mathbf{H}_{0i}^H\bar{\mathbf{R}}_i^{-1}| + f_i(\mathbf{Q}_i, \mathbf{Q}_{-i})\}\end{aligned} \tag{10}$$

where $f_i(\mathbf{Q}_i, \mathbf{Q}_{-i}) = \sum_{l\neq i} \log|\mathbf{I} + \mathbf{H}_{0l}\mathbf{Q}_l\mathbf{H}_{0l}^H\bar{\mathbf{R}}_l^{-1}|$. $\mathbf{Q}_{-i}$ refers to the transmit covariances of all the subcarriers except the $i^{th}$. It is shown in [12] (*Lemma* 1) that $f_i(\mathbf{Q}_i, \mathbf{Q}_{-i})$ is convex in $\mathbf{Q}_i$.Thus, equation (10) is the sum of a concave and convex function and hence the overall capacity is a non-convex function.

As in [4], we replace the non-convex function above by it's minorization which is convex.

$$\begin{aligned}\mathtt{f}_1 : \log|\mathbf{I} + \mathbf{H}_{0i}\mathbf{Q}_i\mathbf{H}_{0i}^H\bar{\mathbf{R}}_i^{-1}| - \mathtt{tr}\left\{\mathbf{B}_i(\mathbf{Q}_i - \bar{\mathbf{Q}}_i)\right\} + \\ f_i(\bar{\mathbf{Q}}_i, \bar{\mathbf{Q}}_{-i}) \\ \mathtt{subject\ \ to} \quad \mathtt{tr}\{\mathbf{Q}_i\} \leq \bar{P}_i\end{aligned} \tag{11}$$

where $\mathbf{B}_i$ is the negative Hermitian of the derivative of $f_i(\mathbf{Q}_i, \mathbf{Q}_{-i})$ with respect to $\mathbf{Q}_i$ evaluated at $\bar{\mathbf{Q}}_i, \bar{\mathbf{Q}}_{-i}$. $\bar{P}_i$ indicates the current value of $P_i$ at any given stage of the algorithm. $\mathbf{B}_i$ is given in equation (12) below (see also [13]).

$$\begin{aligned}\mathbf{B}_i &= -\left[\frac{\partial f_i(\mathbf{Q}_i, \mathbf{Q}_{-i})}{\partial \mathbf{Q}_i}\right]^H \\ &= \sum_{l\neq i} |\Xi_{l,i}|^2\mathbf{H}_{1i}\left\{\bar{\mathbf{R}}_l^{-1} - (\bar{\mathbf{R}}_l + \mathbf{H}_{0l}\mathbf{Q}_l\mathbf{H}_{0l}^H)^{-1}\right\}\mathbf{H}_{1i}^H\end{aligned} \tag{12}$$

Let $\mathbf{A}_i = \mathbf{H}_{0i}^H\bar{\mathbf{R}}_i^{-1}\mathbf{H}_{0i}$, $\mathbf{Q}_i = \mathbf{V}_i\mathbf{\Lambda}_i\mathbf{V}_i^H$ and $\lambda_{ij}$ be the $j^{th}$ diagonal element of $\mathbf{\Lambda}_i$.The optimal solution to this subproblem (see [4])

$$\mathbf{A}_i\mathbf{V}_i = (\mathbf{B}_i + \mu_i\mathbf{I})\,\mathbf{V}_i\mathbf{\Sigma} \tag{13}$$

Let $\mathbf{V}_i^H\mathbf{A}_i\mathbf{V}_i = \mathbf{D}_{1i}$, where $\mathbf{D}_{1i}$ is a diagonal matrix as $\mathbf{V}_i$ is generalized eigenmatrix of $\mathbf{A}_i$, $\mathbf{B}_i + \mu_i\mathbf{I}$. Let $\mathbf{D}_{2i}$ be a diagonal matrix containing the diagonal elements of the matrix $\mathbf{V}_i^H\mathbf{B}_i\mathbf{V}_i$.

$$\begin{aligned}\lambda_{ij} = \left[\frac{1}{\mathbf{D}_{2i}(j,j) + \mu_i} - \frac{1}{\mathbf{D}_{1i}(j,j)}\right]^+ \\ \forall j \text{ such that } \quad \mathbf{D}_{1i}(j,j) > 0\end{aligned} \tag{14}$$

where $[x]^+$ indicates $\max(x, 0)$.

