Transmission of Sporadic Analog Samples over Wireless Channels

Ayşe Ünsal, Raymond Knopp
Mobile Communications Department, Eurecom, Sophia Antipolis, France
{ayse.unsal,raymond.knopp}@eurecom.fr

***Abstract*—A low-latency, parameter modulation-estimation feedback protocol for wideband channels is introduced for both pure line-of-sight and more general fading channels with several degrees of freedom. One round of the protocol consists of a data phase and a control phase and uses non-coherent detection. Asymptotic optimality in energy efficiency of the protocol is analyzed and an upper bound on the distortion level is derived for two-rounds. The proposed scheme as well as known one-way schemes are compared with classical and very recent lower-bounds. Both the lower-bounds and performance evaluation of the feedback protocol are extended to a multi-channel fading model. The improvement of the feedback protocol over one-shot transmission is shown to be very significant on both line-of-sight and fading channels.** [1]

***Index Terms*—Joint source-channel coding, parameter modulation-estimation, non-coherent detection, distortion**

I. Introduction

In this work, we consider simple parameter modulation-estimation strategies applicable to future wireless sensor networks. As an example, the sensor could be sporadically sending samples of analog information (temperature, magnetic field, current, speed, etc.) to a collecting node. The sensors could be seen as analog-to-digital converters which are distributed in space and use a wireless medium to relay their samples to the network. Such traffic is very low-rate, practically zero-rate, since in the majority of cases the sampling rate is very low (a few samples per second) and the available system bandwidth is very large (tens to hundreds of megahertz). The communication link from the samplers to the network often requires low-latency. The latter could arise for two reasons, either reactivity of an actuating element in the network or to minimize energy consumption in the sensing node itself by using discontinuous transmission and reception. Here the latency of the transmission is directly related to the "on"-time of communication circuitry of the sensing node. This example captures the essense of some so-called *machine-type communications*, a term which refers to machines including sensors interconnected via cellular networks and exchanging information autonomously. It is widely believed that this sort of traffic will at least be shared with conventional voice and data communications on current and evolving cellular communication standards. Depending on the evolving usage scenarios, the amount of traffic produced by such low-rate devices could even vastly surpass that of conventional human communications. The purpose of this paper is to study modulation strategies for analog samples applicable to the wireless medium.

Imagine the simplest scenario of one sensor node tracking a slowly time-varying random sequence and sending its observations to a receiver over a wireless channel. The source is denoted by a random variable U of zero mean and variance $\sigma_u^2 = 1$, representing a single realization of the random sequence at a particular time t. The sensor should be seen as a tiny device with strict energy constraints. The communication channel between the sender and the receiver is an additive white Gaussian noise channel. An important question is how to efficiently encode the random variable U for transmission, and what performance can be achieved upon reconstruction as a function of the energy used to achieve this transmission.

For this scenario, the slowly time-varying characteristic of the source has two main impacts on the way the coding problem should be addressed: firstly, the time between two observations is long, and the sensor should not wait for a sequence of observations to encode it. Therefore, the sensor will encode only one observation before sending it through the channel. Secondly, for each source realization the channel can potentially be used over many signal dimensions, for instance by encoding over a wide-bandwidth in the frequency-domain. This corresponds to the case for sensors connected directly to fourth-generation cellular networks. Hence, we can reasonably assume that there is no constraint on the dimensionality of the channel codebook. The latter amounts to saying that very low-rate codes should be used.

[1]This paper was presented [in part] at EUSIPCO 2012, European Signal Processing Conference, August, 27-31, 2012, Bucharest, and SCC 2013, 9th International ITG Conference on Systems, Communications and Coding, January 21-24, 2013, Munich, Germany.

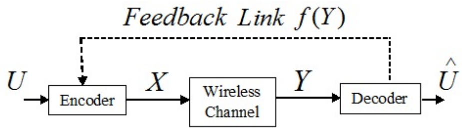

Fig. 1. Single-source Problem

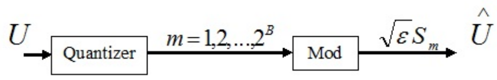

Fig. 2. Goblick's Digital Scheme

A. Addressed Problem and its Background

The single-source model as shown in Figure 1 is described as follows. An encoder maps one realization of the source U into $\mathbf{S}_m$ with a dimension of N where $m(U)$ represents the source message. We make use of causal feedback so that the encoder may also depend on past channel outputs. $\mathbf{S}_m$ is then sent across the channel corrupted by a white Gaussian noise sequence $\mathbf{Z}$, and is received as $\mathbf{Y}$. The receiver is a mapping function which tries to construct an estimate $\widehat{U}$ of U given $\mathbf{Y}$. The fidelity criterion that we wish to minimize is the MSE distortion defined as $\mathcal{D} \triangleq \mathbb{E}[(U - \widehat{U})^2]$, under the mean energy constraint $\mathbb{E}[||\mathbf{S}_m||^2] \leq \mathcal{E}$.

It is well-known that the linear encoder (i.e. $S_m = \sqrt{\mathcal{E}}U$) achieves the best performance under the mean energy constraint for the special case $N = 1$ [1]–[3] and normally-distributed U. In fact, a lower bound on the distortion over all possible encoders and decoders, both with and without a feedback link, is easily derived in [1] using classical information theory, and given by

$$\mathcal{D}_g \geq e^{-2\mathcal{E}/N_0} \tag{1}$$

where $N_0/2$ is the channel noise per dimension. Note that, the form of (1) is adapted to a discrete-time complex Gaussian channel with noise variance $N_0/2$ to make the comparisons easier with lower-bounds to be introduced in the upcoming section. Goblick's information theoretic bound given above was derived through defining the channel capacity and the rate-distortion function in terms of the channel SNR, more precisely the author obtained the minimum distortion in estimating the source message as a function of the channel SNR which leads to the output SNR in a continuous-time channel with limited bandwidth. At the end of the procedure the reconstruction error is composed solely by the quantization process applied to the source. The $e^{-\mathcal{E}/3N_0}$ behaviour for Goblick's digital scheme was described in [4], [5]. Figure 2 is a pictorial representation of Goblick's scheme where the B bit uniform quantization is followed by 2^B-ary orthogonal modulation to transmit the source U using the energy $\mathcal{E}$ which achieves the same performance with [6]. Several schemes can achieve $e^{-\mathcal{E}/3N_0}$ both with and without coherent detection and for both normally and uniformly distributed U. The importance of Goblick's work [1] comes from the method he chose. To the best of our knowledge the first digital scheme for unlimited bandwidth based on scalar quantization and orthogonal modulation was described by Goblick in [1], which is also the quantization/modulation strategy used in this paper, and the performance loss with respect to (1) was heuristically argued to be on the order of 6-9 dB. Unlike Goblick and the achievable scheme proposed in this paper, Wozencraft-Jacobs use [6, pg:623-624] pulse position modulation (PPM).

A comparison in [5] with best-known joint medium-resolution source-channel codes [7] for high channel to source bandwidth ratios shows that simple hybrid yet separated joint-source channel techniques can outperform non-linear mappings. Such optimization for a different power constraint can be found in the literature for example in [8] and [9], where the authors try to bound the optimal number of quantization bits that minimizes distortion.

For the case of a point-to-point channel without feedback, the most recent and significant studies are presented by Merhav in [10], [11] for AWGN channels and discrete-memoryless channels, respectively. In [10], [12] the best-known lower bound for the reconstruction fidelity without feedback, with coherent detection and unlimited channel bandwidth behaves as $e^{-\mathcal{E}/2N_0}$ for uniformly-distributed U where $\mathcal{E}$ is the energy used for transmission of U. In fact, the author achieves this lower-bound on the MSE through the threshold he defines on the maximum exponential rate of error probability decay in estimating $|U - \hat{U}|$ rather than concentrating on the MSE as the performance criterion itself. In order to prove this threshold on the error probability of $|U - \hat{U}|$, he adapts the well-known Ziv-Zakai bound [13] to the case with M hypotheses instead of 2 and the derivation proceeds as the Chazan-Zakai bound [14]. In a quite recent study [11], the author provides both upper and lower bounds for the best achievable exponential decay of $E|\hat{U} - U|^m, \quad m \geq 0$ in a discrete memoryless channel.

