ONLINE ANGLE OF ARRIVAL ESTIMATION IN THE PRESENCE OF MUTUAL COUPLING

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ABSTRACT

A novel algorithm for estimating the Angles of Arrival (AoA) of multiple sources in the presence of mutual coupling is derived. We first formulate an "Equality Constrained Quadratic Optimisation" problem, then derive a suitable MUSIC-like algorithm to solve the aforementioned problem, and thus obtain good estimates of the AoA parameters. Identifiability conditions of the proposed algorithm are also derived. Finally, simulation results demonstrate the Root-Mean-Square Error (RMSE) performance of the algorithm as a function of Signal-to-Noise Ratio (SNR) and number of snapshots, with comparison to an existing method.

1. INTRODUCTION

The estimation of the angles of arrival, or AoAs, of multiple sources is a well known problem in the context of array signal processing. In fact, this problem emanates in many engineering applications such as navigation, tracking of objects, radar, sonar, and wireless communications [1]. Furthermore, numerous high-resolution algorithms were implemented to solve this issue, such as: MUSIC [2], ESPRIT [3], and so forth.

Mutual coupling between antennas is a popular problem in array signal processing. This phenomenon arises when antenna arrays are close to each other [4], and thus the current developed in an antenna element depends on its own excitation and on the contributions from adjacent antennas. As a consequence, an ideal model is no longer valid, and therefore the performance of the above algorithms deteriorate significantly.

Methods that aim on solving the mutual coupling problem are called calibration methods. In fact, there are two types of calibration methods: Offline and Online. In an offline calibration approach, one estimates the mutual coupling parameters in an offline stage independently from the (or knowing at least some) AoAs, such as the technique in [5]. In contrast, online calibration consists of jointly estimating the coupling and AoA parameters. In this paper, we focus on the latter. As a matter of fact, we aim at estimating the AoAs of multiple sources in the presence of mutual coupling. As will be clearer throughout the paper, the optimisation problem maximises the MUSIC cost function with respect to the coupling parameters, then the solution obtained is plugged back in the MUSIC cost function. In other words, the coupling parameters are treated as nuissance parameters.

In the literature, several techniques deal with the online calibration problem, such as those found in [6–8, 10] and references within. In [6], the algorithm is iterative and the sources are assumed to be totally uncorrelated. In [7, 8], the array elements are assumed to be partly calibrated. In other words, one has access to a few coupling parameters. The algorithm in [9] is based on fourth-order cumulants (FOC) to estimate the AoAs in the presence of mutual coupling. We find that the algorithm in [10] is of close nature to the one derived in this paper as it requires a 1D search to estimate the AoAs in the presence of mutual coupling. The difference, however, lies in the algorithm itself. In specific, we propose to the solve the problem in hand using *equality constrained quadratic optimisation*. We observe that we could resolve more sources than the method proposed in [10]. Furthermore, simulations show that our algorithm exhibits a lower Root-Mean-Square Error (RMSE) than that in [10].

Notations: Upper-case and lower-case boldface letters denote matrices and vectors, respectively. $(.)^T$ and $(.)^{\mathrm{H}}$ represent the transpose and the transpose-conjugate operators. The matrix $\mathbf{I}_{\mathrm{N}}$ is the identity matrix of dimensions $N \times N$. The operator $\mathrm{E}\{\mathbf{X}\}$ returns the expectation of a random matrix $\mathbf{X}$. For any matrix $\mathbf{B}$, the k^{th} column of $\mathbf{B}$ is expressed as follows $\mathbf{B}(:, k)$. The symbol ■ indicates the end of a proof.