The optimal $\mu_i$ can now be determined using a bisection search as $\lambda_{ij}$ is monotonic in $\mu_i$. Thus, the convex objective function $\mathtt{f}_1$ can be solved iteratively till $\mathbf{Q}_i$ converges.

### B. Power allocation across the subcarriers

After obtaining one set of updated $\mathbf{Q}_i$ for all the subcarriers, one can now update the power allocation across the various subcarriers. Note that in this step, the optimal transmit directions across all the used subcarriers remain unchanged, and only the power allocation across the various transmit streams of all the used subcarriers is optimized. From [4],

$$\begin{aligned}\lambda_{ij} = \left[\frac{1}{\mathbf{D}_{2i}(j,j) + \eta} - \frac{1}{\mathbf{D}_{1i}(j,j)}\right]^+ \\ \forall i \text{ such that } \quad \mathbf{D}_{1i}(j,j) > 0\end{aligned} \tag{15}$$

The optimal $\eta$ can now be determined using a bisection search as $\lambda_{ij}$ is monotonic in $\eta$. Once all the $\lambda_{ij}$ across all the subcarriers and their transmit streams are obtained, this is in turn used to update the transmit covariance matrix $\mathbf{Q}_i$ and the power allocation $P_i$ of each used subcarrier $i$.

## C. Optimization of window parameters

Once Tx covariance matrices $\mathbf{Q}_i$ have been computed for all the subcarriers, we perform a gradient search to optimize the window parameters. We limit the optimization to the parameters $w_i$, $i \in -N_e, \ldots, -1$ as the window parameters $w_{N+i}$ may be determined to satisfy the Nyquist criterion.

$$\max_{w_i, i \in (-N_e \ldots -1)} C = \sum_{k=0}^{N-1} \log |\mathbf{I} + \mathbf{H}_{0k}\mathbf{Q}_k\mathbf{H}_{0k}^H\bar{\mathbf{R}}_k^{-1}| \tag{16}$$

Following the steps in [13],

$$\frac{\partial C}{\partial w_i^*} = -\mathtt{tr}\{(\mathbf{I} + \mathbf{H}_{0k}\mathbf{Q}_k\mathbf{H}_{0k}^H\bar{\mathbf{R}}_k^{-1})^{-1} \left(\sum_{k=0}^{N-1} \mathbf{H}_{0k}\mathbf{Q}_k \left(\mathbf{H}_{1k}^H \frac{\partial \Xi_{k,k}^H}{\partial w_i^*} - \mathbf{H}_{0k}^H\bar{\mathbf{R}}_k^{-1}\frac{\partial \bar{\mathbf{R}}_k}{\partial w_i^*}\right)\bar{\mathbf{R}}_k^{-1}\right)\} \tag{17}$$

$$\frac{\partial \bar{\mathbf{R}}_k}{\partial w_i^*} = \sum_{l=0, l\neq k}^{N-1} \Xi_{k,l}\frac{\partial \Xi_{k,l}^H}{\partial w_i^*}\mathbf{H}_{1l}\mathbf{Q}_l\mathbf{H}_{1l}^H + e_k^T\mathbf{F}_N\mathbf{T}_{cp}^T\mathbf{D}_w\frac{\partial \mathbf{D}_w^H}{\partial w_i^*}\mathbf{T}_{cp}\mathbf{F}_N^H e_k\mathbf{I}_{N_r} \tag{18}$$

$$\frac{\partial \Xi_{k,l}^H}{\partial w_i^*} = e_k^T\mathbf{F}_N\mathbf{T}_{cp}^T\mathbf{D}_b\frac{\partial \mathbf{D}_w^H}{\partial w_i^*}\mathbf{T}_{cp}\mathbf{F}_N^H e_l\mathbf{I}_{N_r} \tag{19}$$

The matrix $\frac{\partial \mathbf{D}_w^H}{\partial w_i^*}$ is a diagonal matrix with unity at the $i^{th}$ diagonal element and zeros everywhere else. i.e., $\frac{\partial \mathbf{D}_w^H}{\partial w_i^*} = \mathtt{diag}(0, \ldots, 1, \ldots, 0)$. The iterative update of the window parameters is now performed as

$$w_i = w_i + \varepsilon\frac{\partial C}{\partial w_i^*}, i \in -N_e \ldots -1 \tag{20}$$

where $\varepsilon$ is a suitable positive step size for the gradient algorithm.

