Topological Interference Management with Transmitter Cooperation

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Abstract—[1] **Interference networks with no channel state information at the transmitter (CSIT) except for the knowledge of the connectivity graph have been recently studied under the topological interference management (TIM) framework. In this paper, we consider a similar topological knowledge but in a distributed broadcast channel setting, i.e. a network where transmitter cooperation is enabled. We show that the interference topology can also be exploited in this case to strictly improve the degrees of freedom (DoF) as long as the network is not fully connected, which is a reasonable assumption in practice. A fractional graph coloring based interference avoidance and a subspace interference alignment approaches are proposed to characterize the symmetric DoF for so-called regular networks with constant interfering degree, and to identify achievable DoF for arbitrary network topologies.**

I. Introduction

This paper considers the degrees-of-freedom (DoF) characterization for wireless networks with interference. The DoF indicates the system throughput scaling with the signal-to-noise ratio (SNR) in the high SNR regime. Although the DoF as a figure of merit has limitations [1], it has proved useful in understanding the fundamental limits of several cooperative communication protocols, such as interference alignment (IA) [2] and multi-cell MIMO [3] among many others. A common feature behind much of the analysis of cooperation benefits in either interference channels (IC) or broadcast channels (BC) has been the availability of instantaneous channel state information at the transmitters (CSIT), with exceptions dealing with so-called limited feedback schemes. Nevertheless, most efforts on limited, imperfect, or delayed feedback settings rely on the assumption that the transmitters are endowed with an instantaneous form of channel information whose coherence time is similar to that of the actual fading channels, so that a good fraction or the totality of the DoF achieved in the perfect CSIT can be obtained. Such an assumption is hard to realize in many practical scenarios, such as cellular networks [4]. Conversely, it has been reported [5] that a substantial DoF cannot be realized in IC or BC scenario without CSIT. A closer examination of these pessimistic results however reveals that many of the considered networks are fully connected, in that any transmitter interferes with any non-intended receiver in the network.

Owing to the nodes' random placement, the fact that power decays fast with distance, the existence of obstacles and local shadowing effects, we may argue that certain interference links are unavoidably much weaker than others, suggesting the use of a partially-connected graph to model, at least approximately, the network topology. An interesting question then arises as to whether the partial connectivity could be leveraged to allow the use of some relaxed form of CSIT while still achieving a substantial DoF performance. In particular the exploitation of topological information, simply indicating which of the interfering links are weak enough to be approximated by zero interference and which links are too strong to do so, is of great practical interest.

This question was addressed in recent works [6]–[11], in the context of the interference channel and X channel [6]–[8], [11] with topology information, and focusing on the symmetric DoF. These different topological interference management (TIM) approaches arrive at a common conclusion that the symmetric DoF can be significantly improved under the sole topology information, *provided the network is partially connected*. In [7], the TIM problem is bridged with the index coding problem, stating that the optimal solution to the latter is the outer bound of the former, and the linear solution to the former is automatically transferrable to the latter.

Given such promising results, a logical question is whether the TIM framework can somehow be exploited in the context of an interference network where a message exchange mechanism between transmitters pre-exists. For instance, in future LTE-A cellular networks, a backhaul routing mechanism ensures that base stations selected to cooperate under the coordinated multi-point (CoMP) framework receive a copy of the messages to be transmitted. Still, the exchange of timely CSI is challenging due to the rapid obsolescence of instantaneous CSI and the latency of backhaul signaling links. In this case, a broadcast channel over distributed transmitters (a.k.a. network MIMO) ensues, with a lack of instantaneous CSIT. The problem raised by this paper concerns the use of topology information in this setting. We follow the same strategy as [6], [7] in targeting the symmetric DoF as a simple figure of merit. By resorting to interference avoidance and alignment techniques, we characterize the achievable and/or optimal symmetric DoF of the distributed BC with topology information in several scenarios of interest.