2. SYSTEM MODEL

Assume a planar arbitrary array of N antennas. Furthermore, consider $q < N$ narrowband sources attacking the array from different angles, i.e. $\mathbf{\Theta} = [\theta_1 \dots \theta_q]$. Collecting L time snapshots and following [11], we can write

$$\mathbf{X} = \tilde{\mathbf{A}}\mathbf{S} + \mathbf{N} \tag{1}$$

where $\mathbf{X} \in \mathbb{C}^{\mathrm{N \times L}}$ is the data matrix with l^{th} time snapshot, $\mathbf{x}(t_l)$, stacked in the l^{th} column of $\mathbf{X}$. The matrix $\mathbf{S} \in \mathbb{C}^{\mathrm{q \times L}}$ is the source matrix. The steering matrix, or signal manifold, $\tilde{\mathbf{A}} \in \mathbb{C}^{\mathrm{N \times q}}$ is composed of q steering vectors, i.e. $\tilde{\mathbf{A}} = [\tilde{\mathbf{a}}(\theta_1) \dots \tilde{\mathbf{a}}(\theta_q)]$. Each vector $\tilde{\mathbf{a}}(\theta_i)$ is the response of the array to a source impinging the array from direction θ_i. The matrix $\mathbf{N} \in \mathbb{C}^{\mathrm{N \times L}}$ is background noise.

In this paper, we restrict ourselves with Uniform Linear Arrays (ULAs). Furthermore, it is well known that the response of a ULA in the absence of mutual coupling is given as

$$\mathbf{a}(\theta) = [1, z_\theta, \dots z_\theta^{N-1}]^{\mathrm{T}} \tag{2}$$

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where $z_\theta = e^{-j2\pi\frac{d}{\lambda}\sin(\theta)}$, d is the inter-element spacing and λ is the signal's wavelength. Following [10], the response $\tilde{\mathbf{a}}(\theta)$ could be modelled as

$$\tilde{\mathbf{a}}(\theta) = \mathcal{T}_p\mathbf{a}(\theta) \tag{3}$$

where $\mathcal{T}_p \in \mathbb{C}^{\mathrm{N}\times\mathrm{N}}$ is a banded symmetric Toeplitz matrix defined as follows

$$\mathcal{T}_p = \begin{bmatrix} 1 & t_1 & t_2 & \cdots & t_{p-1} & 0 & \cdots & 0 \\ t_1 & 1 & t_1 & \cdots & t_{p-2} & t_{p-1} & \cdots & 0 \\ \vdots & & & \ddots & \ddots & & & \vdots \\ 0 & \cdots & t_{p-1} & t_{p-2} & \cdots & t_1 & 1 & t_1 \\ 0 & \cdots & 0 & t_{p-1} & \cdots & t_2 & t_1 & 1 \end{bmatrix} \tag{4}$$

Note that the matrix $\mathcal{T}_p$ is independent from the AoAs. The model in equations (3) and (4) suggest that antennas that are placed at least p inter-element spacings apart do not interfere, i.e. $t_i = 0$ for all $i \geq p$. This is due to the fact that the mutual coupling is inversely proportional to the distance between antennas.

Before we move on, we shall assume the following:

- **A1**: $\tilde{\mathbf{A}}$ is full column rank.
- **A2**: The noise $\mathbf{n}(t_l)$ is modelled as a white circular complex Gaussian process of zero mean and covariance $\sigma^2\mathbf{I}_{\mathrm{N}}$ and independent from the source signals.
- **A3**: The number of source signals q is known.
- **A4**: The source signals are allowed to be partially correlated, but not coherent.

Assumptions **A1** and **A2** are usually satisfied in practice. As for assumption **A3**, we admit that the number of sources q is known a priori. The problem of estimating the number of sources is, in fact, a *detection* problem in signal processing. Techniques for estimating q are found in [12, 13].
Finally, the source covariance matrix is given as follows:

$$\mathbf{R_{ss}} = \mathrm{E}\{\mathbf{s}(\mathrm{t})\mathbf{s}^{\mathrm{H}}(\mathrm{t})\} \tag{5}$$

It was assumed in [6] that $\mathbf{R_{ss}}$ is diagonal, i.e. all sources are uncorrelated. However, we allow $\mathbf{R_{ss}}$ to be non-diagonal, but full rank (sources are not fully correlated).

