# Adaptive Beamforming in Mobile, Massively Multiuser Satellite Communications: A System Perspective

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***Abstract*—We consider a Mobile Satellite System (MSS) supporting a very large number of beams and providing service to a massive number of Mobile Satellite Terminals (MST). We identify the challenges posed by the design of such a system and address them. More specifically, we propose algorithms to (a) design adaptive beamformers at the gateway and receivers at the MSTs; (b) acquire knowledge on the channel directivity; (c) allocate frequency bands or carriers; and (d) design the Random Access Channel (RACh). Thus, we verify the system feasibility and assess its performance against conventional satellite systems (SS). Simulations shows that the retained selection of algorithms allows to serve simultaneously with the required quality of service (QoS) about four times the number of MSTs served by a conventional system while halving the transmitted power. In general, the proposed system greatly outperforms the conventional one.**

## I. Introduction

There is an increasing interest in developing satellite systems capable to provide high-speed interactive services to a massive number of mobile satellite terminals (MSTs) at a very competitive price, both in terms of equipments and services. From a system point of view, this requires a flexible deployment of a very large number of beams per frequency band (up to the maximum of the antenna resolution) and an intensive exploitation of the available frequency spectrum.

In terrestrial communications, high throughput mass-market communications are enabled by full spatial reuse of the frequency resources across even neighboring cells (nowadays a *de facto* standard approach) and by advanced techniques for inter-cell and intra-cell interference management.

In this contribution, we adapt the concept of full frequency reuse and intensive spatial exploitation of the available spectrum to mobile satellite systems (MSS) by introducing the adaptive beamforming approaches (also known in terrestrial networks as space division multiple access, shortly SDMA) and exploring its feasibility in MSSs.

In fixed SSs, the idea to densify the number of beams and reduce interbeam interference by multiuser detection techniques in the reverse link and precoding techniques in the forward link was introduced in [1] and [2] in the framework of the project [3] and further developed and improved in [4]. In that architecture, fixed dense beams are clustered together and the transmit or the received signals are jointly processed at the gateway. An overview of the techniques related to such network architecture is presented in [5]. Interestingly, that architecture maps the concept of Cooperative MultiPoint (CoMP) systems in terrestrial communications. More specifically, beams correspond to terrestrial cells and a cluster of them to a CoMP. Instead, in adaptive beamforming for MSSs, the satellite/gateway pair maps onto a single enormous cell capable to generate a very large number of beams.

The possibility to utilize the flexibility of digital Beam-Forming Networks (BFN) for the design of satellite systems (SS) based on adaptive beamforming was explored in [6] and [7]. There, a very restricted coverage area (about 1000km wide) with a limited number of fixed users was considered. In [8], the issue of the complexity of the beamformer design for a SS with a very large number of radiating sources at the satellite antenna (SA) is tackled and a low complexity algorithm with fast convergence was proposed.

Compared to the use of fixed beams, designed to cover with their footprints fixed geographical locations and guarantee a target QoS in any location within the beam footprint, the benefits obtained by tailoring beams to the STs' locations are apparent both in terms of energy consumption and interference avoidance. However, the peculiarities of the satellite channel and the global MSS determine very specific prerequisites and challenges in the design and feasibility study of a MSS based on adaptive beamforming.

Objective of our work is to identify and address such challenges, verify the system feasibility, assess its performance against conventional SSs.

Our work is based on the following general qualitative assumptions. The MSS consists of a very large number of beams and MSTs equipped with multiple antennas. The MSTs are mobile phones or laptops of small size equipped with very small antennas. The system is based on bent-pipe satellites and modulation/demodulation and coding/decoding are performed at the gateway or at the MSTs. The satellite supports transmissions at high power, it is equipped with an antenna of large size to compensate for the small antenna size at the MSTs, and it supports a large number of beams. An additional key feature, supported by recent developments in hardware, is the possibility to update the weights of the digital BFN each few seconds. Concerning the mobility aspects, the underlying assumption in our system design is that the movements of the MSTs in the time interval between two successive updates of the digital BFN are smaller than the radius of the beam footprint on the ground.

