On Joint Angle and Delay Estimation in the Presence of Local Scattering

Ahmad Bazzi*†, Dirk T.M. Slock*, and Lisa Meilhac†
*EURECOM Mobile Communications Department, 450 route des Chappes, 06410 Biot Sophia Antipolis, France
Email: {bazzi,slock}@eurecom.fr
†CEVA-RivieraWaves, 400, avenue Roumanille Les Bureaux, Bt 6, 06410 Biot Sophia Antipolis, France
Email:{ahmad.bazzi, lisa.meilhac}@ceva-dsp.com

***Abstract*—We present a novel 2D-MUSIC algorithm that can jointly estimate the angles and times of arrival of multiple signals in the presence of local scattering. We focus on a scenario where the received signal is a sum of "clusters", and each cluster is composed of multi-incident rays that show similar angles and times of arrival. The setting is composed of a Single-Input-Multiple-Output (SIMO) link where the transmitted signal is an OFDM symbol. Simulation results show the potential of the proposed algorithm compared to the traditional 2D-MUSIC algorithm that doesn't take into account local scattering.**

I. Introduction

Recently, indoor localisation has attracted considerable interest from both research and industry communities. Moreover, indoor localisation could be divided into two broad categories: *Radio-based* [1]–[3] and *Fingerprinting* [4], [5]. In *Radio-based* localisation, several reference access points, or "anchors", are used to position a node by first estimating signal parameters, such as Angle-of-Arrival (AoA) or Time-of-Arrival (ToA) of the received signal, then performing triangulation or trilateration techniques to obtain an estimate of the node's location. When "anchors" or nodes cooperate together to provide location estimates, this is referred to as *Cooperative* localisation [6]. However, *Fingerprinting* techniques solely rely on offline training to form a database, which could be readily used online for positioning.

In this paper, we focus on *Radio-based* localisation, and in particular on Joint Angle and Delay Estimation, also known as JADE [7]. Many algorithms have been proposed to perform JADE, such as Maximum Likelihood (ML) estimators found in [8], [9], 2D-subspace techniques derived in [7], [10], 2D-Matrix Pencil algorithms that could jointly estimate ToAs and AoAs using a single snapshot, such as [11], [12]. The problem with subspace algorithms is that they couldn't estimate signal parameters (ToA/AoA) if the signals were coherent, which is the case of multipath propagation. A known remedy for this issue is spatial smoothing [13], or 2D-smoothing for the JADE problem [14].

The difficulty of *Radio-based* techniques arises due to the presence of modelling errors, which can massively deteriorate the performance of signal parameter estimation if these errors were not taken into account [15]. Examples of these errors are antenna calibration [16], [17] and timing synchronisation [18]. In this paper, we focus on an important aspect called local scattering.

In an indoor environment, local scattering is a result of diffuse reflection from rough surfaces. Therefore, the received signal could not be modelled as a sum of discrete paths, or incident "rays". Instead, the received signal is a sum of "clusters" [19]–[21], where each cluster is formed of incident rays that arrive with similar AoAs and ToAs. Hence, each cluster is characterised by a nominal AoA, a nominal ToA, an angular and a temporal spread.

Many work has been done on estimating the nominal AoA in the presence of local scattering, such as those in [22], [23]. In [24], a Capon-based algorithm is proposed to estimation the directions of arrival in the presence of local scattering. In [25], the authors derived AoA estimation algorithms based on covariance matching. In addition, a subspace based algorithm was derived in [26] to estimate the AoAs in the presence of scattering. However, we are interested in JADE; therefore, we extend the model proposed in [22] to the spatio-temporal case and, as a result, a novel 2D-MUSIC algorithm is derived herein to cope with the problem of local scattering in order to jointly estimate the angles and times of arrival.

The rest of this paper is organised as follows: Section II presents the system model, assumptions, and problem statement. A MUSIC-Type algorithm to cope with the local scattering problem is proposed in Section III with its identifiability conditions given is Section IV. Section V demonstrates the potential of our proposed algorithm compared with the conventional MUSIC algorithm that doesn't take local scattering into account. We conclude the paper in Section VI.