We are now ready to address our online calibration problem: *"Given $\mathbf{X}$, q, and p, estimate the angles of arrival $\boldsymbol{\Theta}$ of the incoming signals in the presence of mutual coupling $\mathcal{T}_p$."*

3. AN ONLINE CALIBRATION ALGORITHM

This section makes use of the MUSIC algorithm in order to estimate the angles of arrivals $\boldsymbol{\Theta}$ in the presence of mutual coupling. We start by exploiting the structure of the received signal covariance matrix. Under assumption **A2**, we have

$$\mathbf{R_{xx}} = \mathrm{E}\{\mathbf{x}(\mathrm{t})\mathbf{x}^{\mathrm{H}}(\mathrm{t})\} = \tilde{\mathbf{A}}\mathbf{R_{ss}}\tilde{\mathbf{A}}^{\mathrm{H}} + \sigma^2\mathbf{I}_{\mathrm{N}} \tag{6}$$

Let $\lambda_1 > \lambda_2 > \ldots > \lambda_{\mathrm{N}}$ denote the eigenvalues of $\mathbf{R_{xx}}$. Furthermore, let $\mathbf{u_1}, \mathbf{u_2} \ldots \mathbf{u}_{\mathrm{N}}$ be their corresponding eigenvectors.

It is well known that under assumptions **A1** and **A4**, the following holds:

$$\tilde{\mathbf{a}}^{\mathrm{H}}(\theta_i)\mathbf{U_n}\mathbf{U_n^{\mathrm{H}}}\tilde{\mathbf{a}}(\theta_i) = 0, \qquad \text{for all } i = 1\ldots q. \tag{7}$$

where $\mathbf{U_n} = [\mathbf{u_{q+1}} \ldots \mathbf{u}_{\mathrm{N}}]$ is referred to as the *noise subspace*.

However, in practice, one has access to the sample covariance matrix, i.e.

$$\hat{\mathbf{R}}_{\mathbf{xx}} = \frac{1}{L}\mathbf{X}\mathbf{X}^{\mathrm{H}} \tag{8}$$

Consequently, let $\hat{\lambda}_1 > \hat{\lambda}_2 > \ldots > \hat{\lambda}_{\mathrm{N}}$ and $\hat{\mathbf{u}}_{\mathbf{1}}, \hat{\mathbf{u}}_{\mathbf{2}} \ldots \hat{\mathbf{u}}_{\mathrm{N}}$ denote the sample eigenvalues and eigenvectors of $\hat{\mathbf{R}}_{\mathbf{xx}}$. Finally, the MUSIC algorithm estimates $\boldsymbol{\Theta}$ as follows

$$\{\hat{\theta}_i\}_{i=1}^q = \arg\max_\theta \frac{1}{\tilde{\mathbf{a}}^{\mathrm{H}}(\theta)\hat{\mathbf{U}}_{\mathbf{n}}\hat{\mathbf{U}}_{\mathbf{n}}^{\mathrm{H}}\tilde{\mathbf{a}}(\theta)} \tag{9}$$

However, applying MUSIC (equation (9)) directly to the problem in hand is not possible because the functional form of $\tilde{\mathbf{a}}(\theta)$ is unknown.

Now, in order to proceed, we find the following theorem useful:

Theorem 1: *Let $\mathcal{T}_p \in \mathbb{C}^{N\times N}$ be a symmetric banded Toeplitz matrix defined as in* (4)*, and let $\mathbf{b} \in \mathbb{C}^{N\times 1}$. Therefore, for any $1 \leq p \leq N$, we can say:*

$$\mathcal{T}_p\mathbf{b} = \mathbf{B}\mathbf{t} \tag{10}$$

where

$$\mathbf{t} = [1, t_1 \ \ldots \ t_{p-1}]^{\mathrm{T}} \in \mathbb{C}^{p\times 1} \tag{11}$$

and $\mathbf{B} \in \mathbb{C}^{N\times p}$ is defined as

$$\mathbf{B}(:,k) = \begin{cases} \mathbf{b} & \text{if } k = 1 \\ (\mathbf{P_{k-1}} + \mathbf{P_{k-1}^{\mathrm{T}}})\mathbf{b} & \text{if } k > 1 \end{cases} \tag{12}$$

with $\mathbf{P_k} \in \mathbb{C}^{N\times N}$ being a shift matrix with all ones on its k^{th} sub-diagonal.