More specifically, our contributions are as follows:

- We propose an interference avoidance approach built upon distance-2 fractional graph coloring over the clustered line graph corresponding to the original network topology. Based on this, the optimal symmetric DoF of three-cell networks with all possible topologies is determined.
- We propose an interference alignment based approach to identify the achievable symmetric DoF of so-called *regular* networks. Regular networks correspond to topologies with

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constant degrees of interference.

- We show the sufficient conditions in arbitrary networks to achieve a certain amount of symmetric DoF.

II. System Model and Main Results

A. Channel Model

We consider a network with K transmitters (TX), e.g. cells. In each cell the TX (e.g. base station) is equipped with one antenna and serves one single-antenna user (RX). The partial connectivity of the network is modeled through the received signal equation for RX-j at time instant t by:

$$Y_j(t) = \sum_{i \in \mathcal{T}_j} h_{ji}(t) X_i(t) + Z_j(t) \tag{1}$$

where h_{ji} is the channel coefficient between TX-i and RX-j, the transmitted signal $X_i(t)$ is subject to the individual power constraint, i.e., $\mathbb{E}\left(|X_i(t)|^2\right) \leq P_i$, with P_i being transmit power at TX-i, and $Z_j(t)$ is the Gaussian noise with zero-mean and variance N_0 and is independent of transmitted signals and channel coefficients. We denote by $\mathcal{T}_k$ the transmit set containing the indices of transmitters that are *connected* to RX-k, and by $\mathcal{R}_k$ the receive set consisting of the indices of receivers that are *connected* to TX-k, for $k = \{1, 2, \ldots, K\}$. In practice the partial connectivity may be modeled by taking those interference links that are"weak enough" (due to distance and/or shadowing) to zero. For instance in [7], a reasonable model is suggested whereby a link is disconnected if the received signal power falls below the effective noise floor. However other models maybe envisioned and the study of how robust the derived schemes are with respect to modeling errors is an open problem beyond the scope of this paper.

Conforming with TIM framework, the actual channel realizations are not available at the transmitters, yet the network topology (i.e., $\mathcal{T}_k, \mathcal{R}_k, \forall k$) is known by all transmitters and receivers. A typical transmitter cooperation is enabled, where every transmitter is endowed the messages desired by its connected receivers, i.e., the TX-k has access to a subset of messages $W_{\mathcal{R}_k}$, where W_j $(j \in \mathcal{R}_k)$ denotes the message desired by RX-j. We consider a block fading channel, where the channel coefficients stay constant during a coherence time τ_c. The network topology is fixed throughout the communication.

B. Definitions

We treat the cellular network as a bipartite graph, denoted by $\mathcal{G} = (\mathcal{U}, \mathcal{V}, \mathcal{E})$. A few basic definitions pertaining to graph theory [12] are now recalled, while some more definitions specific to this paper will be given in later sections.

Definition 1 (Basic Graph Theoretic Definitions)**.**

- *A* **line graph** *of $\mathcal{G}$ is another graph, denoted by $\mathcal{G}_e$, that represents the adjacencies between edges in $\mathcal{G}$.*
- *The* **fractional coloring** *refers to assigning each vertex with m colors drawing from a palette of n colors, such that any two adjacent vertices have no colors in common. The* **fractional chromatic number** *$\chi_f(\mathcal{G})$ is the minimum value of $\frac{n}{m}$ among all possible fractional coloring.*
- *A (K, d)-***regular bipartite graph** *$\mathcal{G} = (\mathcal{U}, \mathcal{V}, \mathcal{E})$ is such that $|\mathcal{U}| = |\mathcal{V}| = K$ and $|\mathcal{T}_k| = |\mathcal{R}_k| = d, \ \forall\ k$.*
- *A* **Hamiltonian cycle** *for a graph is a cycle that visits all vertices exactly once.*
- *A* **matching** *of the graph is a set of edges with no common vertices between any two edges. A* **perfect matching** *is a matching contains all vertices.*

In this work, we follow the strategy of [6]–[11] and set the symmetric DoF (i.e., the DoF which can be achieved by all users simultaneously) as our main figure of merit.

$$d_{\text{sym}} \triangleq \limsup_{P \to \infty} \sup_{(R_{\text{sym}}, \ldots, R_{\text{sym}}) \in \mathcal{C}} \frac{R_{\text{sym}}}{\log P} \tag{2}$$

where $\mathcal{C}$ is the set of all achievable rate tuples.