In feedback systems, for example cellular networks, we could clearly imagine the use of reliable feedback from the down-link, with vanishing probability of error, i.e. perfect feedback. The main drawback is the requirement of energy for receiver which impacts the overall energy budget of the sensing node. Although it is difficult to model, protocol latency becomes an issue for overall energy consumption. Some of the earliest work in analog transmission of low-bandwidth sources assumed feedback with the presence of the feedback signal $f(\mathbf{Y})$. Through communication in the presence of feedback, stochastic control approaches [15], [16] can achieve, at least asymptotically, the lower bound on distortion in (1). This comes at the expense of delay, since, as in many adaptive systems, the feedback system must converge to minimize distortion. It is reasonable to assume that both can be extended to non-coherent detection and even broadband frequency-selective channels for diversity. However, the underlying estimation strategies will quickly become quite involved.

As mentioned above following Goblick's original work, with an addition of a noiseless feedback link to the system, using coherent detection and unlimited channel bandwidth, the classical closed-loop schemes described in [15] asymptotically achieves (1). In comparison to [1], the proposed scheme is differentiated by not being quantized neither coded. It should be noted that both [1] and [15] consider unlimited channel bandwidth for a normally distributed source and coherent detection but unlike Goblick's digital scheme non-coherent detection is not applicable to [15]. As a second drawback, the scheme requires perfect feedback to achieve (1). The paper differs from Schalkwijk's previous work [17] with respect to the source statistics where the author uses uniform distribution with band-limited signals and feedback.

An example of a modern feedback-scheme for transmitting small amounts of sporadic information is the random-access procedure [18] in LTE systems, where a 6-bit message is conveyed using an orthogonal signal set occupying a large physical bandwidth (PRACH physical random access channel). The so-called *random-access response* contains the message hypothesized by the decoder, among other information, which serves either as an acknowledgment or an indication to retransmit. Although simplified, such a scheme was originally studied in [19] by Yamamoto which is an adaptation of the earlier work by Schalkwijk-Barron [20]. Note that, in these works the analysis with non-coherent detection is not provided.

B. Contributions and Outline

One of the main contributions of this work is to analyze the use of such a retransmission protocol for the transmission of scalar quantized analog samples in terms of the energy-efficiency as a function of the reconstruction fidelity. It is shown that there is a very significant benefit at the expense of feedback in comparison to a one-shot transmission of the parameter. The efficient use of such a protocol calls for joint optimization of the parameter quantization and modulation. It is important to note that in our scenario we are driven to assume unknown channels, i.e. non-coherent reception, in the formulation of the problem. Since the information content is very small, additional overhead for channel estimation is not warranted and thus, it is unreasonable to assume the channel state (i.e. channel amplitude and phase) be known to either the transmitter or receiver. The analysis is carried out for line-of-sight and non line-of-sight channels and we consider both cases of perfect and imperfect feedback. We furthermore provide new lower-bounds on the performance of such feedback-based schemes as well as numerical evaluation of recent bounds [10] for one-shot transmission. These bounds allow us to assess how close the proposed schemes are to fundamental limits.

In the upcoming section, we describe the system model of the addressed problem. In Section III, we introduce a low-latency feedback protocol for a single source transmitting analog information over a non-coherent AWGN channel. In spirit, this is very similar to the first phase of the LTE random-access procedure described above. The analytical exponential behavior of the protocol with respect to the reconstruction error for estimating the source-message is observed and discussed subject to the energy used by the protocol. This is followed by the discussion regarding the effect of the feedback error on the distortion-energy trade-off made in Subsection III-B. Additionally, for the case of one-shot transmission without feedback, in Subsection III-C we extend Merhav's recent lower-bounds derived in [10] for the problem addressed here, in order to provide the tightest numerical lower-bounds on performance. We proceed with the numerical analysis for a more general wireless channel model in Section IV. In Subsection IV-A, we provide adaptations of Goblick's and Merhav's lower-bounds for the more general fading channel models. The numerical analysis results for a chosen configuration of the fading channel model are given in Section V together with the results of the non-coherent AWGN channel. The numerical results are also contrasted with the best-known theoretical lower-bounds on the reconstruction fidelity. Comparisons between the two channel models are provided in Section VI for both single-round transmission without feedback in addition to the improvement achieved with two rounds of the novel protocol.

II. System Model

In this paper, we consider a general multi-channel wireless model where the channel amplitude and phase correspond to that of a multi-dimensional Ricean channel with a ratio of the non-line-of-sight amplitude total signal amplitude α. As explained in the previous section, the goal is to bound the reconstruction error in estimating the analog source message sent over a wireless channel exploiting a feedback link between the decoder and encoder. A source sample quantized to B bits is encoded into one of 2^B N-dimensional messages $\mathbf{S}_m$, with $m = 1, 2, \cdots, 2^B$ and each message is transmitted with equal energy $\mathcal{E}$. The output signal is given for this channel as

$$\mathbf{Y'}_l = \sqrt{\mathcal{E}/L'}\left(\sqrt{(1-\alpha)}\mathrm{e}^{j\Phi_l} + \sqrt{\alpha}h_l\right)\mathbf{S}_m + \mathbf{Z}_l, \tag{2}$$

for $l = 0, \cdots, L-1$ where $\mathrm{h} \sim N_{\mathbf{C}}(0,1)$ which have the desired statistics in both the frequency and time dimensions and α is a constant defined in the range $[0, 1]$. The random phase sequence Φ is assumed to be i.i.d. with a uniform distribution defined on $[0, 2\pi)$. The N-dimensional vector noise sequence $\mathbf{Z}$ is complex, circularly symmetric with zero-mean and autocorrelation $N_0\mathbf{I}_{N\times N}$. Here L represents the total number of statistically independent observations or diversity order of the transmitted signals and $L' \leq L$ denotes the number of observations over which the average received energy is spread. To a first-order approximation, L' represents the number of coherence bandwidths and L/L' would represent the number of receive antennas. For example, $L = 4$, $L' = 2$ would correspond to a dual-antenna receiver with two coherence bandwidths. Clearly, the channel model given above by (2) boils down to an AWGN channel for $\alpha = 0$ and $L' = 1$ with the N-dimensional channel observation given by

$$\mathbf{Y} = \sqrt{\mathcal{E}}\mathrm{e}^{j\Phi}\mathbf{S}_m + \mathbf{Z} \tag{3}$$

Throughout the paper, the general model given by (2) and the second model (3) will be referred as the fading channel and the AWGN channel, respectively. The following section will introduce a retransmission feedback protocol based on the AWGN channel and provide an analytical evaluation of its asymptotic performance. Due to its complexity the performance of the fading channel will be provided in the following Section IV only through numerical analysis.

III. Asymptotic Performance of a Novel Feedback Protocol with Non-coherent Detection on Line-of-Sight Channels

A. *Reliable feedback without energy cost*

Let us consider now a protocol applied to the transmission of isolated analog samples with non-coherent reception. This will serve as a motivating example for the use of feedback with low-latency achieving asymptotically near-optimal distortion performance. In the analysis, we first focus on a simple non-coherent AWGN channel (3) with a one dimensional source letter.

The protocol consists of two phases, a data phase and a control phase and proceeds as shown in Figure 3. In our adaptation the two phases compose one round of the protocol.During the data phase, the source message is quantized, transmitted and its estimate is fed back from the receiver. The channel observation (3) becomes

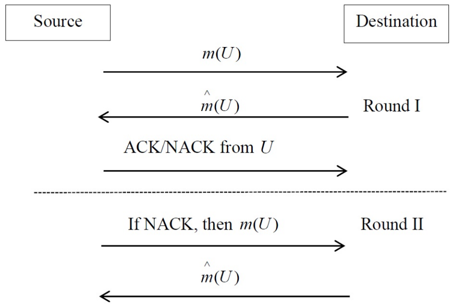

Fig. 3. Pictorial representation of the retransmission protocol.

$$\mathbf{Y}_D = \sqrt{\mathcal{E}_{\mathrm{D},i}}\mathrm{e}^{j\Phi_i}\mathbf{S}_m + \mathbf{Z}_D \tag{4}$$

where the subscripts D and i represent the data phase and the i^{th} round of the protocol, respectively. This type of transmission can exactly model any low-rate transmission strategy based on orthogonal modulation. For instance, to further put this in the context of the random-access procedure LTE systems, the $\mathbf{S}_m$ can represent the so-called *PRACH preamble* [21], where $m = 0, 1, \cdots, 63$, and conveys the 6-bit message (MSG1) described in Section I. The preamble in LTE is a Zadoff-Chu roots-of-unity sequence which usually occupies $N = 839$ signaling dimensions for $B = 6$ information bits. Orthogonality over time-dispersive channels is guaranteed through up to 64 cyclic time-shifts of $\mathbf{S}_m$ coupled with the use of a cyclic extension. For very dispersive channels (i.e. with delay-spreads longer than the cyclic-shift between preambles), fewer than 64 (and hence longer) cyclic time-shifts can be used at the expense of using multiple preamble sequences which are quasi-orthogonal.