Notations: Upper-case and lower-case boldface letters denote matrices and vectors, respectively. $(.)^T$ and $(.)^H$ represent the transpose and the transpose-conjugate operators. $E\{.\}$ is the statistical expectation. $\otimes$ and $\odot$ represent the *Kronecker* and pointwise *Hadamard* products, respectively. For any $N \times M$ matrix $\mathbf{X}$, $vec(\mathbf{X})$ is the vector operator which returns an $NM \times 1$ vector by stacking the columns of $\mathbf{X}$, starting from the first to the last column, and $\mathbf{X}^{\langle i,j \rangle}$ is the $(i,j)^{th}$ entry of $\mathbf{X}$. The vector $\mathbf{e}_1$ is a vector of all-zeros except the first entry set to 1.

II. System Model

A. Analytic Formulation

Consider an OFDM symbol $s(t)$ composed of M subcarriers and centered at a carrier frequency f_c, impinging an array of N antennas via q multipath components. We model each multipath component as a cluster, or "beam", of incident rays. Therefore, the i^{th} component contains K_i rays, where the k^{th} ray arrives at angle $\theta_i + \tilde{\theta}_{ik}$ and delay $\tau_i + \tilde{\tau}_{ik}$. Note that $\tilde{\theta}_{ik}$ and $\tilde{\tau}_{ik}$ are negligible compared to θ_i and τ_i, respectively. In other words, $\tilde{\theta}_{ik}$ and $\tilde{\tau}_{ik}$ take values in small intervals, say $\triangle\theta_i$ and $\triangle\tau_i$, respectively. Therefore, one could model the i^{th} "beam" as a superposition of K_i rays arriving at AoAs in the interval $[\theta_i \pm \frac{\triangle\theta_i}{2}]$ and ToAs in the interval $[\tau_i \pm \frac{\triangle\tau_i}{2}]$. The two quantities $\triangle\theta_i$ and $\triangle\tau_i$ are referred to as the "angular" and "temporal" spread, respectively. Also, note that θ_i and τ_i represent the nominal AoA and ToA of the i^{th} cluster. In baseband, we could write the l^{th} received OFDM symbol at the n^{th} antenna as:

$$r_n^{(l)}(t) = \sum_{i=1}^{q}\sum_{k=1}^{K_i}\alpha_{ik}\gamma_i^{(l)}a_n(\theta_i+\tilde{\theta}_{ik})s(t-\tau_i-\tilde{\tau}_{ik})+n_n^{(l)}(t) \quad (1)$$

where

$$s(t) = \begin{cases}\sum_{m=0}^{M-1} b_m e^{j2\pi m\triangle_f t} & \text{if } t\in[0,T]\\ 0 & \text{elsewhere}\end{cases} \quad (2)$$

where $T=\frac{1}{\triangle_f}$ is the OFDM symbol duration, $\triangle_f$ is the subcarrier spacing, b_m is the modulated symbol onto the m^{th} subcarrier, $a_n(\theta)$ is the n^{th} antenna response to an incoming signal at angle θ. The form of $a_n(\theta)$ depends on the array geometry. $\gamma_i^{(l)}$ is the complex coefficient of the i^{th} cluster and α_{ik} is the complex gain of the k^{th} ray in the i^{th} cluster. The term $n_n^{(l)}(t)$ is background noise. Plugging (2) in (1) and sampling $r_n^{(l)}(t)$ at regular intervals of $k \triangleq k\frac{T}{M}$, then applying an M-point DFT, we observe that the data at the m^{th} subcarrier and n^{th} antenna is given by

$$R_{n,m}^{(l)} = b_m\sum_{i=1}^{q}\sum_{k=1}^{K_i}\alpha_{ik}\gamma_i^{(l)}a_n(\theta_i+\tilde{\theta}_{ik})c_m(\tau_i+\tilde{\tau}_{ik})+N_{n,m}^{(l)} \quad (3)$$

where $c_m(\tau)=e^{-j2\pi m\triangle_f\tau}$.
Now, since $\tilde{\theta}_{ik}\ll\theta_i$, we expand the term $a_n(\theta_i+\tilde{\theta}_{ik})$ using *Taylor* series in the neighborhood of θ_i, viz.