C. Main Results

In what follows, we will present an interference avoidance and an interference alignment approaches, by which we obtain achievable symmetric DoF of a set of network topologies and characterize the optimal symmetric DoF of the three-cell networks and the cyclic Wyner-type networks.

Theorem 1 (Achievable DoF of General Networks)**.** *For the TIM problem with transmitter cooperation, the symmetric DoF*

$$d_{\text{sym}} = \frac{1}{\chi_f^2(\mathcal{G}_e)} \tag{3}$$

can be achieved by an interference avoidance approach built upon distance-2 fractional graph coloring, where

- *$\mathcal{G}_e$: the line graph of $\mathcal{G}$, representing the adjacencies of the edges in $\mathcal{G}$;*
- *χ_f^2: fractional chromatic number corresponding to the distance-2 fractional clustered-graph coloring.*

Proof: See Section III-A for a sketch of proof building on an illustrative example and [13] for the general proof. ■

Corollary 1 (Optimal DoF of Three-cell Networks)**.** *The optimal symmetric DoF of the three-cell TIM problem with transmitter cooperation can be achieved by interference avoidance.*

Proof: The optimality is established by checking all $K = 3$ topologies. The achievability is due to the distance-2 fractional graph coloring based interference avoidance approach, and the outer bounds are based on the concept of generators [8]. Due to the lack of space, the full proof is shown in [13]. ■

Definition 2 (Reference Graph)**.** *A reference (K, d)-regular bipartite graph $\mathcal{G}_r = (\mathcal{U}_r, \mathcal{V}_r, \mathcal{E}_r)$ is characterized by*

$$\mathcal{T}_j = \{j, j+1, \ldots, j+d-1\}. \tag{4}$$

with indices modulo K. A graph $\mathcal{G} = (\mathcal{U}, \mathcal{V}, \mathcal{E})$ is said to be similar to this reference graph, denoted as $\mathcal{G} \simeq \mathcal{G}_r$, if $\mathcal{U}$ and $\mathcal{V}$ in $\mathcal{G}$ can be obtained by reordering the vertices of $\mathcal{U}_r$ and $\mathcal{V}_r$ in $\mathcal{G}_r$, respectively.

Theorem 2 (Achievable DoF of Regular Networks)**.** *For a K-cell regular network representable by a (K, d)-regular bipartite* *graph $\mathcal{G}$, as long as it is similar to the reference one $\mathcal{G}_r$, i.e., $\mathcal{G} \simeq \mathcal{G}_r$, the symmetric DoF*

$$d_{\rm sym}(K,d) = \begin{cases} \frac{2}{d+1}, & d \le K-1 \\ \frac{1}{K}, & d = K \end{cases} \tag{5}$$

can be achieved by an interference alignment approach, when the channel coherence time satisfies $\tau_c \ge d+1$.

Proof: See Section III-B for an illustrative example and Appendix for the general proof. ■

Corollary 2 (Optimal DoF of Cyclic Wyner-type Networks)**.** *For a cyclic Wyner-type network represented by a $(K,2)$-regular bipartite graph, the optimal symmetric DoF of the TIM problem with transmitter cooperation is*

$$d_{\rm sym}(K,2) = \begin{cases} \frac{1}{2}, & K = 2 \\ \frac{2}{3}, & K \ge 3 \end{cases} \tag{6}$$

given that the coherence time $\tau_c \ge 3$.

Proof: The achievability can be obtained from the general (K,d) case by setting $d=2$, and the outer bounds are derived via compound settings [6] and relegated to [13]. ■

In addition to the regular networks, we also identify the sufficient conditions for arbitrary network to achieve a certain amount of symmetric DoF. We start with two definitions.