After the transmission of the source message, the receiver feeds $\hat{m}$ back to the encoder via the noiseless feedback link. Let the corresponding error event be denoted by E_i. Square-law detection of the received signal produces the two possible decision variables assuming m is transmitted as

$$\begin{aligned}\mathbf{U}_{m'} &= |\langle \mathbf{Y}_1, \mathbf{S}_m \rangle|^2 \\ &= |\sqrt{\mathcal{E}_{D,1}} + N_m|^2 + |N_{m'}|^2.\end{aligned} \tag{5}$$

Here $\langle ., . \rangle$ denotes the inner product and N_m and $N_{m'}$ are defined as complex-valued zero-mean Gaussian random variables with a variance of N_0. $|\sqrt{\mathcal{E}_{D,1}} + N_m|^2$ and $|N_{m'}|^2$ are random variables with non-central chi-square distribution (noncentrality parameter $\mathcal{E}_{D,1}$) and central chi-square distribution, respectively. According to (5), the receiver chooses $\hat{m} = \mathrm{argmax}_{\hat{m}'}\,\mathbf{U}_{m'}$.

After the data phase, the encoder enters the control phase and informs the receiver whether or not its decision was correct via a signal $\sqrt{\mathcal{E}_{\mathrm{C},i}}\mathbf{S}_C$ of energy $\mathcal{E}_{\mathrm{C},i}$ if the decision is incorrect and $\mathbf{0}$ if the decision was correct. $\mathcal{E}_{\mathrm{C},i}$ here denotes the energy of the control phase in the i^{th} round since during the control phase the receiver observes $\mathbf{Y}_C = \sqrt{\mathcal{E}_{\mathrm{C},i}}A\mathrm{e}^{j\Phi_i}\mathbf{S}_C + \mathbf{Z}_C$ where A takes the value 0 for an ACK and 1 for a NACK. Let $Y_C = \mathbf{Y_C}^H\mathbf{S}_C$ and assume a detector of the form $\hat{A} = \mathcal{I}\left(|Y_C|^2 > \lambda\mathcal{E}_{\mathrm{c},i}\right)$. Here $\mathbf{Y}_C^H$, $\mathcal{I}(\cdot)$ and λ denote the complex conjugate of $\mathbf{Y}_C$, the indicator function and a threshold to be optimized that is confined to an interval $[0,1)$, respectively. $E_{\mathrm{e}\to\mathrm{c},i}$ corresponds to an uncorrectable error since it acknowledges an error as correct decoding and $E_{\mathrm{c}\to\mathrm{e},i}$ represents a mis-detected acknowledged error declaring correct decoding as incorrect. If the receiver correctly decodes the control signal and it signals that the data phase was correct after the completion of the first round, with probability $\Pr(E_1^c)(1-\Pr(E_{\mathrm{c}\to\mathrm{e},1}))$, the protocol halts, otherwise another identical round is initiated by the receiver. The retransmission probability, i.e. the probability of going on for a second round, is $\Pr(E_1)(1-\Pr(E_{\mathrm{e}\to\mathrm{c},1}))$. This on-off signaling guarantees that with probability $\Pr(E_1^c)(1 - \Pr(E_{\mathrm{c}\to\mathrm{e},1}))$ the transmitter will not expend more than $\mathcal{E}_{\mathrm{D},1}$ joules, which should be close to one. After each data phase, the receiver computes the ML or MAP message $\hat{m}_i(\mathbf{Y}_1,\cdots,\mathbf{Y}_i)$ based on all observations up to round i with error event E_i. The same control phase is repeated and the protocol is terminated after two rounds.

The error probability at the end of the second round is defined and consequently bounded by

$$\begin{aligned} P_{\mathrm{e}} = {} & \Pr(E_1)\Pr(E_{\mathrm{e}\to\mathrm{c},1}) + \\ & \Pr(E_1)(1-\Pr(E_{\mathrm{e}\to\mathrm{c},1}))\Pr(E_2|E_1) + \\ & (1-\Pr(E_1))\Pr(E_{\mathrm{c}\to\mathrm{e},1})\Pr(E_2|E_1^c) \\ & \stackrel{(a)}{\leq} \Pr(E_1)\Pr(E_{\mathrm{e}\to\mathrm{c},1}) + \Pr(E_2). \end{aligned} \tag{6}$$

In step (a) the conclusive expression is obtained through bounding $\Pr(E_{\mathrm{c}\to\mathrm{e},1})$ and $(1-\Pr(E_{\mathrm{e}\to\mathrm{c},1}))$ by 1. The probability of an uncorrectable error in round i, which is defined as $\Pr\left(|\sqrt{\mathcal{E}_{\mathrm{C},i}} + Z_c|^2 \leq \lambda\mathcal{E}_{\mathrm{c},i}\right)$, is obtained as

$$\Pr(E_{\mathrm{e}\to\mathrm{c},i}) = 1 - \mathrm{Q}_1\left(\sqrt{\frac{2\mathcal{E}_{\mathrm{C},i}}{N_0}}, \sqrt{\frac{2\lambda\mathcal{E}_{\mathrm{C},i}}{N_0}}\right), \tag{7}$$

where $\mathrm{Q}_1(\alpha,\beta)$ is the first-order Marcum-Q function and $Z_C = \mathbf{S}_C^H\mathbf{Z}$ is a circularly-symmetric Gaussian zero-mean random variable with variance N_0. Furthermore, we have the recent bound on the $\mathrm{Q}_1(\alpha,\beta)$ for $\alpha > \beta$ from [22, eq:4] which is very useful for bounding (7) as

$$\Pr(E_{\mathrm{e}\to\mathrm{c},i}) \leq 1/2\exp\left(-\frac{(\sqrt{\lambda}-1)^2\mathcal{E}_{\mathrm{C},i}}{N_0}\right). \tag{8}$$

The probability of a mis-detected acknowledged error is obtained as

$$\begin{aligned} \Pr(E_{\mathrm{c}\to\mathrm{e},i}) &= \Pr\left(|Z_C|^2 > \lambda\mathcal{E}_{\mathrm{C},i}\right) \\ &= \mathrm{e}^{-\frac{\lambda\mathcal{E}_{\mathrm{C},i}}{N_0}}. \end{aligned} \tag{9}$$

Let $\mathbf{Y}^{(2)}$ denote the output signal observed in the second round, the second round decision variables can be obtained cumulatively through

$$\mathbf{U}_{m'}^{(2)} = \mathbf{U}_{m'} + |<\mathbf{Y}^{(2)}, \mathbf{S}_m>|^2. \tag{10}$$

The receiver chooses $\hat{m} = \mathrm{argmax}_{\hat{m}'}\ \mathbf{U}_{m'}^{(2)}$ over all possible sequences as in the first round. The probability of error for binary orthogonal signaling is defined in [23, eq:12.1-24] as

$$P_2(j) \leq \frac{1}{2^{2j-1}}\mathrm{e}^{-\gamma/2}\sum_{n=0}^{j-1} c_n\left(\frac{\gamma}{2}\right)^n \tag{11}$$

where $c_n = 1/n!\sum_{k=0}^{j-1-n}\binom{2j-1}{k}$ and γ represents the signal to noise ratio. The probability of making an error on a particular round j, $\Pr(E_j) \leq 2^B P_2(j)$ can be derived using (11) and given for the first and the second rounds by

$$\Pr(E_1) \leq 2^{B-1}\mathrm{e}^{-\frac{\mathcal{E}_{\mathrm{D},1}}{2N_0}}, \tag{12}$$

$$\Pr(E_2) \leq 2^{B-3}\left(1 + 3\frac{\mathcal{E}_{\mathrm{D},1}+\mathcal{E}_{\mathrm{D},2}}{N_0}\right)\mathrm{e}^{-\frac{\mathcal{E}_{\mathrm{D},1}+\mathcal{E}_{\mathrm{D},2}}{2N_0}}. \tag{13}$$

Naturally, (12) and (13) are obtained using the first round decision variables (5) and the second round decision variables (10), respectively.

The reconstruction error of the source message is obtained by calculating the mean squared error distortion through $\mathcal{D} = \mathcal{D}_q(1-P_e) + \mathcal{D}_e P_e$ where P_e, $\mathcal{D}_q$ and $\mathcal{D}_e$ represent the total probability of error, the quantization distortion and the MSE distortion for the case where an error was made, respectively. For a uniform source U on $(-\sqrt{3},\sqrt{3})$ (i.e. a source with zero mean and unit variance) the distortion is bounded by

$$\mathcal{D}\left(\mathcal{E}, N_0, N, \lambda\right) \leq 2^{-2B}(1-P_e) + 2P_e. \tag{14}$$

A detailed derivation of the distortion terms in (14) is provided in Appendix VII-A. The following subsections III-A1 and III-A2 discuss the asymptotic performance of the bound given above by (14) in the absence and presence of a feedback link in the system, respectively.