$$a_n(\theta_i+\tilde{\theta}_{ik})\simeq a_n(\theta_i)+\tilde{\theta}_{ik}\frac{\partial a_n(\theta_i)}{\partial\theta_i}+\mathcal{O}(\tilde{\theta}_{ik}^2) \quad (4)$$

Similarly, we can approximate the term $c_m(\tau_i+\tilde{\tau}_{ik})$ in the neighborhood of τ_i, since $\tilde{\tau}_{ik}\ll\tau_i$ i.e.

$$c_m(\tau_i+\tilde{\tau}_{ik})\simeq c_m(\tau_i)+\tilde{\tau}_{ik}\frac{\partial c_m(\tau_i)}{\partial\tau_i}+\mathcal{O}(\tilde{\tau}_{ik}^2) \quad (5)$$

Compensating the term b_m in $R_{n,m}^{(l)}$ (equation (3)) by multiplying with $\frac{b_m^*}{|b_m|^2}$, we re-write equation (3) in a compact matrix form taking into account the approximations in equations (4) and (5)

$$\mathbf{x}(l)\simeq\mathbf{H}\boldsymbol{\gamma}(l)+\mathbf{n}(l),\quad l=1\dots L \quad (6)$$

where $\mathbf{x}(l)$ and $\mathbf{n}(l)$ are $MN\times 1$ vectors

$$\mathbf{x}(l)=vec\{\mathbf{R}\},\qquad \mathbf{R}^{\langle m,n\rangle}=R_{n,m}^{(l)} \quad (7)$$

$$\mathbf{n}(l)=vec\{\mathbf{N}\},\qquad \mathbf{N}^{\langle m,n\rangle}=N_{n,m}^{(l)} \quad (8)$$

The $q\times 1$ vector $\boldsymbol{\gamma}(l)$ is composed of the multipath coefficients

$$\boldsymbol{\gamma}(l)=[\gamma_1^{(l)}\dots\gamma_q^{(l)}]^T \quad (9)$$

$\mathbf{H}$ is an $MN\times q$ matrix given as

$$\mathbf{H}=[\mathbf{h}(\theta_1,\tau_1)\dots\mathbf{h}(\theta_q,\tau_q)] \quad (10)$$

where

$$\begin{aligned}\mathbf{h}(\theta_i,\tau_i)=&\Big(\sum_{k=1}^{K_i}\alpha_{ik}\Big)\Big(\mathbf{a}(\theta_i)\otimes\mathbf{c}(\tau_i)\Big)\\&+\Big(\sum_{k=1}^{K_i}\alpha_{ik}\tilde{\theta}_{ik}\Big)\Big(\frac{\partial\mathbf{a}(\theta_i)}{\partial\theta_i}\otimes\mathbf{c}(\tau_i)\Big)\\&+\Big(\sum_{k=1}^{K_i}\alpha_{ik}\tilde{\tau}_{ik}\Big)\Big(\mathbf{a}(\theta_i)\otimes\frac{\partial\mathbf{c}(\tau_i)}{\partial\tau_i}\Big)\end{aligned} \quad (11)$$

and $\mathbf{a}(\theta)$ and $\mathbf{c}(\tau)$ are vectors of size $N\times 1$ and $M\times 1$, respectively. The n^{th} entry of $\mathbf{a}(\theta)$ is $a_n(\theta)$. Also, the m^{th} entry of $\mathbf{c}(\tau)$ is $c_m(\tau)$. We refer the reader to Appendix A for a general form of $\mathbf{a}(\theta)$ and $\frac{\partial\mathbf{a}(\theta)}{\partial\theta}$ and Appendix B for the form of $\mathbf{c}(\tau)$ and $\frac{\partial\mathbf{c}(\tau)}{\partial\tau}$.