Proof: The matrix $\mathcal{T}_p$ could be re-written as

$$\mathcal{T}_p = \mathbf{I_N} + \sum_{i=1}^{p-1} t_i(\mathbf{P}_i + \mathbf{P}_i^{\mathrm{T}}) \tag{13}$$

Plugging the expression of $\mathcal{T}_p$ in (10), we have

$$\begin{aligned} \mathcal{T}_p\mathbf{b} &= \Big(\mathbf{I_N} + \sum_{i=1}^{p-1} t_i(\mathbf{P}_i + \mathbf{P}_i^{\mathrm{T}})\Big)\mathbf{b} \\ &= \Big[\mathbf{b}\Big|(\mathbf{P}_1 + \mathbf{P}_1^{\mathrm{T}})\mathbf{b}\Big|\ldots\Big|(\mathbf{P}_{p-1} + \mathbf{P}_{p-1}^{\mathrm{T}})\mathbf{b}\Big]\mathbf{t} \\ &= \mathbf{B}\mathbf{t} \qquad \blacksquare \end{aligned} \tag{14}$$

Note that **Theorem 1** is the same as Lemma 3 in [14] but written in a more compact way. Now, using **Theorem 1**, we can say that $\tilde{\mathbf{a}}(\theta) = \mathcal{T}_p\mathbf{a}(\theta) = \mathbf{B}(\theta)\mathbf{t}$, where $\mathbf{B}(\theta)$ is defined as in (12). Now, the MUSIC function in (9) could be re-written as

$$\{\hat{\theta}_i\}_{i=1}^q = \arg\max_{\theta,\mathbf{t}} \frac{1}{\mathbf{t}^{\mathrm{H}}\mathbf{B}^{\mathrm{H}}(\theta)\hat{\mathbf{U}}_{\mathbf{n}}\hat{\mathbf{U}}_{\mathbf{n}}^{\mathrm{H}}\mathbf{B}(\theta)\mathbf{t}} \tag{15}$$

where the maximisation problem is also done over the vector $\mathbf{t}$. Now, let $\mathbf{K}(\theta) \triangleq \mathbf{B}^{\mathrm{H}}(\theta)\hat{\mathbf{U}}_{\mathbf{n}}\hat{\mathbf{U}}_{\mathbf{n}}^{\mathrm{H}}\mathbf{B}(\theta)$, we propose to solve the following *equality constrained quadratic problem* for a given θ:

$$\begin{aligned} &\min_{\mathbf{t}} \mathbf{t}^{\mathrm{H}}\mathbf{K}(\theta)\mathbf{t} \\ &\text{subject to} \quad \mathbf{e_1^{\mathrm{H}}}\mathbf{t} = 1 \end{aligned} \tag{16}$$

where $\mathbf{e_1}$ is the 1^{st} column of $\mathbf{I}_{\mathrm{p}}$. The Lagrangian function corresponding to the optimisation problem in (16) is the following:

$$\mathcal{L}(\mathbf{t},\alpha) = \mathbf{t}^{\mathrm{H}}\mathbf{K}(\theta)\mathbf{t} - \alpha(\mathbf{e_1^{\mathrm{H}}}\mathbf{t} - 1) \tag{17}$$