Conventional SSs with fixed beams do not require any knowledge of the links between satellite and STs. However, the studies in [3] have already pointed out that such a information is critical to attain effective interference cancelation in the forward link by precoding techniques, as discussed in [5]. In the case of a MSS with adaptive beamforming, the channel state information (CSI) between satellite and STs is essential for beamforming design. Compared to terrestrial system, in SSs the issues of both CSI acquisition and beamforming design is further exacerbated by the fact that the channel coherence time is too short compared to the propagation delay. Thus, instantaneous CSI feedback to the gateway is stale and cannot be utilized for the design of tailored beams. We model the link as a cascade, i.e. analytically a multiplication, of a directivity vector and propagation components. The directivity vector represents the direction of the line of sight (LOS) depending on the ST's position and changes slowly depending on the ST's movements. The propagation coefficients model the fast fading components accounting for shadowing and atmospheric losses. Then, in this work we propose a heuristic algorithm for the design of a beamformer based solely on the estimates of the directivity vectors and on the statistical knowledge of the propagation coefficients. The design criterion is based on the minimization of the transmitted total power under target QoS constraints. We resort to the Parametric Least Squared Estimation (PLSE) in [9] for the estimation of the directivity components at the gateway.

As well known, the performance of a SDMA system strongly benefits from an opportunistic allocation of the spectral resources to the users and a careful exploitation of user diversity. In contrast to other multiple access schemes such as time division and frequency division multiple accesses (TDMA and FDMA), in SDMA an optimal allocation of the frequency bands to each user requires performance analysis of the system under any possible allocation (exponential complexity in the number of STs) which, in turn, requires typically beamforming design (see e.g. [10]). In [10], greedy algorithms for a large variety of SDMA systems are proposed. In the MSS considered here, characterized by a massive number of beams and MSTs, even greedy algorithms requiring beamformer performance analysis are computationally too intensive to be feasible. Then, we propose here low complexity spectral resource allocation algorithms based on metrics depending only on the crosscorrelations between directivity vectors.

An additional crucial aspect in the design of the proposed MSS is the design of a RACh capable to support the high load of service requests expected in a mass-market interactive MSS and the particular constraints of an adaptive beamforming MSS. In contrast to conventional systems, from the previous choices on the design of an adaptive-beamforming-based MSS, the RACh should be designed to perform an initial estimation of the directivity components with a level of accuracy able to guarantee a successful introduction of a ST in the system with an appropriate allocation of the frequency band and an accurate design of the corresponding tailored beam. In this contribution, we adopt the RACh design proposed in [11].

Objective of this work is to propose practical solutions to the above mentioned challenges and assess the performance of the global MSS based on adaptive beamforming in terms of maximum number of MSTs that can be simultaneously served by the the system with a given QoS when a certain probability of outage is tolerated. Such a metric is used to compare the proposed MSS to a conventional SS based on fixed beams. Numerical simulations show that the selection of algorithms proposed here to (a) design the beamformer at the gateway and the receivers at the STs; (b) acquire knowledge on the directivity components; (c) allocate frequency bands; (d) design the RACh component; allow to serve simultaneously four times the number of STs served by a conventional system while halving the transmitted power. In general, the proposed system greatly outperforms the conventional one.

The paper is organized as follows. Section II describe the system. It is structured in two parts, the first one describes the system at a network level and the user mobility and the second defines the physical model of the system. Section III focuses on the description of the algorithms adopted to answer the challenges of MSS based on adaptive beamforming. Section IV describes the design of the conventional system with fixed beams utilized as benchmark in the simulations. Section V presents and discusses the performance of the global system obtained by numerical simulations. Finally, Section VI draws the conclusions.