B. Assumptions and Problem Statement

Throughout the paper, we assume the following:

- **A1**: $\mathbf{H}$ is full column rank.
- **A2**: The complex coefficients $\boldsymbol{\gamma}(l)$ are fixed within a snapshot, and are uncorrelated over OFDM symbols.
- **A3**: The number of clusters q is known. However, the number of rays per cluster (K_i) is unknown.
- **A4**: The vector $\mathbf{n}(l)$ is additive Gaussian noise of zero mean and variance $\sigma^2\mathbf{I}$, assumed to be white over space, frequencies, and symbols; we also assume that the noise is independent from the multipath coefficients.

Even though algorithms exist for estimating the number of sources, such as Minimum Description Length (MDL) [27], [28], Modified MDL (MMDL) [29], bootstrap [30], etc, these methods couldn't be directly used when the model includes local scattering. The reason behind this is that all these algorithms assume that the environment is modeled as a discrete number of rays. The problem of estimating the number of clusters q is beyond the scope of this paper.
Any further assumptions will be mentioned. Now, we address our problem:
"Given $\{\mathbf{x}(l)\}_{l=1}^L$ and q, estimate the nominal signal parameters $\{(\theta_i,\tau_i)\}_{i=1}^q$ in the presence of local scattering."

III. A MUSIC-Type Algorithm in the Presence of Local Scattering

We start by defining the covariance matrix of the received vector $\mathbf{x}(l)$, i.e.

$$\mathbf{R}_{xx}=E\{\mathbf{x}(l)\mathbf{x}^{\mathrm{H}}(l)\} \quad (12)$$

In practical scenarios, this matrix is computed using a sample average as follows

$$\hat{\mathbf{R}}_{xx}=\frac{1}{L}\sum_{l=1}^{L}\mathbf{x}(l)\mathbf{x}^{\mathrm{H}}(l) \quad (13)$$

Let $\hat{\lambda}_1>\hat{\lambda}_2>\dots>\hat{\lambda}_{\mathrm{MN}}$ and $\hat{\mathbf{u}}_1,\hat{\mathbf{u}}_2\dots\hat{\mathbf{u}}_{\mathrm{MN}}$ denote the eigenvalues and their corresponding eigenvectors of $\hat{\mathbf{R}}_{xx}$. Under assumptions **A1**, **A2** and **A4**, one could jointly estimate $\{(\theta_i,\tau_i)\}_{i=1}^q$ by evaluating the peaks of the following 2D-MUSIC cost function

$$(\hat{\theta}_i,\hat{\tau}_i)=\arg\max_{\theta,\tau}\frac{1}{\mathbf{h}^{\mathrm{H}}(\theta,\tau)\hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^{\mathrm{H}}\mathbf{h}(\theta,\tau)} \quad (14)$$

where $\hat{\mathbf{U}}_n=[\hat{\mathbf{u}}_{\mathrm{q+1}}\dots\hat{\mathbf{u}}_{\mathrm{MN}}]$ is called the "noise" subspace. The MUSIC cost function exploits the orthogonality between vectors in the signal and noise subspaces [31]. Using equation (11), we re-write $\mathbf{h}(\theta,\tau)$ in a more compact form as follows

$$\mathbf{h}(\theta_i,\tau_i)=\mathbf{B}(\theta_i,\tau_i)\mathbf{w}_i \quad (15)$$

where $\mathbf{B}(\theta_i,\tau_i)$ is $MN\times 3$ given as

$$\mathbf{B}(\theta_i,\tau_i)=[\mathbf{a}(\theta_i)\otimes\mathbf{c}(\tau_i)|\frac{\partial\mathbf{a}(\theta_i)}{\partial\theta_i}\otimes\mathbf{c}(\tau_i)|\mathbf{a}(\theta_i)\otimes\frac{\partial\mathbf{c}(\tau_i)}{\partial\tau_i}] \quad (16)$$

and $\mathbf{w}_i = [1, w_{i1}, w_{i2}]^{\mathrm{T}}$ where

$$w_{i1} = \frac{\sum_{k=1}^{K_i} \alpha_{ik}\tilde{\theta}_{ik}}{\sum_{k=1}^{K_i} \alpha_{ik}} \tag{17a}$$

and

$$w_{i2} = \frac{\sum_{k=1}^{K_i} \alpha_{ik}\tilde{\tau}_{ik}}{\sum_{k=1}^{K_i} \alpha_{ik}} \tag{17b}$$