Setting the derivative of $\mathcal{L}(\mathbf{t},\boldsymbol{\alpha})$ with respect to $\mathbf{t}$ to 0, we get

$$\frac{\partial}{\partial\mathbf{t}}\mathcal{L}(\mathbf{t},\alpha) = 2\mathbf{K}(\theta)\mathbf{t} - \alpha\mathbf{e_1} = 0 \tag{18}$$

Equation (18) gives the optimal coupling parameters, $\mathbf{t}^*$, for a given θ, in terms of the optimal Lagrangian multiplier α^* as

$$\mathbf{t}^* = \frac{\alpha^*}{2}\mathbf{K}(\theta)^{-1}\mathbf{e_1} \tag{19}$$

Now plugging the expression of $\mathbf{t}^*$ in the constraint of the optimisation problem in (16) yields the optimal value of α^*

$$\alpha^* = \frac{2}{\mathbf{e_1^{\mathrm{H}}}\mathbf{K}(\theta)^{-1}\mathbf{e_1}} \tag{20}$$

Therefore, $\mathbf{t}^*$ is now given as

$$\mathbf{t}^* = \frac{\mathbf{K}(\theta)^{-1}\mathbf{e_1}}{\mathbf{e_1^{\mathrm{H}}}\mathbf{K}(\theta)^{-1}\mathbf{e_1}} \tag{21}$$

Finally, plugging the expression of $\mathbf{t}^*$ in the MUSIC cost function in (15), we get

$$\{\hat{\theta}_i\}_{i=1}^{q} = \arg\max_{\theta} \mathbf{e_1^{\mathrm{H}}}\mathbf{K}(\theta)^{-1}\mathbf{e_1} \tag{22}$$

To prove the existance and uniqueness of $\mathbf{t}^*$, we need the following Lemma:

Lemma [15]: *Consider the "Equality Constrained Quadratic Optimisation" problem given in equation* (16). *Equations* (18) *and the constraint in equation* (16) *together are written in matrix form as:*

$$\underbrace{\begin{bmatrix} 2\mathbf{K}(\theta) & -\mathbf{e_1} \\ \mathbf{e_1}^H & 0 \end{bmatrix}}_{\triangleq \mathcal{M}} \begin{bmatrix} \mathbf{t} \\ \alpha \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} \tag{23}$$

The coefficient matrix $\mathcal{M}$ is referred to as the KKT matrix [15].
Let $[\mathbf{t}^*, \alpha^*]^T$ denote a solution of (23).
The following holds:

- *The KKT matrix $\mathcal{M}$ is nonsingular, and therefore invertible.*
- *The solution $[\mathbf{t}^*, \alpha^*]^T$ is the unique global solution of the equality constrained quadratic problem in equation* (16).

if and only if:

- *Assumption 1: The matrix $\mathbf{e_1}^H$ has linearly independent rows.*
- *Assumption 2: The matrix $\mathbf{K}(\theta)$ is positive definite in the null space of $\mathbf{e_1^H}$, i.e. $\mathbf{z}^H\mathbf{K}(\theta)\mathbf{z} > 0$ for all $\mathbf{z} \neq 0$ satisfying $\mathbf{e_1^H}\mathbf{z} = 0$.*

Using the above **Lemma**, we have the following Theorem:

Theorem 2: *The solution $[\mathbf{t}^*, \alpha^*]^T$ is the unique global solution if and only if $q + p < N + 1$ and $p \leq \frac{N}{2}$.*

Proof: We shall seek the conditions under which assumptions 1 and 2 of the above **Lemma** hold true. Clearly, *Assumption 1* is satisfied for any p. As for *Assumption 2*, let $\mathbf{z} \in \mathbb{C}^{\mathrm{p}\times 1}$ be a vector such that $\mathbf{e_1^H}\mathbf{z} = \mathbf{0}$, then:

$$\mathbf{z} \in \mathrm{span}\{\mathbf{e_2}, \ldots, \mathbf{e_p}\} = \mathcal{N}(\mathbf{E}) \tag{24}$$