Definition 3. *The **Alignment-Feasible Graph (AFG)**, denoted by $\mathcal{G}_{AFG}$, refers to a graph with vertices representing the messages and with edges between two messages indicating if they are alignment-feasible. The two messages W_i and W_j ($\forall\ i \neq j$) are said to be alignment-feasible if*

$$\mathcal{T}_i \nsubseteq \mathcal{T}_j, \quad \text{and} \quad \mathcal{T}_j \nsubseteq \mathcal{T}_i. \tag{7}$$

Definition 4. *A partition of $\mathcal{K} \triangleq \{1,2,\dots,K\}$, $\mathcal{K} = \{\mathcal{P}_1, \mathcal{P}_2, \dots, \mathcal{P}_\kappa\}$, is called a **Proper Partition**, if, for every portion $\mathcal{P}_i = \{i_1, i_2, \dots, i_{p_i}\}$ with $p_i \triangleq |\mathcal{P}_i|$ ($i = 1,\dots,\kappa$),*

$$\mathcal{T}_{i_k} \bigcap \left(\bigcup_{i_j \in \mathcal{P}_i \setminus i_k} \mathcal{T}_{i_j} \right)^c \neq \emptyset, \quad \forall\ i_k \in \mathcal{P}_i, \tag{8}$$

where $\mathcal{T}^c$ is the complementary set of $\mathcal{T}$. Messages with indices in the same portion can be aligned in the same subspace.

Theorem 3 (Achievable DoF of Arbitrary Networks)**.** *For a K-cell network with arbitrary topologies, the following symmetric DoF is achievable*

- $d_{\rm sym} = \frac{2}{K}$, *if there exists a Hamiltonian cycle or a perfect matching in $\mathcal{G}_{AFG}$;*
- $d_{\rm sym} = \frac{1}{\kappa}$, *if there exists a proper partition with size κ.*

Proof: The achievability proofs are based on the interference alignment and are relegated to [13]. ■

III. Schemes and Illustrative Examples

Due to the page limit, we present here some illustrative examples and sketches of proofs. More results and the general proofs are relegated to [13].

A. Interference Avoidance Approach

We focus on the network topology studied in [8], as shown in Fig. 1, but with a key difference that message sharing across transmitters is enabled. As in [7], [8], [10], the optimal symmetric DoF is pessimistically $\frac{1}{3}$ without message sharing. In contrast, if transmitter cooperation is allowed, the symmetric DoF can be remarkably improved to $\frac{2}{5}$ even with a simple interference avoidance scheme according to Theorem 1.

Without message sharing, the interference avoidance scheme consists in scheduling transmitters to avoid mutual interferences. For instance, by delivering W_1, TX-1 will cause interferences to RX-2, 3, and consequently TX-2, 3 should be deactivated, because W_2, W_3 cannot be delivered free of interference. In contrast, with message sharing, the desired message W_1 can be sent either from TX-1 or TX-4. Hence, scheduling can be done across links rather than across transmitters. For instance, if the link TX-4 $\to$ RX-1 (denoted by e_{41}) is scheduled, the links adjacent to e_{41} (i.e., e_{11}, e_{42}, and e_{44}) as well as the links adjacent to e_{11}, e_{42} and e_{44} (i.e., e_{12}, e_{13}, e_{22}, e_{32}, e_{34} and e_{54}) should not be scheduled, because activating TX-1 will interfere RX-1 and RX-2, 4 will overhear interferences from TX-4 such that any delivery from TX-1 or to RX-2, 4 causes mutual interferences. A possible link scheduling associated with Fig. 1 is shown in Table I. It can be found that each message is able to be independently delivered twice during five time slots, and hence symmetric DoF of $\frac{2}{5}$ is achievable.