## D. Overall Algorithm and Convergence

The overall algorithm that solves $\mathtt{f}_0$ is summarized in Table I. The overall algorithm alternates between the transmit beamformer optimization and the window coefficient optimization. At every iteration of the transmit beamformer , a convex sub-problem $\mathtt{f}_1$ is created and optimized based on the updated value of $\mathbf{Q}_i, \mathbf{Q}_{-i}$ from the last iteration. A power allocation across all the subcarriers is performed at the end of one round of transmit covariance update for all subcarriers. The window optimization is based on the gradient search method.

To ensure convergence of this maximization algorithm, we observe that the algorithm is non-decreasing from the following.

- $\mathtt{f}_1$ is a minorization ([11]) function for $\mathtt{f}_0$ at any $\mathbf{Q}_i, \bar{\mathbf{Q}}_i, \bar{\mathbf{Q}}_{-i}$. For more detailed explanation, see [4].
- The iterations for optimization of $\mathbf{Q}_i$ and power allocations are steps in cyclic minimization (actually maximization in this problem, also see [11]).

TABLE I. OVERALL ALGORITHM TO SOLVE OBJECTIVE FUNCTION $\mathtt{f}_0$

| |
|---|
| Initialize window parameters using raised cosine filter coefficients |
| $\mathbf{\Xi} = \mathbf{F}_N\mathbf{T}_{cp}^T\mathbf{D}_w\mathbf{D}_b\mathbf{T}_{cp}\mathbf{F}_N^{-1}$ |
| for $k$ = 0,1 $\ldots$ $N-1$ |
| Initialize $P_k = \frac{P}{N_u}\mathbf{I}$ and $\mathbf{Q}_k = \frac{P_k}{N_t}\mathbf{I}$ |
| Initialize $\mathbf{R}_{\mathbf{v}_k} = (e_k \otimes \mathbf{I}_{N_r})^T\mathbf{F}_{N,N_r}\mathbf{T}_{cp}^T\mathbf{D}_w\mathbf{D}_w^H\mathbf{T}_{cp}\mathbf{F}_{N,N_r}^H(e_k \otimes \mathbf{I}_{N_r})$ |
| Initialize $\bar{\mathbf{R}}_k = \mathbf{R}_{\mathbf{v}_k} + \sum_{l=0,l\neq k}^{N-1} \lvert\Xi_{k,l}\rvert^2\mathbf{H}_{1k}\mathbf{Q}_l\mathbf{H}_{1k}^H$ |
| Initialize $\mathbf{H}_{0k} = \{\tilde{\mathbf{H}}_{0k} + \mathbf{H}_{1k}\Xi_{k,k}\}$ |
| Repeat until convergence |
| **Perform Tx beamformer optimization** |
| Repeat until convergence |
| Update Tx covariance matrix, perform power allocation, see [4], Table I |
| **Perform window coefficient update** |
| Repeat until convergence |
| for $i = -N_e, \ldots, -1$ |
| compute $\frac{\partial C}{\partial w_i^*}$ as in (17) |
| $w_i = w_i + \varepsilon\frac{\partial C}{\partial w_i^*}$ and $w_{N+i} = 1 - w_i$ |
| $\mathbf{\Xi} = \mathbf{F}_N\mathbf{T}_{cp}^T\mathbf{D}_w\mathbf{D}_b\mathbf{T}_{cp}\mathbf{F}_N^{-1}$ |
| for $k$ = 0,1 $\ldots$ $N-1$ |
| $\mathbf{R}_{\mathbf{v}_k} = e_k^T\mathbf{F}_N\mathbf{T}_{cp}^T\mathbf{D}_w\mathbf{D}_w^H\mathbf{T}_{cp}\mathbf{F}_N^H e_k\mathbf{I}_{N_r}$ |
| $\bar{\mathbf{R}}_k = \mathbf{R}_{\mathbf{v}_k} + \sum_{l=0,l\neq k}^{N-1} \lvert\Xi_{k,l}\rvert^2\mathbf{H}_{1k}\mathbf{Q}_l\mathbf{H}_{1k}^H$ |
| $\mathbf{H}_{0k} = \{\tilde{\mathbf{H}}_{0k} + \mathbf{H}_{1k}\Xi_{k,k}\}$ |

- The optimization of the window coefficients is again a step in cyclic minimization with respect to the above two operations.