Without loss of generality, we have factored out the term $\sum_{k=1}^{K_i} \alpha_{ik}$ from $\mathbf{h}(\theta, \tau)$. Note that if w_{i1} is zero, then there is no angular spread. Similarly, if w_{i2} is zero, then temporal spread is not present. Plugging equation (15) in (14), we have the following 2D-MUSIC cost function

$$(\hat{\theta}_i, \hat{\tau}_i) = \arg\max_{\theta,\tau,\mathbf{w}} \frac{1}{\mathbf{w}^{\mathrm{H}}\mathbf{B}^{\mathrm{H}}(\theta,\tau)\hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^{\mathrm{H}}\mathbf{B}(\theta,\tau)\mathbf{w}} \tag{18}$$

Now, for each pair (θ, τ), the cost function in (18) is maximised with respect to $\mathbf{w}$, i.e.

$$\begin{aligned} &\min_{\mathbf{w}} \mathbf{w}^{\mathrm{H}}\mathbf{B}^{\mathrm{H}}(\theta,\tau)\hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^{\mathrm{H}}\mathbf{B}(\theta,\tau)\mathbf{w} \\ &\text{subject to} \quad \mathbf{w}^{\mathrm{H}}\mathbf{e_1} = 1 \end{aligned} \tag{19}$$

The Lagrangian function corresponding to the problem stated in (19) is the following

$$\mathcal{L}(\mathbf{w}, \lambda) = \mathbf{w}^{\mathrm{H}}\mathbf{B}^{\mathrm{H}}(\theta,\tau)\hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^{\mathrm{H}}\mathbf{B}(\theta,\tau)\mathbf{w} - \lambda\big(\mathbf{w}^{\mathrm{H}}\mathbf{e_1} - 1\big) \tag{20}$$

Setting the derivative of $\mathcal{L}(\mathbf{w}, \lambda)$ with respect to $\mathbf{w}$ to 0, we get

$$\frac{\partial}{\partial \mathbf{w}}\mathcal{L}(\mathbf{w}, \lambda) = 2\mathbf{B}^{\mathrm{H}}(\theta,\tau)\hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^{\mathrm{H}}\mathbf{B}(\theta,\tau)\mathbf{w} - \lambda\mathbf{e_1} = 0 \tag{21}$$

Equation (21) yields in

$$\mathbf{w} = \frac{\lambda}{2}\Big(\mathbf{B}^{\mathrm{H}}(\theta,\tau)\hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^{\mathrm{H}}\mathbf{B}(\theta,\tau)\Big)^{-1}\mathbf{e_1} \tag{22}$$

Substituting (22) in the constraint of (19), we get

$$\lambda = \frac{2}{\mathbf{e_1}^{\mathrm{H}}\Big(\mathbf{B}^{\mathrm{H}}(\theta,\tau)\hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^{\mathrm{H}}\mathbf{B}(\theta,\tau)\Big)^{-1}\mathbf{e_1}} \tag{23}$$

Therefore

$$\mathbf{w} = \frac{\Big(\mathbf{B}^{\mathrm{H}}(\theta,\tau)\hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^{\mathrm{H}}\mathbf{B}(\theta,\tau)\Big)^{-1}\mathbf{e_1}}{\mathbf{e_1}^{\mathrm{H}}\Big(\mathbf{B}^{\mathrm{H}}(\theta,\tau)\hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^{\mathrm{H}}\mathbf{B}(\theta,\tau)\Big)^{-1}\mathbf{e_1}} \tag{24}$$