TABLE I: Link Scheduling

Slot	Scheduled Links (e_{ij}: TX-i $\to$ RX-j)	Delivered Messages
A	e_{41}, e_{55}, e_{66}	W_1, W_5, W_6
B	e_{12}, e_{54}, e_{66}	W_2, W_4, W_6
C	e_{13}, e_{54}	W_3, W_4
D	e_{41}, e_{33}	W_1, W_3
E	e_{12}, e_{55}	W_2, W_5

1) Reinterpretation as a Graph Coloring Problem: Although the above link scheduling solution provides an achievable scheme for the example in Fig. 1, the generalization is best undertaken by reinterpreting the link scheduling into a graph coloring problem, such that the rich graph theoretic toolboxes can be directly utilized to solve our problem. The nature of our problem calls for a distance-2 fractional clustered-graph coloring scheme, which consists of the following ingredients:

- Distance-2 fractional coloring: Both the adjacent links and the adjacency of the adjacent links (resp. edges with distance less than 2) should be scheduled in difference time slots (resp. assigned with different colors).
- Clustered-graph coloring: Only the total number of messages delivered by links with the common receiver (resp. colors assigned to the edges with the same vertex) matters. Thus, the number of assigned colors should be counted by clusters where the edges with common vertices are grouped together.

In what follows, we reinterpret the link scheduling as a distance-2 fractional graph coloring. To ease presentation, we transform graph edge-coloring into graph vertex-coloring of the line graph. As shown in Fig. 1, we first transform the

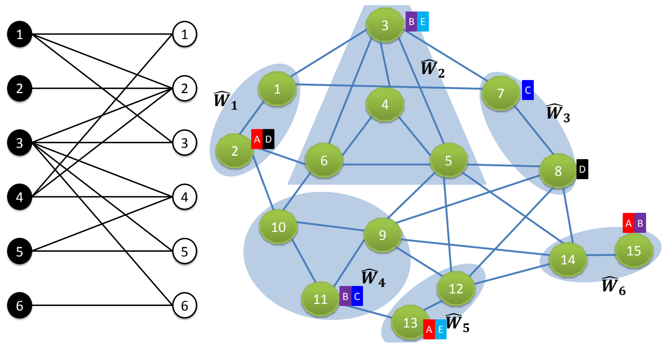

Fig. 1: A 6-cell network. On the left is the network topology graph $\mathcal{G}$, and on the right is its line graph $\mathcal{G}_e$.

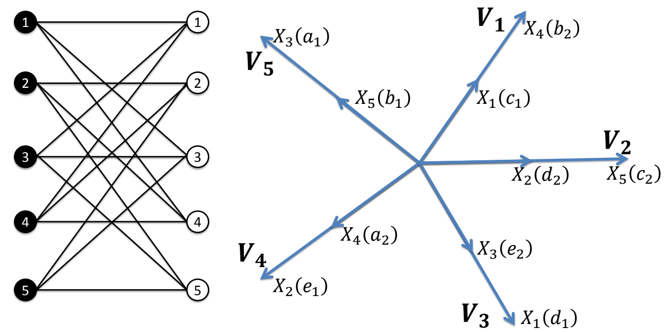

Fig. 2: A $(5,3)$-regular cellular network. On the right is the illustration of the interference alignment scheme.

topology graph $\mathcal{G}$ (left) into its line graph $\mathcal{G}_e$ (right) and map the links connected to each RX in $\mathcal{G}$ to the vertices in $\mathcal{G}_e$. For instance, the four links to RX-2 in $\mathcal{G}$ are mapped to Vertices 3,4,5,6 in $\mathcal{G}_e$. Then, we group relevant vertices in $\mathcal{G}_e$ as clusters, e.g., Vertices $3,4,5,6$ in $\mathcal{G}_e$ corresponding to the links to RX-2 are grouped as one cluster. By now, a clustered-graph is generated. The graph coloring can be performed as follows. For instance, if Vertex 2 in $\mathcal{G}_e$ receives a color indicated by 'A', then Vertices 13 and 15 can receive the same color, because the distance between any two of them is no less than 2. Try any possible coloring assignment until we obtain a proper one, where each cluster receives m distinct colors out of total n ones, such that any two vertices with distance less than 2 receive distinct colors. There may exist many proper coloring assignments. The fractional chromatic number $\chi_f^2(\mathcal{G}_e)$ refers to the minimum of $\frac{n}{m}$ among all proper coloring assignments. In this example, we have $m=2$ and $n=5$. The vertices (i.e., links in $\mathcal{G}$) with the same color can be scheduled in the same time slot. Accordingly, each cluster receives two out of five colors means every message is scheduled twice during five time slots, yielding the symmetric DoF of $\frac{2}{5}$. According to the connection between link scheduling and graph coloring, the inverse of the fractional chromatic number, i.e., $\frac{1}{\chi_f^2(\mathcal{G}_e)}$, can serve as an inner bound of symmetric DoF of the general cellular networks, although its computation is NP-hard.