1) The performance of the protocol without feedback : For the case of $N = 1$, i.e. the protocol terminates without retransmission, we obtain the bound on the reconstruction error in estimating the message of U as given in the following. The error probability defined by (6) consists of the probability of making an error in the first round $\Pr(E_1)$ solely, since there is no use of the control phase given that there will not be a second round to retransmit the message. Thus, through substitution of $P_e \leq 2^{B-1}\mathrm{e}^{-\frac{\mathcal{E}_{\mathrm{D},1}}{2N_0}}$ into the distortion bound given by (14), we get the following bound

$$\mathcal{D}(\mathcal{E}, N_0, 1, \lambda) \leq \mathrm{e}^{-2B\ln 2} + \mathrm{e}^{B\ln 2 - \frac{\mathcal{E}_{\mathrm{D},1}}{2N_0}}. \quad (15)$$

By setting the two exponentials in (15) equal, it can be seen that 2^{-B} is in the same order of $\mathrm{e}^{-\frac{\mathcal{E}_{\mathrm{D},1}}{6N_0}}$. In other words, the upper bound (15) is obtained as $\mathcal{D}(\mathcal{E}, N_0, 1, \lambda) \leq 2\mathrm{e}^{-\frac{\mathcal{E}_{\mathrm{D},1}}{3N_0}}$ for a single round.

2) The performance of the protocol exploiting feedback : At the end of the second round the resulting distortion is given by

$$\begin{aligned}
&\mathcal{D}(\mathcal{E}, N_0, 2, \lambda) \leq \\
&\mathrm{e}^{-2B\ln 2} + \mathrm{e}^{(B-1)\ln 2 - \frac{\mathcal{E}_{\mathrm{D},1}}{2N_0} - \left(1+\lambda-2\sqrt{\lambda}\right)\frac{\mathcal{E}_{\mathrm{C},1}}{N_0}} \\
&+ \left(1 + 3\frac{\mathcal{E}_{\mathrm{D},1} + \mathcal{E}_{\mathrm{D},2}}{N_0}\right)\mathrm{e}^{(B-2)\ln 2 - \frac{\mathcal{E}_{\mathrm{D},1}+\mathcal{E}_{\mathrm{D},2}}{2N_0}} \quad (16)
\end{aligned}$$

through substituting (6) with (12), (13) and (8) for $i = 1$ into the distortion (14). By equating the three exponentials of (16) we have that $\mathcal{E}_{\mathrm{C},1} = \frac{\mathcal{E}_{\mathrm{D},2}}{2(1+\lambda-2\sqrt{\lambda})}$. In order for $\Pr(E_1)$ to be exponentially bounded away from zero so that $\mathcal{E}$ can be made arbitrarily close to $\mathcal{E}_{\mathrm{D},1}$, we define $\mathcal{E}_{D,2} = (2-\mu)\mathcal{E}_{D,1}$ where μ is an arbitrary constant satisfying $\mu \in (0, 2)$. Finally, we obtain the bound on the distortion at the end of the second round as given by

$$\mathcal{D}(\mathcal{E}, N_0, 2, \lambda) \leq \mathrm{e}^{-\frac{\mathcal{E}_{\mathrm{D},1}(1-\mu/3)}{N_0}}\left(3 + 3\frac{\mathcal{E}_{\mathrm{D},1} + \mathcal{E}_{\mathrm{D},2}}{N_0}\right). \quad (17)$$

Note that, at the end of the second round, 2^{-B} is in the same order of $\mathrm{e}^{-\frac{\mathcal{E}_{\mathrm{D},1}(1-\mu/3)}{2N_0}}$.

The average energy used by the protocol after two rounds is

$$\begin{aligned}
\mathcal{E} &= \mathcal{E}_{\mathrm{D},1} + \Pr(E_1)\mathcal{E}_{\mathrm{C},1} + (\Pr(E_1)(1 - \Pr(E_{\mathrm{e}\to\mathrm{c},1})) \\
&+ (1 - \Pr(E_1))\Pr(E_{\mathrm{c}\to\mathrm{e},1}))\mathcal{E}_{\mathrm{D},2}. \quad (18)
\end{aligned}$$

$\mathcal{E}_{\mathrm{D},2}$ here denotes the required energy for retransmission, which is the energy to be used in the data phase of the second round. Clearly if $\Pr(E_1)$ is small, then the protocol achieves marginally more than $\mathcal{E}_{\mathrm{D},1}$ joules per source symbol. It is worth mentioning that (1) and the limiting expression in [15, eq.15] is achieved to within a factor of 1/2 in the energy using only two rounds and, moreover, with non-coherent reception. Even though it is possible to obtain $\mathrm{e}^{-\frac{2\mathcal{E}_{\mathrm{D},1}}{N_0}}$ (i.e. twice better than the performance in (17)) by changing the relationship between the energies used in the different rounds, this causes the average energy used by the protocol to exceed $\mathcal{E}_{\mathrm{D},1}$, the energy used in the data phase of the first round. As a result, the proposed protocol cannot achieve the error exponent in (1).

In Section III-B, we investigate the case when the feedback link from the decoder to the encoder is not perfect and discuss the effect of a possible error in feedback on the exponential behavior of the reconstruction error. Note that, for modeling systems where both the transmitter and receiver are subject to the constraints on energy usage, one would have to consider the energy consumption of the feedback link, and we also shed some light on this issue in Section III-B.

B. Unreliable feedback with and without energy cost

One might consider the case of an imperfect feedback link in the system described and analyzed above. Let $P_{\mathrm{fb},1}$ denote the following error probability $\Pr(\hat{\hat{m}} = m|\hat{m} \neq m)$ whereas $P_{\mathrm{fb},2} = \Pr(\hat{\hat{m}} \neq m|\hat{m} = m)$. Here m denotes the transmitted message, $\hat{m}$ and $\hat{\hat{m}}$ denote the messages decoded at the receiver and transmitter (after the feedback phase) respectively. The overall energy used by the protocol in this scenario becomes

$$\begin{aligned}
\mathcal{E} &= \mathcal{E}_{\mathrm{D},1} + \mathcal{E}_{\mathrm{C},1}\Pr(E_1)(1 - P_{\mathrm{fb},1}) \\
&+ \mathcal{E}_{\mathrm{D},2}\left[\Pr(E_1)(1 - P_{\mathrm{fb},1}) + (1 - \Pr(E_1))P_{\mathrm{fb},2}\right] \quad (19)
\end{aligned}$$

whereas the error probability at the end of the second round yields (20) as given on the top of the next page. In step (a) of (20), $(1 - P_{\mathrm{fb},1})$, $(1 - \Pr(E_{\mathrm{e}\to\mathrm{c},1}))$ and $\Pr(E_{\mathrm{c}\to\mathrm{e},1})$ is upper bounded by 1. Clearly, if $P_{\mathrm{fb},1} = P_{\mathrm{fb},2} = 0$ this case boils down to the perfect feedback scenario studied above and the expressions on average energy (19) given above and the error probability (20) yield (18) and (6), respectively. Now, we apply the modified error probability given above to the overall distortion term (14). In order to obtain the same exponential behavior of $\mathrm{e}^{-\mathcal{E}_{\mathrm{D},1}/N_0}$ like in (17), $P_{\mathrm{fb},1}$ should be upper-bounded by the uncorrectable error $\frac{1}{2}\mathrm{e}^{-\frac{(\sqrt{\lambda}-1)^2\mathcal{E}_{C,1}}{N_0}}$ given earlier by (7). With respect to the energy consumption, we can say that in addition to the error probability in the first round, vanishing $P_{\mathrm{fb},2}$ guarantees the energy consumed by two rounds of the protocol to be upper bounded by the energy which is used by the data phase of the first round.