In other words, there exists $\beta_2 \ldots \beta_p \in \mathbb{C}$ such that

$$\mathbf{z} = [0, \beta_2 \ldots \beta_p]^{\mathrm{T}} \tag{25}$$

Now, we seek a condition under which a vector $\mathbf{z} \in \mathcal{N}(\mathbf{e_1^H})$ satisfies $\mathbf{z}^{\mathrm{H}}\mathbf{K}(\theta)\mathbf{z} = 0$. Since $\mathbf{B}(\theta)$ is full column rank for any p satisfying $p \leq \frac{N}{2}$, then

$$\begin{aligned}\mathrm{rank}\Big(\mathbf{K}(\theta)\Big) = \mathrm{rank}\Big(\mathbf{B}^{\mathrm{H}}(\theta)\hat{\mathbf{U}}_{\mathbf{n}}\hat{\mathbf{U}}_{\mathbf{n}}^{\mathrm{H}}\mathbf{B}(\theta)\Big) &= \mathrm{rank}\Big(\hat{\mathbf{U}}_{\mathbf{n}}\hat{\mathbf{U}}_{\mathbf{n}}^{\mathrm{H}}\Big) \\ &= N - q\end{aligned} \tag{26}$$

Therefore, $\mathbf{K}(\theta)$ admits $N-q$ linearly independent columns. Recall that the number of possibly non-zero elements of $\mathbf{z}$ is $p-1$. This immediately implies that there exists a vector $\mathbf{z}$ such that $\mathbf{z}^{\mathrm{H}}\mathbf{K}(\theta)\mathbf{z} = 0$ if and only if

$$p - 1 \geq N - q \tag{27}$$

Finally, for every $\mathbf{z} \in \mathcal{N}(\mathbf{E})$ such that $\mathbf{z}^{\mathrm{H}}\mathbf{K}(\theta)\mathbf{z} \neq 0$ is satisfied if and only if $p + q < N + 1$ and $p \leq \frac{N}{2}$. And the proof is done. ■

Note that when $p = 1$ (absence of mutual coupling), we get the traditional identifiability, i.e. $q < N$.

4. DISCUSSION

Theorem 2 provides a sufficient and necessary condition for the existance and uniqueness of the coupling parameters $\mathbf{t}^*$ using the proposed algorithm, i.e. $p + q < N + 1$ and $p \leq \frac{N}{2}$. However, the identifiability condition in [10] is the following: $2p + q \leq N + 1$. One could, thus, easily verify that the proposed algorithm could resolve more sources.

We would strongly like to note that we have not addressed the coupling estimation part as the optimisation was first done over $\mathbf{t}$, then the solution of $\mathbf{t}$ (i.e. $\mathbf{t}^*$) was substituted back in the MUSIC cost function. In other words, the vector $\mathbf{t}^*$ was treated as a nuisance parameter. The problem of estimating the coupling parameters $\mathbf{t}$ is beyond the scope of this paper. Once again, our aim is estimating the AoAs of multiple sources in the presence of mutual coupling.

5. SIMULATION RESULTS

In this section, we present our simulation results and compare with the method presented by Liao *et Al.* [10]. In the first experiment, consider a ULA array that is composed of $N = 7$ antennas spaced at $\frac{\lambda}{2}$. Furthermore, assume two sources impinging the array at $\theta_1 = 10^\circ$ and $\theta_2 = 30^\circ$. As for the mutual coupling, we fix $p = 3$, with $t_1 = -0.95 - 1.29j$ and $t_2 = -0.05 + 0.25j$. The SNR is set to 9 dB and the number of snapshots $L = 100$. Figure 1 depicts the spectrum of our method versus Liao's method for this situation. The vertical dashed lines correspond to the true AoAs. We can clearly see that our method peaks at the true AoAs, whereas Liao's method is biased away from the true values.