B. Interference Alignment Approach

Let us consider the $(5,3)$-regular network shown in Fig. 2. By enabling transmitter cooperation, the symmetric DoF is improved from $\frac{2}{5}$ (as reported in [6]) to $\frac{1}{2}$ according to Theorem 2. In what follows, we will show an interference alignment scheme to achieve this.

By the network topology, we have transmit and receive sets $\mathcal{T}_1=\mathcal{R}_1=\{1,3,4\}, \mathcal{T}_2=\mathcal{R}_2=\{2,4,5\}, \mathcal{T}_3=\mathcal{R}_3=\{1,3,5\}, \mathcal{T}_4=\mathcal{R}_4=\{1,2,4\}, \mathcal{T}_5=\mathcal{R}_5=\{2,3,5\}$. For notational convenience, we denote by a,b,c,d,e the messages desired by five receivers, with the subscript distinguishing different symbols for the same receiver. Given five random vectors $\boldsymbol{V}_1,\boldsymbol{V}_2,\boldsymbol{V}_3,\boldsymbol{V}_4,\boldsymbol{V}_5 \in \mathbb{C}^{4\times 1}$, any four of which are linearly independent, the transmitters send signals with precoding

$$\boldsymbol{X}_1=\boldsymbol{V}_1c_1+\boldsymbol{V}_3d_1, \quad \boldsymbol{X}_2=\boldsymbol{V}_2d_2+\boldsymbol{V}_4e_1 \tag{9}$$

$$\boldsymbol{X}_3=\boldsymbol{V}_5a_1+\boldsymbol{V}_3e_2, \quad \boldsymbol{X}_4=\boldsymbol{V}_4a_2+\boldsymbol{V}_1b_2 \tag{10}$$

$$\boldsymbol{X}_5=\boldsymbol{V}_5b_1+\boldsymbol{V}_2c_2 \tag{11}$$

within four time slots, where $\boldsymbol{X}_i \in \mathbb{C}^{4\times 1}$ is the vector of the concatenated transmit signals from TX-i.

Assuming the coherence time $\tau_c \geq 4$, the received signal at RX-1 for example within four time slots, with $\mathcal{T}_1=\{1,3,4\}$, can be written as

$$\begin{aligned} \boldsymbol{Y}_1 &= \sum_{i\in\mathcal{T}_1} h_{1i}\boldsymbol{X}_i+\boldsymbol{Z}_1 \qquad (12)\\ &= h_{11}\boldsymbol{X}_1+h_{13}\boldsymbol{X}_3+h_{14}\boldsymbol{X}_4+\boldsymbol{Z}_1 \qquad (13)\\ &= \underbrace{h_{13}\boldsymbol{V}_5a_1+h_{14}\boldsymbol{V}_4a_2}_{\text{desired signal}} \\ &\quad + \underbrace{\boldsymbol{V}_1(h_{11}c_1+h_{14}b_2)+\boldsymbol{V}_3(h_{11}d_1+h_{13}e_2)}_{\text{aligned interferences}}+\boldsymbol{Z}_1. \qquad (14) \end{aligned}$$

Recall that $\{\boldsymbol{V}_i, i=1,\ldots,5\}$ are 4×1 linearly independent vectors spanning four-dimensional space, by which it follows that the interferences are aligned in the two-dimensional subspace spanned by $\boldsymbol{V}_1$ and $\boldsymbol{V}_3$, leaving two-dimensional interference-free subspace spanned by $\boldsymbol{V}_4$ and $\boldsymbol{V}_5$ to the desired symbols a_1,a_2. Hence, the desired messages of RX-1 can be successfully recovered, almost surely. Applying this to all other receivers, all receivers can decode two messages within four time slots, yielding the symmetric DoF of $\frac{1}{2}$.