## II. System Description

### A. Communication Architecture

The communication framework consists of two independent parts, the RACh where new service requests are processed and the data transmission part supporting connection oriented communications both in the forward and reverse link. Figure 1 illustrates the system architecture. New incoming service requests or calls are presented to the gateway on the frequency bands assigned to the RACh. In this system component, the incoming requests are detected, eventual contentions are possibly resolved, and the directivity vectors of the incoming STs are estimated. Such estimation is crucial to accept the request and assign an initial frequency band to STs. Otherwise, outage events occur. If an initial frequency band can be associated to a service request, a connection for data transmission can be established both in the forward and reverse link. Based on the directivity estimation and the allocated frequency band, an adaptive beamforming including beams tailored to new incoming users are designed and data are transmitted from the gateway to the final users. Similarly, frequency bands are allocated also in the reverse link and communications can take place to support both data transmission and directivity vector estimation. The periodical updates of the directivity vectors are utilized to manage STs' mobility by updating the beamformer and eventually reallocating frequency bands.

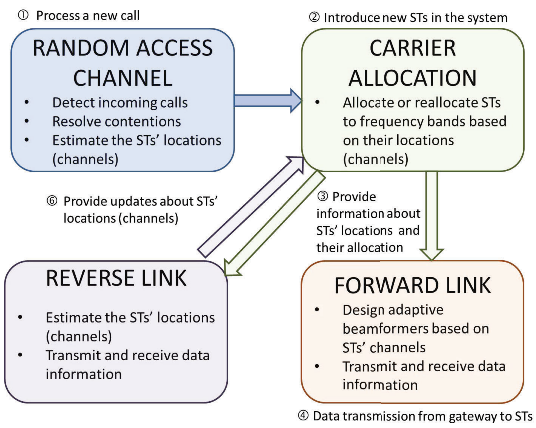


Figure 1. System Architecture

### B. System Model at Network Level

Both in the RACh and in the connection oriented communications for data transmission in the reverse link we assume synchronous access. The time dimension is organized in frames and each frame is structured in time slots. The frame duration is directly related to the frequency of the digital BFN updates. The slot duration is significantly longer than the coherence time of the link between satellite and MSTs. In the slotted synchronized RACh, as in slotted ALOHA, packet transmissions for new service requests start at the beginning of each time slot. Frequency band allocation for new MSTs entering in the system and reallocation for the existing ones, as well as beamforming updates, are performed at the end of each frame.

The MSTs arrive in the system according to a Poisson process and are homogeneously distributed over all the coverage area of the system. The MSTs' positions are updated assuming that the MSTs move from their last position in a random direction uniformly distributed with a random speed uniformly selected in the range $[v_{\mathrm{MIN}}, v_{\mathrm{MAX}}]$.

### C. System Model at Physical Layer

The modeling of a multi-antenna satellite system channels with ST mobility is currently object of intense research. An overview of the ongoing studies and recent results about the channel modeling can be found in [5].

Relevant for the definition of our channel model where the measurement campaign carried out by ESA in [12], University of Surrey [13], and CNES in [14] and more recent campaigns sponsored by CNES. Although reference channel models for the targeted scenarios are not standardized yet, we adopted a model defined with CNES which captures the main features of satellite channels while keeping a reasonable level of complexity. In the following, we describe it jointly with the system whole communications model.

We consider a satellite system consisting of a gateway, a bent-pipe satellite equipped with $N$ antennas and $K$ STs. Each MST is endowed with $R = 2$ antennas. In current satellite systems, the channel for a single carrier can be modeled as flat fading. In the following we focus on a single carrier. The baseband received signal $\mathbf{y}^f$ in the forward link is given by

$$\mathbf{y}^f = \mathbf{H}^f(\mathbf{F}\mathbf{x}^f + \mathbf{e}^f) + \mathbf{n}^f \tag{1}$$

where $\mathbf{x}^f$ is the $2K \times 1$ vector of transmitted symbols such that $\mathrm{E}[\mathbf{x}^{f\dagger}\mathbf{x}^f] = \mathbf{I}$, (for $R = 2$, two information streams are transmitted to each MST), $\mathbf{H}^f$ denotes the $2KR \times 2N$ transfer matrix of the channel between the satellite and the $K$ MSTs. $\mathbf{e}^f$ is the $2N \times 1$ vector of intermodulation noise, $\mathbf{n}^f$ is the $2KR\times$ 1 column vector of zero mean additive Gaussian noise with variance $\sigma_n^2$ and $\mathbf{F}^f$ is the $2N \times 2K$ beamformer matrix.