Thus, by design, the algorithm is non-decreasing and ensures convergence.

# IV. NUMERICAL RESULTS

We consider a MIMO fading channel based on equation (3). A single user MIMO scenario with signal to AWGN noise ratio of 25dB is considered. For every Tx-Rx pair,FIR Rayleigh fading channels are generated independently with the power delay profile (PDP) as [0 -5 -5] in dB for $\mathbf{h}_0$ and $\mathbf{h}_1$. An LTE OFDM system operating at unlicensed 2.4GHz band is considered with 15KHz of channel spacing. A Doppler frequency corresponding to 450kmph is assumed. The entries of $\mathbf{h}_1$ are scaled such that the overall ICI power experienced at any receive antenna corresponds to a Doppler frequency shift of 450kmph. The capacity of the iterative scheme under different scenarios is considered. In the simulation results presented, all subcarriers are assumed to be used. For the gradient search, a step size of 0.01 is used. Four rounds of overall iterations is considered in the simulation. In each round, 10 iterations of the Tx beamformer optimization at the end of which, one iteration for the window optimization is done. A single iteration of the window optimization itself has 100 steps of equation (20). Thus, in the simulations, the window optimizations happen at iterations 10, 20, 30 and 40. The raised cosine window used for initialization is the same as in [8].

Figure 2 shows a scenario with $N_t = 3$ transmit antennas and $N_r = 3$ receive antennas.In this case, we consider $N = 16, N_e = 8$. The first curve (”WF, No ExCP”) gives the waterfilling performance in the absence of cyclic prefix exploitation. Also, the transmitter covariance matrices are designed as though there were no ICI. This is the standard waterfilling algorithm where $\mathbf{H}_1$ is all zeros and no excess CP is exploited. Also given is a curve (”ICI aware WF,no