In order to characterize the amount energy required for feedback we consider an explicit scheme for feedback. The receiver uses waveform $\mathbf{S}_{\hat{m}}$ on the feedback link with energy $\mathcal{E}_{\mathrm{fb}}$ so that the received signal is

$$\mathbf{Y}_{\mathrm{fb}} = \sqrt{\mathcal{E}_{\mathrm{fb}}}\mathrm{e}^{j\Phi}\mathbf{S}_{\hat{m}} + \mathbf{Z} \quad (21)$$

$$P_{\rm e} = \Pr(E_1)(1-P_{\rm fb,1})\Pr(E_{\rm e\to c,1}) + \Pr(E_1)P_{\rm fb,1} + \Pr(E_1)(1-P_{\rm fb,1})(1-\Pr(E_{\rm e\to c,1}))\Pr(E_2|E_1)$$
$$+ \ [(1-\Pr(E_1))(1-P_{\rm fb,2})\Pr(E_{\rm c\to e,1}) + (1-\Pr(E_1))P_{\rm fb,2}(1-\Pr(E_{\rm e\to c,1})]\Pr(E_2|E_1^c)$$
$$\stackrel{(a)}{\leq} \Pr(E_1)\left(\Pr(E_{\rm e\to c,1}) + P_{\rm fb,1}\right) + \Pr(E_2). \qquad (20)$$

In order to determine if message m was received correctly, the transmitter projects on waveform $\mathbf{S}_m$ and computes the statistic $U = |\langle \mathbf{Y}_{\rm fb}, \mathbf{S}_m\rangle|^2$ which is compared to a threshold $\lambda_{\rm fb}\mathcal{E}_{\rm fb}$ where $\lambda_{\rm fb} \in [0,1)s$. The important feedback probability is then

$$P_{\rm fb,1} = \Pr\left(|\langle \mathbf{Y}_{\rm fb}, \mathbf{S}_m\rangle|^2 \geq \lambda_{\rm fb}\mathcal{E}_{\rm fb}\right) = {\rm e}^{-\frac{\lambda_{\rm fb}\mathcal{E}_{\rm fb}}{N_0}}. \qquad (22)$$

As a result, in order for $P_{\rm fb,1}$ to be on the same exponential order as $\Pr(E_{\rm e\to c,1})$ we require that $\mathcal{E}_{\rm fb} = \frac{1-\mu/2}{\lambda_{\rm fb}}\mathcal{E}_{d,1}$ and that the energy used by the protocol approaches $\frac{\lambda_{\rm fb}+1-\mu/2}{\lambda_{\rm fb}}\mathcal{E}_{d,1}$. The main conclusion is that when we account for the energy consumption required by the feedback link, it reduces the reconstruction fidelity in a non-negligible manner under a total energy constraint. In the primary application scenario considered here, namely energy-constrained sensors transmitting to cellular basestations, we believe that this does not pose a significant problem. Basestations are power constrained and not short-term energy constrained and if the aggregate downlink traffic dedicated to feedback for sensors is an order of magnitude less than other downlink services, this energy consumption is insigificant. If such schemes were to be used for transmission between energy-constrained devices, the benefits may be significantly reduced.

C. Lower-bounds on Distortion

The first set of bounds all rely on channel state knowledge at the receiving end which clearly is also a bound for the case where the channel phases are unknown. The simplest bound is Goblick's bound which in our case of a uniform random variable on $[-\sqrt{3},\sqrt{3})$ is given by

$$\mathcal{D}_G(\mathcal{E},N_0) \geq \frac{6}{\pi e}{\rm e}^{-\frac{2\mathcal{E}}{N_0}}. \qquad (23)$$

For the case of a single round without feedback we use the recent bounds from Merhav in [10] which are adaptations of the Ziv-Zakai lower-bound [13] on mean-squared error for parameter modulation-estimation. We consider only the case of zero-rate transmission in the context of [10] and adapt the results to the normalized uniform distribution considered here. We have the following bound on the distribution of distortion

$$\Pr\left(|U-\hat{U}| > \frac{\sqrt{3}}{M}\right) \geq \frac{\sqrt{3}}{M}Q\left(\sqrt{\frac{\mathcal{E}}{N_0}\frac{M}{M-2}}\right). \qquad (24)$$

The right-hand side of (24) is the weakest version of Shannon's lower-bound on M-ary transmission over an AWGN channel [24, eq. 82]. Through the use of the Chebyshev inequality, this results in the following lower-bound on the distortion

$$\mathcal{D}_{M1}(\mathcal{E},N_0) \geq \max_M \frac{3\sqrt{3}}{M^3}Q\left(\sqrt{\frac{\mathcal{E}}{N_0}\frac{M}{M-2}}\right). \qquad (25)$$

A tighter version makes use of Shannon's best bound [24, eq. 81] yielding

$$\mathcal{D}_{M2}(\mathcal{E},N_0) \geq \max_M \frac{6\sqrt{3}}{M^4}\sum_{n=2}^{M} Q\left(\sqrt{\frac{\mathcal{E}}{N_0}\frac{n}{n-1}}\right). \qquad (26)$$

As suggested in [10, eq.23] an even tighter version based on [24, eq. 81] is derived using (27) as given on the top of the current page for any suitably large M. All of the lower-bounds introduced above are numerically evaluated in comparison to the proposed transmission strategies in Section V.

1) Relationships with classical conjectures on optimal signal sets: It is worth pointing out that certain classical and more recent results on the validity of conjectures on optimal signal sets are strongly related to the problem at hand and could provide tighter numerical lower-bounds on the reconstruction fidelity. In Merhav's bounding technique for the parameter modulation-estimation problem he relies on zero-rate lower-bounds on the probability of error (e.g. in [10, eq. 21]) in characterizing the tail-function of the estimation error at discrete values of its argument. For coherent detection on AWGN channels, it was long conjectured that the regular simplex was an optimal signal set for M-ary signaling in $M-1$ dimensions (i.e. without a bandwidth constraint). This was disproved by Steiner in [25] for the so-called *Strong Simplex Conjecture* which corresponds to the average energy constraint used here. The so-called *Weak Simplex Conjecture* is the classical conjecture [26] for equal-energy signaling which still has not been disproved and is valid for $M = 2,3$. It is largely considered to be true for all M, and from a numerical perspective, was shown to be valid for

$$\mathcal{D}_{M3}(\mathcal{E}, M, N_0) = 2\int_0^{2\sqrt{3}} d\Delta \cdot \Delta(2\sqrt{3} - (\left\lfloor 2\sqrt{3}/\Delta \right\rfloor - 1)\Delta) \cdot \Pr\left(|U - \hat{U}| > \Delta\right)$$
$$\geq \; 2\left(\int_0^{\sqrt{3}/M} d\Delta \cdot \Delta(2\sqrt{3} - (M-1)\Delta) \Pr\left(|U - \hat{U}| > \sqrt{3}/M\right)\right.$$
$$\left. + \; \sum_{i=3}^{M} \int_{\sqrt{3}/i}^{\sqrt{3}/(i-1)} d\Delta \cdot \Delta(2\sqrt{3} - (i-1)\Delta) \Pr\left(|U - \hat{U}| > \sqrt{3}/(i-1)\right)\right)$$
$$= \; \frac{\sqrt{3}}{M^4}(5M+1)\sum_{n=2}^{M} Q\left(\sqrt{\frac{\mathcal{E}}{N_0}\frac{n}{n-1}}\right) + \sqrt{3}\sum_{i=2}^{M-1}\left(\frac{5i-4}{(i-1)^4} - \frac{5i+1}{i^4}\right)\sum_{n=2}^{i} Q\left(\sqrt{\frac{\mathcal{E}}{N_0}\frac{n}{n-1}}\right). \quad (27)$$

$M \leq 8$ in [27]. From a numerical perspective, the use of the constructive techniques in [27] for finding optimal signal sets could be used instead of Shannon's lower bound in (27). Although this will not provide an asymptotic difference, it could lead to tighter bounds for low signal-to-noise ratios. For the equal-energy case, it may be sufficient to use the error probability of the regular simplex in (27), at least if we limit the sum to $M \leq 8$. Even if the Weak Simplex Conjecture is false, it is highly unlikely that any other signal set will provide a noticeable numerical difference in (27).

The equivalent equal-energy conjecture for non-coherent detection [28] also remains unproven. But it is reasonable for numerical purposes to use the error probability of orthogonal modulation with non-coherent detection as an approximate lower-bound. Using [28, eq. 28] instead of Shannon's lower bound in (27) we obtain

$$\mathcal{D}_{M4}(\mathcal{E}, M, N_0) \geq \frac{\sqrt{3}}{M^3}(5M+1)P_M + \sqrt{3}\sum_{i=2}^{M-1}\left(\frac{5i-4}{(i-1)^3} - \frac{5i+1}{i^3}\right)P_i \quad (28)$$

where

$$P_i = \sum_{n=1}^{i-1}(-1)^{n+1}\binom{i-1}{n}\frac{1}{n+1}\exp\left[-\frac{n}{n+1}\frac{\mathcal{E}}{N_0}\right] \quad (29)$$

which, strictly speaking, is only a true bound for equal-energy signaling and $M = 2$, subject to the validity of the classical conjecture. Note that (28) will have the same asymptotic behavior as (27).