Thanks to the assumption of massively loaded MSS, the intermodulation noise is modeled here as a white Gaussian noise with variance $\sigma_e^2 = \frac{10^{CIM/10}}{N}\mathrm{E}[\mathbf{x}^{f\dagger}\mathbf{x}^f]$, proportional to the transmitted signal power. Here, CIM is the average signal to intermodulation noise, which we typically set to $\mathrm{CIM} = 15$ dB. The forward link channel matrix is modeled as,

$$\mathbf{H}^f = \mathbf{C}^f\mathbf{P}^f\mathbf{D}^f \tag{2}$$

where $\mathbf{C}^f$ is a block diagonal matrix with $K$ identical blocks,

$$\mathbf{C}_k^f = \begin{pmatrix} \mathbf{C}(1,b,0) & \mathbf{C}(a,b,1) \\ \mathbf{C}(a,b,1) & \mathbf{C}(1,b,0) \end{pmatrix} \tag{3}$$

and,

$$\mathbf{C}(a,b,\delta) = \begin{pmatrix} a\sqrt{1-\varepsilon} & ab & ab\delta & a\delta\sqrt{1-\varepsilon} \\ a\delta\sqrt{1-\varepsilon} & ab\delta & ab & a\sqrt{1-\varepsilon} \end{pmatrix} \tag{4}$$

$a = [0, -22]$ dB, and $b = 15$ dB. It describes the antenna correlation in left and right polarization at the MST's receive antennas. Note that $\varepsilon$ should be normalized such that $(1 + 2a^2)\sqrt{1-\varepsilon} + b^2 + 2a^2b^2 = 1$.

The $4KR \times 2K$ matrix $\mathbf{P}^f$ represents the propagation matrix, is block diagonal with $K$ blocks of dimensions $4R\times 2$, and accounts for the propagation losses between the SAs and MSTs. Its structure is detailed in [9]. Each propagation block $\mathbf{P}_k^f$, $k = 1, \ldots K$, is independent from the others and is generated by a simulator based on the model in [13] and offered by CNES.

The matrix $\mathbf{D}^f$ represents the $2K \times 2N$ directivity matrix whose $2 \times 2N$ block rows (directivity vectors) describe the propagation of the signal between $N$ SAs and each MSTs in deep space. The factor 2 appearing in the dimensions accounts for left and right polarization at the transmitting antennas and co- and cross-polarization at the receive antennas. Each block depends on the radiation pattern at the satellite and the MTSs' positions. The directivity matrix changes slowly due to the assumption of relatively slow speed of the MSTs. Due to space limitation, the reader is referred to [9] for detailed description for the directivity vectors and their generation. The radiation of a satellite covering Europe and utilized in this work was provided by CNES.

When we focus on the receive signal of ST $k$, we can write (1) as

$$\mathbf{y}_k^f = \mathbf{H}_k^f\mathbf{F}\mathbf{x}^f + \mathbf{H}_k^f\mathbf{e}^f + \mathbf{n}_k^f \tag{5}$$

$$= \mathbf{C}_k^f\mathbf{P}_k^f\mathbf{D}_k^f\mathbf{F}\mathbf{x}^f + \mathbf{z}_k^f \tag{6}$$

where $\mathbf{z}_k^f$ is an equivalent additive colored noise with zero mean and covariance matrix $\mathbf{C}_k^z = \sigma_n^2\mathbf{I} + \mathbf{C}_k^e$, where $\mathbf{C}_k^e = \sigma_e^2\mathrm{E}[\mathbf{C}_k\mathbf{P}_k\mathbf{D}_k\mathbf{D}_k^\dagger\mathbf{P}_k^\dagger\mathbf{C}_k^\dagger]$ is the covariance of the equivalent intermodulation noise at receiver $k$.