In the second experiment (i.e. Figure 2), we fix $N = 10$ antennas, $q = 3$ sources arriving at $\theta_1 = 10^\circ$, $\theta_2 = 20^\circ$, and $\theta_3 = 30^\circ$. The number of coupling parameters is $p = 3$. The number of snapshots $L = 100$. The number of Monte-Carlo trials is $M = 500$. In addition, at each trial, the coupling parameters are chosen randomly to assess generality of our RMSE curves. We notice that our proposed method exhibits an improvement of around 1.5°, in average, in terms of RMSE when 5 dB $<$ SNR $<$ 20 dB. Interestingly, when SNR $>$ 22 dB, our method coincides with MUSIC ("coupling-free" MUSIC, that is) and the RMSE is 0, whereas Liao's method still shows some error of around 0.75° RMSE.

In the third experiment (i.e. Figure 3) ,we plot RMSE vs. number of snapshots (L) at fixed SNR. The parameters q, $\mathbf{\Theta}$, N, M, and p are the same as those in the 2^{nd} experiment. The SNR is set to 30 dB. Again, we observe that our proposed method performs better than Liao's. When the number of snapshots exceeds 20, our method shows zero RMSE and coincides with "coupling-free" MUSIC. However, Liao's method shows error even when the number of snapshots reach 100.

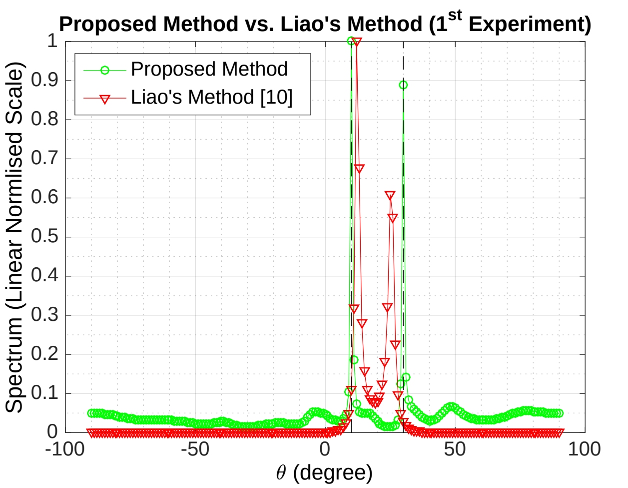

Fig. 1: Comparison of Spectra of different methods ($N = 7, p = 3, q = 2, L = 100$). Vertical dashed lines correspond to the true AoAs.

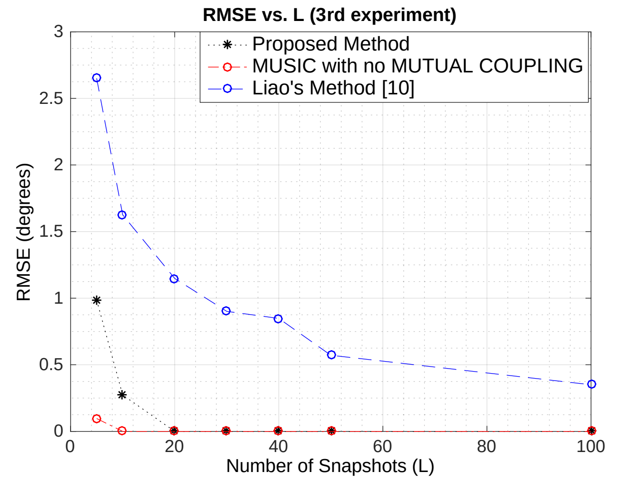

Fig. 3: RMSE on a linear scale vs. Number of Snapshots of experiment 3.