To illustrate the interference alignment, we describe the transmitted signals geometrically as shown in Fig. 2. In this figure, we depict the subspace spanned by $\{\boldsymbol{V}_i, i=1,\ldots,5\}$ as a four-dimensional space, where any four of them are sufficient to represent this space. We also denote the message for example W_j sent from TX-i by $X_i(W_j)$. Let us still take RX-1 for example. Because of $\mathcal{T}_1=\{1,3,4\}$, the transmitted signals from the transmitters that do not belong to $\mathcal{T}_1$ will not reach RX-1, and hence the vector $\boldsymbol{V}_2$ is disappeared. In addition, we have the interference-free signals in the directions of $\boldsymbol{V}_4$ and $\boldsymbol{V}_5$, and the aligned interferences carrying messages other than a_1,a_2 in the subspace spanned by $\boldsymbol{V}_1$ and $\boldsymbol{V}_3$. Recall that vectors $\{\boldsymbol{V}_1,\boldsymbol{V}_3,\boldsymbol{V}_4,\boldsymbol{V}_5\}$ are linearly independent, almost surely. As such, the interference alignment is feasible at RX-1, and also it can be checked to be feasible at all other receivers.

IV. Conclusion

The topological interference management (TIM) problem with transmitter cooperation is considered in this work. A fractional graph coloring based interference avoidance and an interference alignment approaches have been proposed to exploit the benefits of both topological knowledge and transmitter cooperation, with which the achievable symmetric DoF are identified for a class of network topologies.

APPENDIX: PROOF OF THEOREM 2

For the transmit sets $\{\mathcal{T}_j, j = 1, \ldots, K\}$, we have $|\mathcal{T}_j| = d$. As we know, when $d = K$, the network is fully connected and therefore the optimal symmetric DoF is $\frac{1}{K}$ by time division. So, in what follows, we will consider the general achievability proof when $d \leq K - 1$.

Since the cellular network graph is assumed to be similar to the reference one by reordering the transmitters and/or receivers, we directly consider the reference one, because they are equivalent in terms of symmetric DoF with transmitter cooperation. For the reference network topology, the transmit set of RX-j is given by

$$\mathcal{T}_j = \{j, j+1, \ldots, j+d-1\}. \tag{15}$$

Note here that all the receiver indices are modulo K, e.g., $j - K = j$ and $0 = K$. Thus, at TX-i we send symbols with careful design

$$\boldsymbol{X}_i = \boldsymbol{V}_{i+1} X_i(W_i^1) + \boldsymbol{V}_{i+2} X_i(W_{i-d+1}^2), \forall\ i = 1, \ldots, K$$

where $X_i(W_j)$ denotes the signal sent from TX-i carrying the message W_j, W_j^1 and W_j^2 are two realizations (symbols) of message W_j, and $\{\boldsymbol{V}_i,\ i = 1, \ldots, K\}$ are $(d+1) \times 1$ random vectors and linearly independent among any $d+1$ vectors, almost surely. The received signals at RX-j during $d+1$ time slots, with $\tau_c \geq d+1$, can be given in a compact form as