Plugging the expression of $\mathbf{w}$ in the cost function in (19), one has the following 2D-MUSIC function

$$(\hat{\theta}_i, \hat{\tau}_i) = \arg\max_{\theta,\tau} \mathbf{e_1}^{\mathrm{H}}\Big(\mathbf{B}^{\mathrm{H}}(\theta,\tau)\hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^{\mathrm{H}}\mathbf{B}(\theta,\tau)\Big)^{-1}\mathbf{e_1} \tag{25}$$

Therefore, one could search for the q highest peaks of the cost function given in equation (25). A summary of this algorithm is provided in **Table 1**.

IV. Identifiability conditions of the proposed MUSIC-Type Algorithm

At each point (θ_i, τ_j), a matrix inversion of $\mathbf{B}^{\mathrm{H}}(\theta_i,\tau_j)\hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^{\mathrm{H}}\mathbf{B}(\theta_i,\tau_j) \in \mathbb{C}^{3\times 3}$ is required. This matrix is invertible under two conditions:

1) The matrix $\mathbf{B}(\theta_i, \tau_j)$ is full column rank.
2) The rank of the projector matrix onto the "noise" subspace, i.e. $\hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^{\mathrm{H}}$, should be greater than 3.

The first condition occurs when none of the columns of $\mathbf{B}(\theta_i, \tau_j)$ are all zeros. The 1^{st} and 3^{rd} columns of $\mathbf{B}(\theta_i, \tau_j)$ are never all-zeros, but the 2^{nd} column might be all-zeros in some cases. For example, consider a Uniform Linear Array (ULA) where all $\bar{y}_1 = \ldots = \bar{y}_N = 0$ and $\bar{x}_k = (k-1)d$ where d is the inter-element spacing and the quantities $\bar{x}_i$ and $\bar{y}_i$ are defined in Appendix A. Then, the 2^{nd} column of $\mathbf{B}(\theta_i, \tau_j)$ is all-zeros when $\theta_i = \pm 90°$. So, in this particular case, the cost function in (25) should not be evaluated at $\theta_i = \pm 90°$.

As for the second condition, the rank of $\hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^{\mathrm{H}}$ could be easily verified to be $MN - q$. Therefore, one must have $MN \geq q + 3$.

Table 1: Summary of the proposed MUSIC-Type Algorithm

<u>**INITIALISATION**</u>:

Step 1. Given the data $\{\mathbf{x}(l)\}_{l=1}^{L}$, compute:

$$\hat{\mathbf{R}}_{xx} = \frac{1}{L}\sum_{l=1}^{L}\mathbf{x}(l)\mathbf{x}^{\mathrm{H}}(l)$$

Step 2. Perform an Eigenvalue Decomposition of $\hat{\mathbf{R}}_{xx}$ and extract the noise subspace as:

$$\hat{\mathbf{U}}_n = [\hat{\mathbf{u}}_{\text{q+1}} \ldots \hat{\mathbf{u}}_{\text{MN}}]$$

<u>**MAIN LOOP**</u>:

Step 3. On a 2D discretized grid, find the q highest peaks of the following cost function:

$$(\hat{\theta}_i, \hat{\tau}_i) = \arg\max_{\theta,\tau} \mathbf{e_1}^{\mathrm{H}}\Big(\mathbf{B}^{\mathrm{H}}(\theta,\tau)\hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^{\mathrm{H}}\mathbf{B}(\theta,\tau)\Big)^{-1}\mathbf{e_1}$$

V. Simulation Results

In both experiments, the array was linear and uniform with $N = 3$ antennas spaced half a wavelength apart. The transmitted signal was an OFDM symbol where we use only $M = 5$ subcarriers. The subcarrier spacing was chosen to be $\triangle_f = 3.125$ MHz. We would like to note here that in a real system, a Wi-Fi OFDM comprises of at least 64 subcarriers, but we have chosen to use only 5 out of these 64 subcarriers.