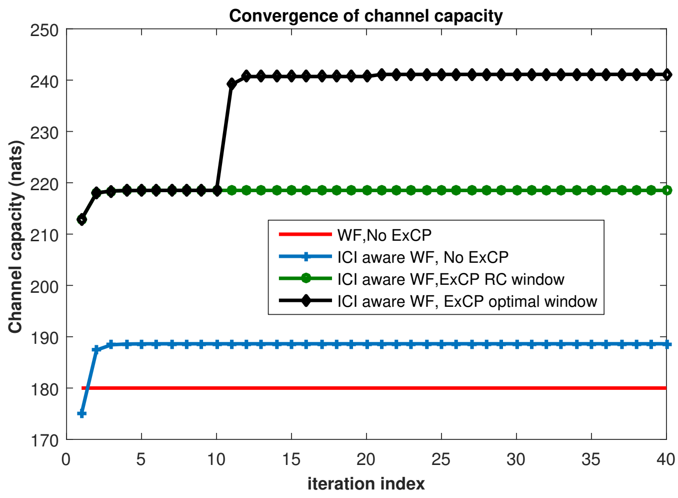


Fig. 2. Simulation Results with $N_t = 3$,$Nr = 3$ and Doppler of 450Kmph

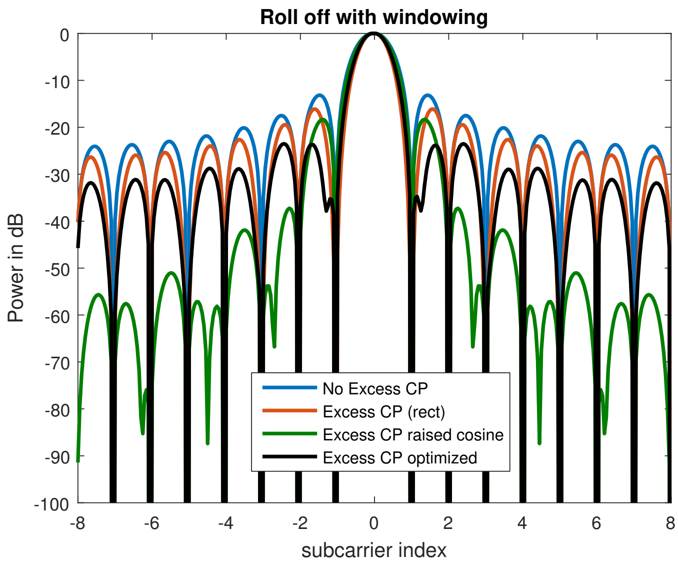


Fig. 3. Comparison of windows used to exploit excess CP,$N = 16$, $N_e = 8$

ExCP") that does the iterative optimization of the transmit beamformer with the knowledge of ICI, again in the absence of excess CP. The curve "ICI aware WF, ExCP RC window" shows the performance of the ICI aware transmit beamforming optimization that uses a raised cosine window to exploit the excess CP. Finally, the curve ("ICI aware WF, ExCP optimal window") shows the performance with ICI aware transmit beamformer optimization and optimized window coefficients for excess CP. Figure 3 gives the roll-off obtained for the optimized window in comparison with other windows for the scenario $N = 16, N_e = 8$. "No Excess CP" corresponds to the scenario where excess CP is not exploited. "Excess CP (rect)" refers to all window coefficient values being set to $0.5$. "Excess CP raised cosine" refers to raised cosine window being used for Excess CP. It is very clearly seen that the optimal window does a good side lobe reduction for the closest side lobes and does not over attenuate the farther side lobes, as done by the raised cosine window. This is quite intuitive too and explains why the optimal window performs superior to the raised cosine window.

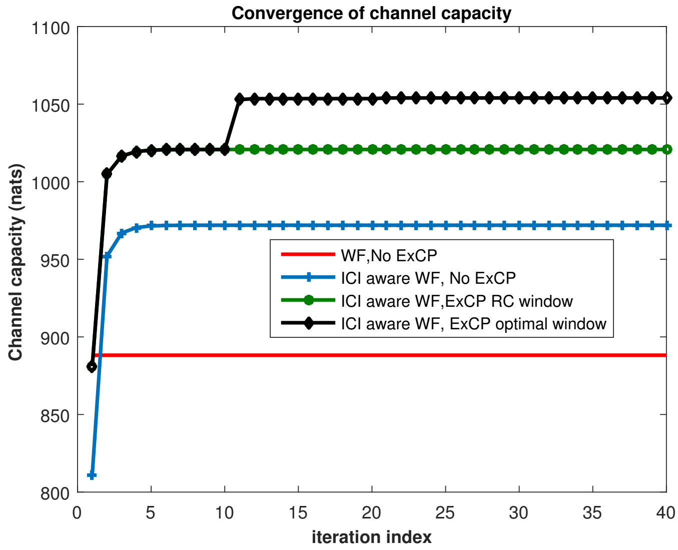


Fig. 4. Simulation Results with $N_t = 4$,$Nr = 3$ and Doppler of 450Kmph

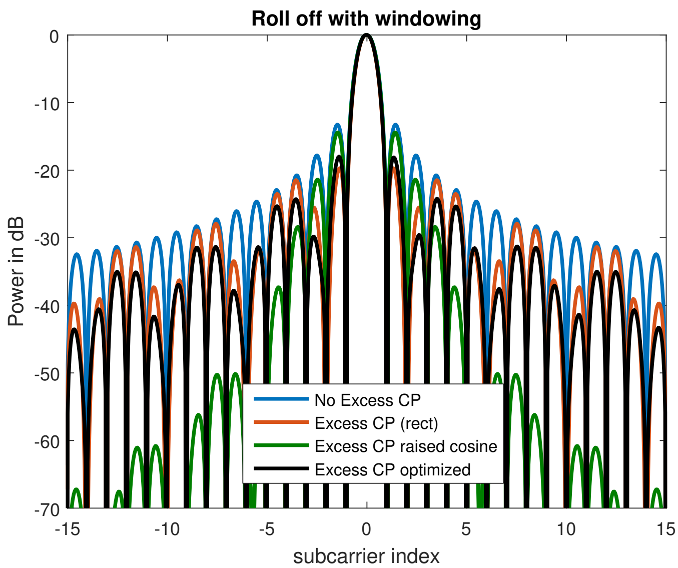


Fig. 5. Comparison of windows used to exploit excess CP,$N = 64$, $N_e = 16$

Figure 4 shows a scenario with $N_t = 4$ transmit antennas and $N_r = 3$ receive antennas.In this case, we consider $N = 64, N_e = 16$. The corresponding window roll off is given in Figure 5. To improve the clarity of the figure, only subcarrier numbers from -15 to +15 are displayed. The optimal window can be observed to strike a better balance in side lobe reduction compared to the raised cosine window.