## III. Algorithms Description and Selection

### A. Adaptive Beamforming Based on Directivity Estimates

In this section, we design an adaptive beamforming matrix $\mathbf{F}$ based solely on the knowledge of the estimated directivity matrix and the statistics of the product of the correlation and propagation matrices, but not on the exact channel realizations.

The beamformer is designed to minimize the total transmit power under some target SINR $\gamma_k$ for each MST $k$ and a maximum total average transmit power constraint. The solution of such a problem is well known in the case of receiver equipped with single antenna and complete CSI at the transmitter based on the duality between broadcast and multiple access channel [15], [16]. When the receivers are equipped with multiple antennas and perfect CSI is available, the same problem become much more complex. In order to keep the complexity of the algorithm low, we propose a heuristic approach for the beamforming design based on the use of an equivalent deterministic channel known at the transmitter and approximating the behavior of the fading channel. With this aim, we can rewrite the model in (6) in an equivalent form in terms of the SINR, as follows,

$$\overline{\mathbf{y}}_k^f = \mathbf{\Gamma}_k^{-1/2}\mathbf{U}_k\mathbf{C}_k^f\mathbf{P}_k^f\tilde{\mathbf{x}}_k^f + \mathbf{w}_k^f \tag{7}$$

where $\tilde{\mathbf{x}}_k^f = \mathbf{D}_k^f\mathbf{F}\mathbf{x}^f$, $\mathbf{w}^f$ is an additive Gaussian noise with covariance matrix $\mathbf{I}$, and $\mathbf{\Gamma}_k$ and $\mathbf{U}_k$ are obtained from the eigenvalue decomposition of the noise covariance matrix, i.e., $\mathbf{C}_k^z = \mathbf{U}_k^\dagger\mathbf{\Gamma}_k\mathbf{U}_k$. The system model can be further simplified as follows,

$$\tilde{\mathbf{y}}_k^f = \sqrt{\frac{\kappa_k}{2}}\tilde{\mathbf{C}}_k^f\tilde{\mathbf{P}}_k^f\tilde{\mathbf{x}}_k^f + \mathbf{w}_k^f \tag{8}$$

where $\tilde{\mathbf{C}}_k = \mathbf{\Gamma}_k^{-1/2}\mathbf{U}_k\mathbf{C}_k$, $\tilde{\mathbf{P}}_k = \frac{\mathbf{P}_k}{\sqrt{\kappa_k}}$, such that $\kappa_k$ represents the SNR over all antennas and polarizations and is given by

$$\kappa_k = \frac{1}{2}\mathrm{tr}\left(\mathrm{E}\left[\mathbf{C}_k\mathbf{P}_k\mathbf{P}_k^\dagger\mathbf{C}_k^\dagger\right]\mathbf{C}_z^{-1}\right) \tag{9}$$

and $\mathrm{E}[\tilde{\mathbf{C}}_k\tilde{\mathbf{P}}_k\tilde{\mathbf{P}}_k^\dagger\tilde{\mathbf{C}}_k^\dagger] = 4R$.