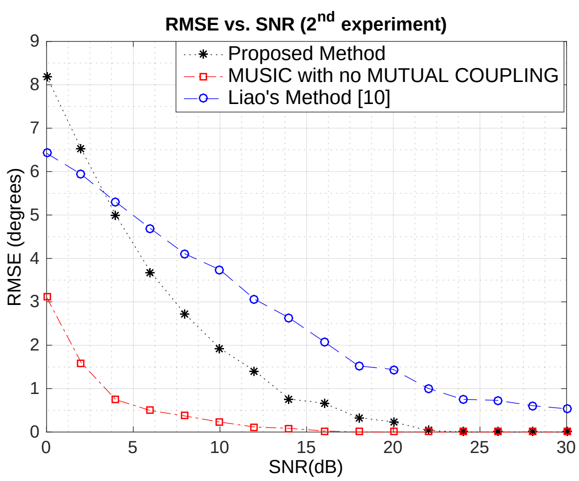

Fig. 2: RMSE on a linear scale vs. SNR of experiment 2.

6. CONCLUSION

We have presented a novel online mutual coupling algorithm that could estimate the Angles of Arrival of multiple sources in the presence of mutual coupling. Simulation results demonstrate the potential of the proposed algorithm, especially when compared to the one in [10]. In particular, our algorithm resolves more sources and exhibits a lower RMSE than that of [10].

7. REFERENCES

[1] Tuncer, E. and B. Friedlander, "Classical and Modern Direction of Arrival Estimation," *Elsevier*, Burlington, MA, 2009.

[2] R. O. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Transactions on Antennas and Propagation*, vol. 34, no. 3, pp. 276-280, Mar. 1986.

[3] R. Roy and T. Kailath, "ESPRIT-Estimation of signal parameters via rotational invariance techniques," *IEEE Transactions on Acoustics, Speech, Signal Processing*, vol.37, no. 7, pp. 984-995, July 1989.

[4] Stutzman, W.L. Thiele, "G.A. Antenna Theory and Design," 2nd Edition. New York: *Wiley*, 1998, p. 124-125.

[5] A. Leshem and M. Wax, "Array Calibration in the Presence of Multipath," *IEEE Transactions on Signal Processing*, 48(1):53-59, Jan 2000.

[6] F. Sellone and A. Serra, "A novel online mutual coupling compensation algorithm for uniform and linear arrays," *IEEE Transactions on Signal Processing*, 55, 2, 560-573, 2007.

[7] M. Pesavento, A. B. Gershman, and K. M. Wong, "Direction finding in partly calibrated sensor arrays composed of multiple subarrays," *IEEE Transactions on Signal Processing*, vol. 50, no. 9, pp. 2103-2115, Sep. 2002.

[8] C. M. S. See and A. B. Gershman, "Direction-of-arrival estimation in partly calibrated subarray-based sensor arrays," *IEEE Transactions on Signal Processing*, vol. 52, no. 2, pp. 329-338, Feb. 2004.

[9] B. Liao and S.-C. Chan, "A cumulant-based method for direction finding in uniform linear array with mutual coupling," *IEEE Antennas and Wireless Propagation Letters*, vol. 13, pp. 1717-1720, 2014.

[10] B. Liao, Z.G. Zhang, S.C. Chan, "DOA Estimation and Tracking of ULAs with Mutual Coupling," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 48, 2012, 891-905.

[11] H. L. Van Trees, "Detection, Estimation, and Modulation Theory," Part IV, Optimum Array Processing. New York: *Wiley*, 2002.

[12] M. Wax and I. Ziskind, "Detection of the number of coherent signals by the MDL principle," *IEEE Trans. Acoustics, Speech, Signal Processing*, vol. 37, pp. 1190-1196, Aug. 1989.

[13] A. Bazzi, D. T.M. Slock, L. Meilhac, "Detection of the Number of Superimposed Signals using Modified MDL Criterion: A Random Matrix Approach," *International Conference on Acoustics, Speech, and Signal Processing*, March, 2016.

[14] B. Friedlander and A. J. Weiss, "Direction Finding in the Presence of Mutual Coupling," *IEEE Transactions on Antennas and Propagation*, Vol. 39, No. 3. March 1991.

[15] S. Boyd, L. Vandenberghe, "Convex Optimization," *Cambridge University Press*, 2009.