2) Comments on variable-energy signaling: It is reasonable to expect that the use of variable-energy signaling (even orthogonal) can help close the 1.76dB asymptotic gap between (15) and (27) and the 3dB gap between (16) and (23). This is because with equal-energy signaling, erroneous decisions can lead to distortions at the peak or on the order of a bit with equal probability. A more judicious choice of energy distribution across the signal set would choose the energy difference between points according to their pairwise distortion. High distortion error events would then be less likely than low distortion error events.

IV. More General Wireless Channels

We consider now the fading channel introduced in Section II by (2) and adapt this system to our re-transmission feedback protocol proposed and analyzed in the previous section. In this fading channel model the output signal (2) in the data phase of round i on channel l becomes

$$\mathbf{Y}'_{D,l} = \sqrt{\mathcal{E}_{\mathrm{D},i}/L'}\left(\sqrt{(1-\alpha)}\mathrm{e}^{j\Phi_{i,l}} + \sqrt{\alpha}h_{i,l}\right)\mathbf{S}_m + \mathbf{Z}_l, \quad (30)$$

where $\mathrm{h_i} \sim N_{\mathbf{C}}(0,1)$. For this model, only the statistics of the mis-detected acknowledged error event is unchanged and is as given by (9). The probability of an uncorrectable error is given by (31) on the top of the next page. The error probabilities $\Pr(E_1)$ and $\Pr(E_2)$ corresponding to the first and second rounds, respectively are derived using an adaptation of [23, eq:12.1-22], which is given by (32) where j is the round index, I_n is the modified Bessel function of order n, $v = \frac{u}{2\mathcal{E}(N_0+\alpha\mathcal{E})}$ and $\gamma = \mathcal{E}/L'N_0$. u is the first decision variable with a non-central chi-square distribution having $2L$ degrees of freedom and non-centrality parameter $s^2 = \mathcal{E}(1-\alpha)$. Note that above probability reduces to [23, eq:12.1-22] for $\alpha = 0$. In the fading channel case, the protocol provides a more significant improvement when going from one to two rounds, due to the added diversity. Here it should be expected that the use of more than two rounds could be even more beneficial, unlike the AWGN case. The use of many rounds, however, will incur a non-coherent combining loss, despite the added diversity.

The upper bound on the reconstruction error given in Section III by (14) is adapted to the current model and by substituting (32) and (31) we obtain the following bound on the distortion at the end of the second round.

$$\mathcal{D}\left(\mathcal{E}, N_0, 2, \lambda\right) \leq 2^{-2B}(1-P_e) + 2P_e$$
$$\leq 2^{-2B} + 2\left[P_M(1)\Pr(E_{\mathrm{e}\to\mathrm{c},1}) + P_M(2)\right] \quad (33)$$

$$\Pr(E_{\mathrm{e}\to\mathrm{c},i}) = \Pr\left(\sum_{l=0}^{L-1}|\sqrt{(1-\alpha)\mathcal{E}_{\mathrm{C},i}/L'}\mathrm{e}^{j\Phi_{i,l}} + \sqrt{\alpha\mathcal{E}_{\mathrm{C},i}/L'}h_{i,l} + z_{c,l}|^2 \leq \lambda L\mathcal{E}_{\mathrm{C},i/L'}\right)$$

$$= 1 - \mathrm{Q}_L\left(\sqrt{\frac{2L(1-\alpha)\mathcal{E}_{\mathrm{C},i}}{\alpha\mathcal{E}_{\mathrm{C},i}+L'N_0}}, \sqrt{\frac{2L\lambda(1-\alpha)\mathcal{E}_{\mathrm{C},i}}{\alpha\mathcal{E}_{\mathrm{C},i}+L'N_0}}\right). \tag{31}$$

$$P_M(j) = 1 - \int_0^\infty \left(1 - e^{-v(1+\alpha\gamma)}\sum_{k=0}^{jL-1}\frac{(v(1+\alpha\gamma))^k}{k!}\right)^{M-1}$$

$$\left[v\left(\frac{1+\alpha\gamma}{\gamma(1-\alpha)}\right)\right]^{\frac{jL-1}{2}} e^{-v-\frac{\gamma(1-\alpha)}{(1+\alpha\gamma)}} I_{jL-1}\left(2\sqrt{\frac{v\gamma(1-\alpha)}{1+\alpha\gamma}}\right) \tag{32}$$

In the upcoming section, we provide numerical evaluation results of the upper bound given above for different values of α for 0.5 and 0.1 since it is not possible to give an analytical result and discuss the improvement to be gained in two rounds through comparing (33) versus the distortion to be achieved in a single round without feedback, i.e. $\mathcal{D} \leq 2^{-2B} + 2P_M(1)$.

A. *Lower-bounds on Distortion*

We consider the same two lower bounds on performance considered in the previous sections. Merhav's bounding technique must be computed numerically in this case, as the upper bound introduced above and the other one is presented in an analytical form. Merhav's results were derived for Rayleigh fading which is generalized here to channels with a line-of-sight component and more degrees-of-freedom. Secondly we adapt the classical bound from Goblick [1] for a fading channel. Both of these techniques assume that the channel is known to the receiver and the distortion is averaged over all realizations of the random channel coefficients.

Merhav's bound (27) is

$$\overline{\mathcal{D}}_{M3}(\mathcal{E}, M, N_0) \geq \mathbb{E}_a D_{M3}(a\mathcal{E}, M, N_0) \tag{34}$$

where $a = \sum_{i=0}^{L-1}|\sqrt{1-\alpha}+\sqrt{\alpha}h_{i,l}|^2$ is a non-central chi-square distributed random variable with the non-centrality parameter $(1-\alpha)L$, $2L$ degrees of freedom and with the variance of the $2L$ underlying Gaussian random variables given by $\sigma^2 = \alpha/2$. Its p.d.f. is given below.

$$f(a) = \frac{1}{\alpha}\left(\frac{a}{(1-\alpha)L}\right)^{\frac{L-1}{2}} \exp\left(-\frac{a+(1-\alpha)L}{\alpha}\right)$$

$$I_{L-1}\left(2\frac{\sqrt{a(1-\alpha)L}}{\alpha}\right). \tag{35}$$

The behavior of the lower-bound (34) will be presented numerically in the upcoming section.

The wireless adaptation of the Goblick bound given by (1) tries to capture the scenario considered in the achievable scheme above, namely that a finite number of channel of realizations (or block-fading model) is exploited by the transmission strategy. To this end, we consider observations comprising N signaling dimensions split into R blocks of size N/R. Let $\mathbf{x}_i$ be the codeword in block i and constrain its energy as $\mathbb{E}||\mathbf{x}_i||^2 \leq \mathcal{E}/R$. Each block witnesses an independent and identically distributed fading amplitude. We show in Appendix VII-B that the distortion is bounded below by

$$\mathcal{D} \geq (1+4\alpha\mathcal{E}/RN_0)^{-LR}\exp\left\{-\frac{2(1-\alpha)L\mathcal{E}/N_0}{1+4\alpha\mathcal{E}/RN_0}\right\}. \tag{36}$$

V. Numerical Evaluation

In this section, we provide numerical evaluation results for the bounds introduced in Sections III and IV. In Figure (4) we show the bound given by (16) for two rounds and different values of B from 2 to 10. The convex hull of these curves should be compared with the Goblick-bound given by (23) which is valid for systems with feedback. The curves labeled as the single-round scheme without feedback represent (15). The convex hull of these curves should be compared with the Merhav bounds which are valid only without feedback. Note that, in Figure (4) Merhav bound 1, 2 and 3 represent the lower bounds given by equations (25), (26) and (27) , respectively. Firstly we see the significant effect of using the novel feedback protocol with respect to the reconstruction fidelity. The latter clearly provides an improvement in terms of distortion or approximately 3 dB in energy efficiency. We do not quite see the predicted 3dB gap (around 4.5 dB for 14-bits) in energy-efficiency with respect to the outer-bound with a known channel, even with a very high-resolution quantization level. Tighter bounding techniques for the case with feedback in addition to variable-energy schemes should therefore be considered for future work. The tightest of the Merhav bounds is clearly (27) but also does not quite predict the 1.7 dB asymptotic gap. Although not shown, numerical analysis also confirmed the asymptotic result given in Section III by (17) regarding the use of twice as much energy in the second round in comparison to the first.