$$\begin{aligned}
\boldsymbol{Y}_j &= \sum_{i \in \mathcal{T}_j} h_{ji} \boldsymbol{X}_i + \boldsymbol{Z}_j \\
&= \sum_{i=j}^{j+d-1} h_{ji}(\boldsymbol{V}_{i+1} X_i(W_i^1) + \boldsymbol{V}_{i+2} X_i(W_{i-d+1}^2)) + \boldsymbol{Z}_j \\
&= h_{j,j} \boldsymbol{V}_{j+1} X_j(W_j^1) + h_{j,j+d-1} \boldsymbol{V}_{j+d+1} X_{j+d-1}(W_j^2) \\
&\quad + \sum_{i=j+1}^{j+d-1} h_{ji} \boldsymbol{V}_{i+1} X_i(W_i^1) + \sum_{i=j}^{j+d-2} h_{ji} \boldsymbol{V}_{i+2} X_i(W_{i-d+1}^2) + \boldsymbol{Z}_j \\
&= \underbrace{h_{j,j} \boldsymbol{V}_{j+1} X_j(W_j^1) + h_{j,j+d-1} \boldsymbol{V}_{j+d+1} X_{j+d-1}(W_j^2)}_{\text{desired signal}} \\
&\quad \underbrace{+ \sum_{i=j}^{j+d-2} \boldsymbol{V}_{i+2}(h_{j,i+1} X_{i+1}(W_{i+1}^1) + h_{j,i} X_i(W_{i-d+1}^2))}_{\text{aligned interferences}} + \boldsymbol{Z}_j.
\end{aligned}$$

It is shown that the interferences occupy $d - 1$ dimensional subspace out of the total $d + 1$ dimensional space, leaving 2-dimensional interference-free subspace spanned by $\{\boldsymbol{V}_{j+1}, \boldsymbol{V}_{j+d+1}\}$ to the desired signals, such that the desired messages for RX-j, W_j^1 and W_j^2, can be successfully recovered. This philosophy applies to all other receivers. During $d+1$ time slots, every receiver can decode two messages, yielding symmetric DoF of $\frac{2}{d+1}$.

The concept of interference alignment can be illustrated in Fig. 3, and also interpreted as follows. Transmitted signals

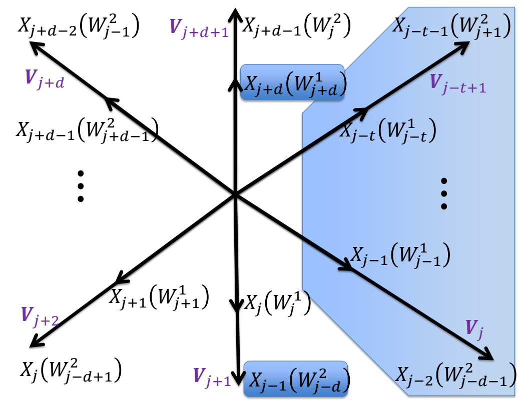

Fig. 3: Interference alignment for the general (K, d) regular cellular networks.

$X_{j-1}(W_{j-1}^1)$ and $X_{j-2}(W_{j-d-1}^2)$ are aligned in the same subspace spanned by vector $\boldsymbol{V}_j$, which is absent to RX-k ($\forall\ k \in \{j, \ldots, j+K-3\}$). Note that $t \triangleq K - d - 1$ and $j - t = j + d + 1$ modulo K. By deduction, the subspace spanned by $\{\boldsymbol{V}_{j+d+2}, \ldots, \boldsymbol{V}_j\}$ (i.e., the shadow in Fig. 3) are absent to RX-j, leaving $d+1$ linearly independent vectors $\{\boldsymbol{V}_{j+1}, \ldots, \boldsymbol{V}_{j+d+1}\}$ to span the space. In addition, as $X_j(W_j^1)$ and $X_{j+d-1}(W_j^2)$ are aligned with $X_{j-1}(W_{j-d}^2)$ and $X_{j+d}(W_{j+d}^1)$ in the subspace spanned by $\boldsymbol{V}_{j+1}$ and $\boldsymbol{V}_{j+d+1}$, respectively, and the signals from TX-$(j-1)$ and TX-$(j+d)$ cannot be heard by RX-j according to the network topology, it follows that $X_j(W_j^1)$ and $X_{j+d-1}(W_j^2)$ are free of interference, and retrievable from overall $d+1$ dimensional subspace.

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