In the first experiment (Figures 1 and 2), the number of clusters is $q = 1$. The source impinges the antenna at $\theta_1 = 30°$ and $\tau_1 = 0$ nsec. The number of incident rays is $K_1 = 200$ with angular spread $\triangle\theta_1 = 5°$, whereas the temporal spread is $\triangle\tau_1 = 10$ nsec. Note that the $K_1 = 200$ rays are chosen randomly with AoAs $[\theta_1 \pm \frac{\triangle\theta_1}{2}]$ and ToAs $[\tau_1 \pm \frac{\triangle\tau_1}{2}]$. We observe a clear improvement when the local scattering is taken into account using our proposed method. In this case and when SNR exceeds 10 dB, the gain in Mean Squared Error (MSE) in angular domain was about 6 dB (see Figure 1) and almost 8 dB in the temporal domain (see Figure 2).

In the second experiment, i.e. Figures 3 and 4, the above parameters are the same except for the following: The number of arriving clusters

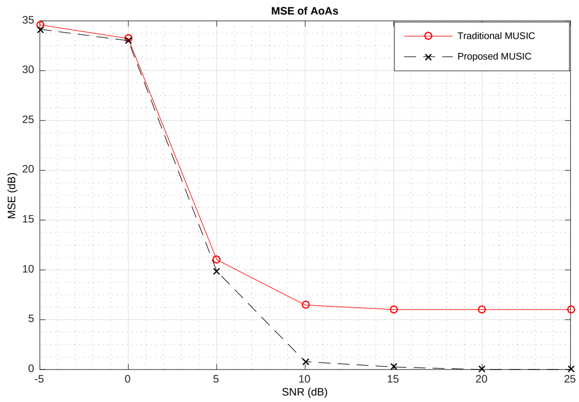

Fig. 1: MSE of the AoAs as a function of SNR of the Traditional MUSIC vs. the Proposed one. (1^{st} Experiment)

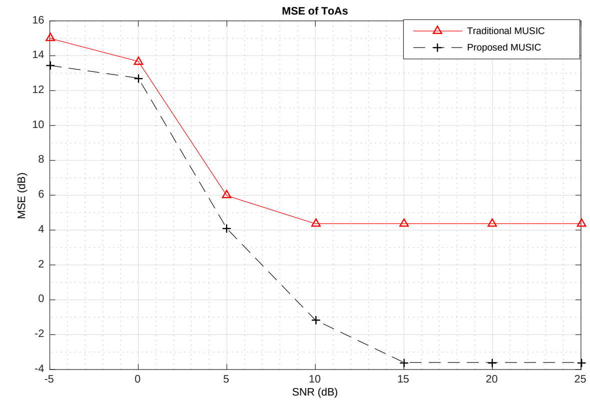

Fig. 2: MSE of the ToAs as a function of SNR of the Traditional MUSIC vs. the Proposed one. (1^{st} Experiment)

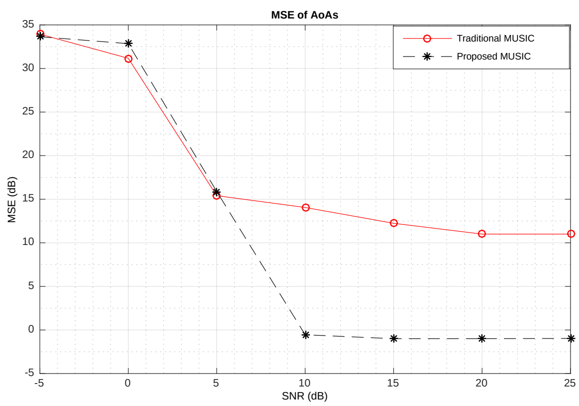

Fig. 3: MSE of the AoAs as a function of SNR of the Traditional MUSIC vs. the Proposed one. (2^{nd} Experiment)