The upper-bound in (33) is depicted in Figures (5), (6) and (7) for several values of B for the cases $\alpha = 0.1$ and $\alpha = 0.5$ and both high ($L = 4$) and low diversity orders $L = 1$. In all cases we see a very significant effect ($\geq$10dB in energy-efficiency) in using a two-round feedback protocol compared to a one-shot transmission, and this even in the case of a strong line-of-sight component ($\alpha = 0.1$). Bound types of lower-bounds are looser in the case of the fading channels, and especially in the high-diversity case (Figure (7)). This can be attributed to the non-coherent combining loss which is not captured by the bounds which assume known channels. This motivates the search for better lower-bounds assuming unknown channels in their formulation.

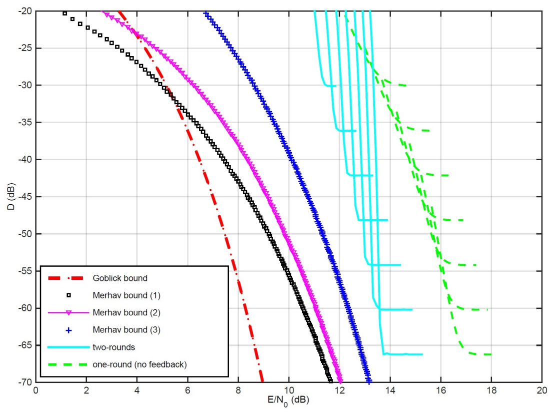

Fig. 4. Numerical evaluation of the upper and lower bounds on distortion for different values of B in an AWGN channel.

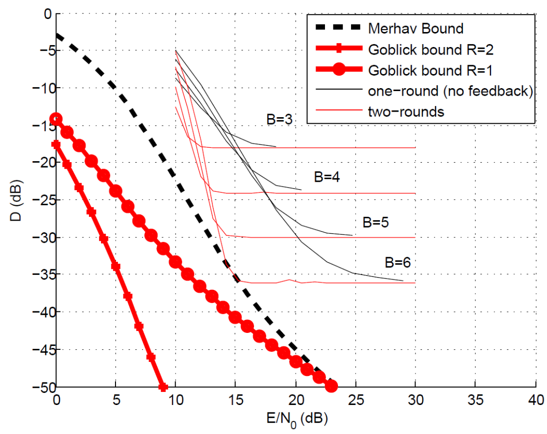

Fig. 5. Numerical evaluation of the distortion for B from 3 to 6 in a wireless channel for $\alpha = 0.1, L_p = 1, L = 1$

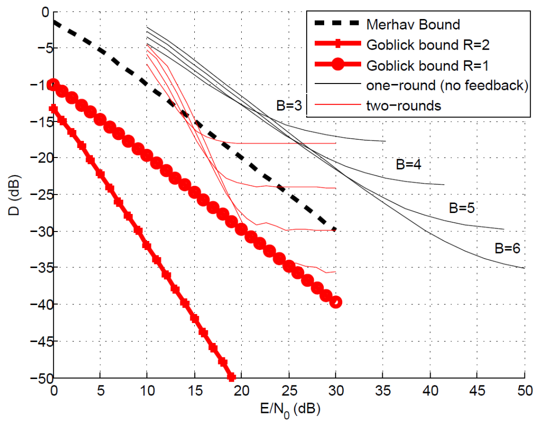

Fig. 6. Numerical evaluation of the distortion for B from 3 to 6 in a wireless channel for $\alpha = 0.5, L_p = 1, L = 1$

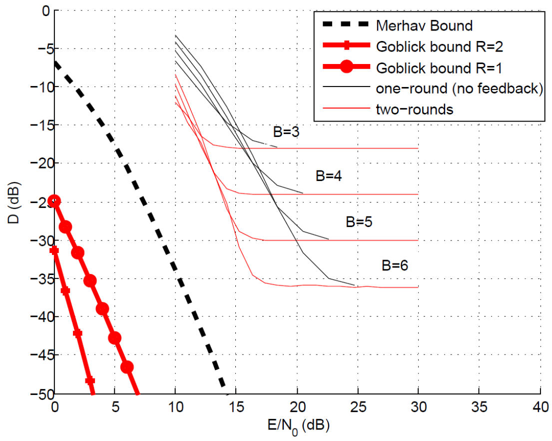

Fig. 7. Numerical evaluation of the derived bounds for B from 3 to 6 in a wireless channel for $\alpha = 0.5, L_p = 2, L = 4$

VI. Conclusion

We introduced a low-latency feedback protocol for the transmission of a single random variable over a wide-band channel and analyzed its asymptotic behavior with non-coherent detection on both pure line-of-sight and more general fading channels. The protocol and transmission strategy can be used for future energy-limited sensors making use of broadband cellular networks. We showed that the improvement over a one-shot transmission is on the order of 3-4 dB and asymptotically 4.7 dB. We have also included a discussion regarding the case of imperfect feedback and its effect on the trade-off between the required energy for the protocol and the reconstruction error in estimating the source message. We showed that in this case, if the energy consumption required by the feedback link is accounted, this reduces the reconstruction fidelity. Additionally, numerical evaluation of Merhav's recent lower-bounds for one-shot transmission are included and the tightest variant using his techniques is determined. Both the bounds and performance evaluation of the feedback protocol have been extended to a multi-channel fading model. The improvement of the feedback protocol over one-shot transmission is even more significant than in the line-of-sight case. We further suggest that tighter bounding techniques which rely on unknown channels should be found for the fading channel. Furthermore, schemes using variable-energy transmission should be considered to close the gap with the lower-bounds.

VII. Appendix

A. Derivation of the Distortion $\mathcal{D}_e$

$\mathcal{D}_e = E[u-\hat{u}]^2$ is bounded as follows

$$\begin{aligned} E[u-\hat{u}]^2 &= E[u^2] + E[\hat{u}]^2 - 2E[u\hat{u}] \\ &= E[u^2] + E[\hat{u}]^2 \\ &= 1 + 2^{1-B} \sum_{i=0}^{2^{B-1}-1} \left(\frac{i\sqrt{3}}{2^{B-1}} + \frac{\sqrt{3}}{2^{B-2}} \right)^2 \\ &\leq 2 \end{aligned} \tag{37}$$

On the other hand, the quantization distortion $\mathcal{D}_q$ is simply the variance within a single bin which is $\left(\frac{1}{2^B}\right)^2$.

B. Wireless Adaptation of the Goblick Bound

In order to derive a lower bound the distortion level of the wireless channel with feedback, we begin with the model $Y_{r,i} = \sqrt{h_r} X_{r,i} + Z_{r,i}$, $i = 1, \cdots, N/R$, $r = 1, \cdots, R$ where $Y_{r,i}$, $X_{r,i}$, h_r and $Z_{r,i}$ are the channel output, input, complex fadig amplitude and the noise terms, respectively. We start with two different expansions of the mutual information $I(\mathbf{U};\mathbf{Y}|\{\mathbf{H}_r = h_r, r = 1, \cdots, R\})$ which are equated and given as follows.

$$\begin{aligned} & I(\mathbf{U};\mathbf{Y}|\{\mathbf{H}_r = h_r\}) = h(\mathbf{U}|\{\mathbf{H}_r = h_r\}) \\ & - h(\mathbf{U} - \hat{\mathbf{U}}(\{\mathbf{H}_r = h_r\})|\mathbf{Y}, \{\mathbf{H}_r = h_r\}) \\ & \stackrel{(a)}{=} h(\mathbf{U}) - h(\mathbf{U} - \hat{\mathbf{U}}(\{\mathbf{H}_r = h_r\})|\mathbf{Y}, \{\mathbf{H}_r = h_r\}, \\ & \qquad \hat{\mathbf{U}}(\{\mathbf{H}_r = h_r\})) \\ & \geq h(\mathbf{U}) - h(\mathbf{U} - \hat{\mathbf{U}}(\{\mathbf{H}_r = h_r\})) \\ & = \frac{1}{2}\log 2\pi e - \frac{1}{2}\log(2\pi e \mathcal{D}(h)) \\ & = \frac{1}{2}\log(1/\mathcal{D}(h)) \end{aligned} \tag{38}$$

where $\mathcal{D}(h)$ represents $\mathcal{D}(\{\mathbf{H}_r = h_r\})$. For the second expansion we have