In the simulation scenarios considered, we see that the iterations always exhibit a non-decreasing behaviour in the capacity as is predicted by the theory (section III-D).

## V. CONCLUSION

In this paper, we formulated the waterfilling problem for an OFDM system, in the presence of ICI. The channel variations resulting in ICI are assumed to be linear across the OFDM symbol length. The waterfilling algorithm also takes into account the excess CP that can be exploited at the receiver. In fact, the optimal window weights at the receiver are also derived. The ICI roll off for the optimally derived window is compared with other windows and the observations are intuitively appealing too. To solve the joint problem of transmit precoder design and the window weights, we followed the alternating minimization approach. The window weights were arrived at using the gradient descent algorithm. We were able to show analytically, the convergence of the iterative method and then give numerical results that show the convergence behaviour.

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## References

[1] M. Faulkner, L. Wilhelmsson, and J. Svensson, "Low-Complex ICI Cancellation for Improving Doppler Performance in OFDM Systems," in *IEEE Vehicular Technology Conference*, Sept 2006.

[2] Y. Peng, W. Wang, and Y. I. Kim, "Performance Analysis of OFDM System Over Time-Selective Fading Channels," in *IEEE Wireless Communications and Networking Conference (WCNC)*, April 2009, pp. 1–5.

[3] K. Gopala and D. Slock, "Doppler compensation and Beamforming for High Mobility OFDM transmissions in multipath," in *EAI International Conference on Cognitive Radio Oriented Wireless Networks*, June 2016.

[4] K.Gopala and D.Slock, "MIMO OFDM Capacity Maximizing Beamforming for Large Doppler Scenarios," in *IEEE Workshop on Signal Processing Advances in Wireless Communications (SPAWC '16)*, July 2016.

[5] S. H. Muller-Weinfurtner, "Optimum Nyquist windowing in OFDM receivers," *IEEE Transactions on Communications*, vol. 49, no. 3, pp. 417–420, Mar 2001.

[6] C. Y. Ma, S. W. Liu, and C. C. Huang, "On Optimum Segment Combining Weight for ICI Self-Cancellation in OFDM Systems under Doubly Selective Fading Channels," in *IEEE Vehicular Technology Conference (VTC Spring)*, May 2012.

[7] C. Y. Ma, C. Y. Wu, and C. C. Huang, "A Simple ICI Suppression Method Utilizing Cyclic Prefix for OFDM Systems in the Presence of Phase Noise," *IEEE Transactions on Communications*, vol. 61, no. 11, pp. 4539–4550, November 2013.

[8] A. Farhang, N. Marchetti, L. E. Doyle, and B. Farhang-Boroujeny, "Low Complexity CFO Compensation in Uplink OFDMA Systems With Receiver Windowing," *IEEE Transactions on Signal Processing*, vol. 63, no. 10, pp. 2546–2558, May 2015.

[9] Y. Mostofi and D. Cox, "ICI mitigation for pilot-aided OFDM mobile systems," *IEEE Transactions on Wireless Communications*, vol. 4, no. 2, pp. 765–774, March 2005.

[10] S. Serbetli, "Doppler compensation for mobile OFDM systems with multiple receive antennas," in *IEEE 19th Symposium on Communications and Vehicular Technology in the Benelux (SCVT)*, 2012, pp. 1–6.

[11] P. Stoica and Y. Selen, "Cyclic minimizers, majorization techniques, and the expectation-maximization algorithm: a refresher," *IEEE Signal Processing Magazine*, vol. 21, no. 1, pp. 112–114, Jan 2004.

[12] S.-J. Kim and G. Giannakis, "Optimal Resource Allocation for MIMO Ad Hoc Cognitive Radio Networks," *IEEE Transactions on Information Theory*, vol. 57, no. 5, pp. 3117–3131, May 2011.

[13] A. Hjorungnes and D. Gesbert, "Complex-Valued Matrix Differentiation: Techniques and Key Results," *IEEE Transactions on Signal Processing*, vol. 55, no. 6, pp. 2740–2746, June 2007.