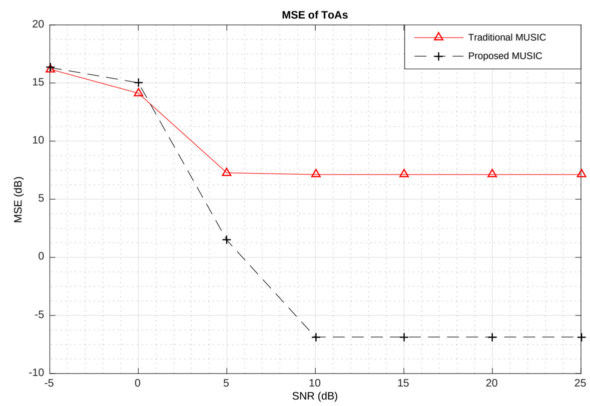

Fig. 4: MSE of the ToAs as a function of SNR of the Traditional MUSIC vs. the Proposed one. (2^{nd} Experiment)

is now $q = 2$. Their corresponding nominal AoAs and ToAs are fixed to $\theta_1 = 30^\circ$, $\tau_1 = 0$ nsec and $\theta_2 = 0^\circ$, $\tau_2 = 50$ nsec. The number of rays per cluster are set to $K_1 = 200$ rays and $K_2 = 300$ rays. As for the angular and temporal spread, we fix $\triangle\theta_1 = 5^\circ$ and $\triangle\tau_1 = 12$ nsec and $\triangle\theta_2 = 7^\circ$ and $\triangle\tau_1 = 15$ nsec. We observe a larger gain in MSE. By referring to Figure 3, one could observe an improvement of around 10 dB, when SNR $>$ 10dB. Similarly, Figure 4 shows a 13 dB gain in MSE.

VI. Conclusion

In this contribution, we have presented a spatio-temporal model that takes into account both angular and temporal spread of different "clusters". Furthermore, a 2D-MUSIC algorithm has been derived based on that model in order to jointly estimate the nominal angles and times of arrival of the different clusters. Simulation results show the potential of our proposed algorithm when compared to the traditional 2D-MUSIC when the scattering is not taken into account.

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Appendix A

For an arbitrary array of N antennas, the form of $\mathbf{a}(\theta)$ is given as:

$$\mathbf{a}(\theta) = \begin{bmatrix} e^{-j\frac{w_c}{c}(\bar{x}_1\sin(\theta)+\bar{y}_1\cos(\theta))} \\ \vdots \\ e^{-j\frac{w_c}{c}(\bar{x}_N\sin(\theta)+\bar{y}_N\cos(\theta))} \end{bmatrix} \quad (26)$$

where $(\bar{x}_i, \bar{y}_i)$ is the position of the i^{th} antenna. The term $w_c = 2\pi f_c$ is the angular frequency, and c is the speed of light in vacuum. The derivative of $\mathbf{a}(\theta)$ with respect to θ is

$$\frac{\partial \mathbf{a}(\theta)}{\partial \theta} = \begin{bmatrix} -j\frac{w_c}{c}(\bar{x}_1\cos(\theta) - \bar{y}_1\sin(\theta)) \\ \vdots \\ -j\frac{w_c}{c}(\bar{x}_N\cos(\theta) - \bar{y}_N\sin(\theta)) \end{bmatrix} \odot \mathbf{a}(\theta) \quad (27)$$

Appendix B

The vector $\mathbf{c}(\tau) \in \mathbb{C}^{M\times 1}$ is given as follows:

$$\mathbf{c}(\tau) = \begin{bmatrix} 1 \\ e^{-j2\pi\triangle_f\tau} \\ \vdots \\ e^{-j2\pi(M-1)\triangle_f\tau} \end{bmatrix} \quad (28)$$

The derivative of $\mathbf{c}(\tau)$ with respect to τ is

$$\frac{\partial \mathbf{c}(\tau)}{\partial \tau} = \begin{bmatrix} 0 \\ -j2\pi \triangle_f \\ \vdots \\ -j2\pi(M-1) \triangle_f \end{bmatrix} \odot \mathbf{c}(\tau) \qquad (29)$$

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