$$\begin{aligned} & I(\mathbf{U};\mathbf{Y}|\{\mathbf{H}_r = h_r\}) = h(\mathbf{Y}|\{\mathbf{H}_r = h_r\}) \\ & - h(\mathbf{Y}|\mathbf{U}, \{\mathbf{H}_r = h_r\}) \\ & = \sum_{r=1}^{R}\sum_{i=1}^{N/R} h(Y_{r,i}|Y_r^{i-1}, Y_1^N, \cdots, Y_{r-1}^N, \{\mathbf{H}_r = h_r\}) \\ & - \sum_{r=1}^{R}\sum_{i=1}^{N/R} h(Y_{r,i}|Y_r^{i-1}, Y_1^N, \cdots, Y_{r-1}^N, \mathbf{U}, \{\mathbf{H}_r = h_r\}) \\ & = \sum_{r=1}^{R}\sum_{i=1}^{N/R} h(Y_{r,i}|Y_r^{i-1}, Y_1^N, \cdots, Y_{r-1}^N, \{\mathbf{H}_r = h_r\}) \\ & - \sum_{r=1}^{R}\sum_{i=1}^{N/R} h(Y_{r,i}|Y_r^{i-1}, Y_1^N, \cdots, Y_{r-1}^N, \mathbf{U}, \mathbf{X}, \{\mathbf{H}_r = h_r\}) \end{aligned}$$

$$\begin{aligned} & = \sum_{r=1}^{R}\sum_{i=1}^{N/R} h(X_{r,i}\sqrt{h_r} + Z_{r,i}|Y_r^{i-1}, Y_1^N, \cdots, Y_{r-1}^N, \\ & \qquad \{\mathbf{H}_r = h_r\}) - \sum_{r=1}^{R}\sum_{i=1}^{N/R} h(Z_{r,i}) \\ & \leq \sum_{r=1}^{R}\sum_{i=1}^{N/R} \log 2\pi e \left(N_0 + E_{r,i}|h_r|^2\right) \\ & - \sum_{r=1}^{R}\sum_{i=1}^{N/R} \log 2\pi e N_0 \\ & \stackrel{(b)}{=} \sum_{r=1}^{R} \frac{N}{R} \log\left(1 + \frac{R}{N}\frac{E_r|h_r|^2}{N_0}\right) \\ & \leq \sum_{r=1}^{R} \frac{E_r|h_r|^2}{N_0}. \end{aligned} \tag{39}$$

In step (a) given the independence between $\mathbf{U}$ and $\{\mathbf{H}_r = h_r\}$, the conditional entropy equals the entropy of the source. And in step (b) we used the following property $\log(1+x) \leq x$. Equating the two expansions (38) and (39) yields

$$\mathcal{D}(h) \geq e^{-2\sum_{r=1}^{R} E_r|h_r|^2/N_0} \tag{40}$$

which can be re-written as $\mathcal{D}(h) \geq e^{-2\frac{\mathcal{E}}{RN_0}\sum_{r=1}^{R}|h_r|^2}$ with $E_r = \mathcal{E}/R \ \forall r$. Let us define the right-hand side of the inequality as the moment generating function of $|h|^2$ with $t = -2\mathcal{E}/N_0$.

$$\begin{aligned} & M_{|h|^2}(t) = \\ & \prod_{r=1}^{R} (1 + 4\alpha\mathcal{E}/RN_0)^{-L} \exp\left\{-\frac{2(1-\alpha)L\mathcal{E}/RN_0}{1 + 4\alpha\mathcal{E}/RN_0}\right\} \end{aligned} \tag{41}$$

The final form of the lower bound (40) is given in Section IV-A by (36).

REFERENCES

[1] T. Goblick, "Theoretical limitations on the transmission of data from analog sources," *IEEE Transactions on Information Theory*, vol. 11, pp. 558–567, October 1965.

[2] P. Elias, "Networks of Gaussian channels with applications to feedback systems," *IEEE Transactions on Information Theory*, vol. 13, pp. 493–501, July 1967.

[3] M. Gastpar, "To code or not to code," Ph.D. dissertation, EPFL, Dec. 2002.

[4] F. Abdallah and R. Knopp, "Source-channel coding for very-low bandwidth sources," in *Information Theory Workshop, 2008. ITW '08. IEEE*, may 2008, pp. 184 –188.

[5] F. Abdallah, "Source-channel coding techniques applied to wireless networks," Ph.D. dissertation, University of Nice-Sophia Antipolis, Dec. 2008.

[6] J. Wozencraft and I. M. Jacobs, *Principles of Communication Engineering*. Wiley, New York, 1965.

[7] V. Vaishampayan and I. Costa, "Curves on a sphere, shift-map dynamics, and error control for continuous alphabet sources," *IEEE Transactions on Information Theory*, vol. 49, pp. 1658–1672, July 2003.

[8] B. Hochwald and K. Zeger, "Tradeoff between source and channel coding," *IEEE Transactions on Information Theory*, vol. 43, pp. 1412–1424, Sept. 1997.

[9] B. Hochwald, "Tradeoff between source and channel coding on a Gaussian channel," *IEEE Transactions on Information Theory*, vol. 44, pp. 3044–3055, Nov. 1998.

[10] N. Merhav, "On optimum parameter modulation-estimation from a large deviations perspective," *IEEE Transactions on Information Theory*, vol. 58, pp. 7215–7225, December 2012.

[11] ——, "Exponential error bounds on parameter modulation-estimation for discrete memoryless channels," *IEEE Transactions on Information Theory*, vol. 60, pp. 832–841, February 2014.

[12] ——, "Data processing inequalities based on a certain structured class of information measures with application to estimation theory," *IEEE Transactions on Information Theory*, vol. 58, pp. 5287–5301, July 2012.

[13] J. Ziv and M. Zakai, "Some lower bounds on signal parameter estimation," *IEEE Transactions on Information Theory*, vol. 15, pp. 386–391, November 1969.

[14] D. Chazan, M. Zakai, and J. Ziv, "Improved lower bounds on signal parameter estimation," *IEEE Transactions on Information Theory*, vol. 15, pp. 386–391, November 1969.

[15] J. Schalkwijk and L. Bluestein, "Transmission of analog waveforms through channels with feedback," *IEEE Transcations on Information Theory*, vol. 13, pp. 617–619, October 1967.

[16] J. Omura, "Optimum linear transmission of analog data for channels with feedback," *IEEE Transactions on Information Theory*, vol. 14, pp. 38–43, January 1968.

[17] J. Schalkwijk, "A coding scheme for additive noise channels with feedback–II: Band-limited signals," *IEEE Transactions on Information Theory*, vol. 12, pp. 183–189, April 1966.

[18] 3GPP TS 36.213 V11.3.0 (2013-06), "3rd Generation Partnership Project; Technical Specification Radio Access Network Aspects; Evolved Universal Terrestrial Radio Access Network (E-UTRA); Physical Layer Procedures(Release 11)," no. 36.213, June 2013.

[19] H. Yamamoto and K. Itoh, "Asymptotic performance of a modified Schalkwijk-Barron scheme for channels with noiseless feedback," *IEEE Transactions on Information Theory*, vol. 25, pp. 729–733, November 1979.

[20] J. Schalkwijk and M. Barron, "Sequential signaling under a peak power constraint," *IEEE Transactions on Information Theory*, vol. 17, pp. 278–282, May 1971.

[21] 3GPP TS 36.211 V11.3.0 (2013-06), "3rd Generation Partnership Project; Technical Specification Radio Access Network Aspects; Evolved Universal Terrestrial Radio Access Network (E-UTRA); Physical Channels and Modulation(Release 11)," no. 36.211, June 2013.

[22] M. Simon and M.-S. Alouini, "Exponential-type bounds on the generalized Marcum Q-function with application to error probability analysis over fading channels," *IEEE Trans. on Communications*, vol. 48, no. 3, pp. 359 –366, march 2000.

[23] J. Proakis, *Digital Communications*. McGraw-Hill, Third Ed., 1995.

[24] C. E. Shannon, "Probability of error for optimal codes in a Gaussian channel," *The Bell System Technical Journal*, vol. 38, pp. 611–656, May 1959.

[25] M. Steiner, "The strong simplex conjecture is false," *IEEE Transactions on Information Theory*, vol. 40, no. 3, pp. 721–731, 1994.

[26] L. MacColl, "Signalling in the presence of thermal noise, i, ii, and iii," *Bell Laboratories internal memoranda issued May 27, June 30, and September 13*, 1948.

[27] Y. Sun, "Stochastic iterative algorithms for signal set design for Gaussian channels and optimality of the l2 signal set," *IEEE Transactions on Information Theory*, vol. 43, no. 5, pp. 1574–1587, 1997.

[28] R. Scholtz and C. Weber, "Signal design for phase-incoherent communications," *IEEE Transactions on Information Theory*, vol. 12, no. 4, pp. 456–463, 1966.