From (5), (6), (7) that $\mathbf{R}_k = \mathbf{C}_k\mathbf{P}_k$ is the fading part of the channel for the $k$-th ST. We can express $\mathbf{R}_k = \tilde{\mathbf{R}}_k\sqrt{\mathbf{G}_k}$, where the Frobenius norm $\|\tilde{\mathbf{R}}_k\|_F = 2R$, and $\mathbf{G}_k = \mathrm{diag}(g_{k,1}, g_{k,2})$ where $g_{k,i} = \frac{\mathrm{E}[\tilde{\mathbf{y}}_{k,i}^{f\dagger}\tilde{\mathbf{y}}_{k,i}^f]}{2}$. Therefore, the equivalent colored noise $\mathbf{z}_k$ in (6) has zero mean and covariance matrix $\mathbf{C}_k^z = \sigma_n^2\mathbf{C}_k^w + \mathbf{C}_k^e$, with $\mathbf{C}_k^e = \sigma_e^2\sqrt{\mathbf{G}_k}\mathbf{D}_k\mathbf{D}_k^\dagger\sqrt{\mathbf{G}_k}^\dagger$, and $\mathbf{C}_k^w = \sqrt{\mathbf{G}_k}\mathrm{E}[\mathbf{R}_k^\dagger\mathbf{R}_k]\sqrt{\mathbf{G}_k}^\dagger$. In this case, $\tilde{\mathbf{D}}_k = \mathbf{\Gamma}^{-1/2}\mathbf{U}_k^\dagger\sqrt{\mathbf{G}_k}\mathbf{D}_k$, where the matrices $\mathbf{\Gamma}^{-1/2}$ and $\mathbf{U}_k^\dagger$ are obtained via the eigenvalue decomposition of the noise covariance matrix. Notice that $\mathbf{D}_k$ is the block row of the directivity matrix $\mathbf{D}$ which is estimated via the reverse link according to the PLSE algorithm in [9]. Therefore, the matrix $\tilde{\mathbf{D}}_k$ is not only able to account for the random attenuation introduced by the propagation matrix and the normalized correlation matrix (the local fading effects), but also for the colored noise. We assume that the receiver ST $k$, which has knowledge of the channel between the SA and the receive antennas, is able to compensate for the neglected local effects of the fading.

When we consider the global forward link for all STs, the deterministic equivalent system utilized for the beamforming design is based on the equivalent matrix $\tilde{\mathbf{D}} = \mathbf{K}\mathbf{D}$, where $\mathbf{K} = \mathrm{diag}(\boldsymbol{\kappa}\otimes(1,1))$ and $\boldsymbol{\kappa} = [\kappa_1, \kappa_2, ..., \kappa_k]$, and the forward link model discussed previously can be written as

$$\tilde{\mathbf{y}}^f = \tilde{\mathbf{D}}\overline{\mathbf{F}}\boldsymbol{\mathcal{Q}}^{1/2}\mathbf{x}^f + \mathbf{w}^f \tag{10}$$

where $\boldsymbol{\mathcal{Q}}^{1/2}$ is $2K \times 2K$ power allocation diagonal matrix of the power levels of the transmitted signals, $\overline{\mathbf{F}}$ is the matrix normalized such that $\mathrm{diag}(\overline{\mathbf{F}}^\dagger\overline{\mathbf{F}}) = \mathbf{I}$, and $\mathbf{F} = \overline{\mathbf{F}}\boldsymbol{\mathcal{Q}}^{1/2}$.

To apply the duality method between broadcast and multiple access channel, we can write the corresponding dual reverse link model as follows,

$$\tilde{\mathbf{y}}^r = \tilde{\mathbf{D}}^\dagger\boldsymbol{\mathcal{P}}^{1/2}\mathbf{x}^r + \mathbf{w}^r \tag{11}$$

where $\boldsymbol{\mathcal{P}}^{1/2}$ is the $2K\times 2K$ diagonal matrix of the power levels for the transmitted signals and $\mathbf{x}^r$ is the transmitted signal with unit covariance, and $\mathbf{w}^r$ is the white additive Gaussian noise with unit variance. Note that if $\mathbf{M}^\dagger$ is any multiuser detector at the reverse link, the transmitted signal can be estimated as $\hat{\mathbf{x}}^r = \mathbf{M}^\dagger\mathbf{y}^r$ with normalization $\mathrm{diag}(\mathbf{M}^\dagger\mathbf{M}) = \mathbf{I}$.

Assuming the system in (10), we can formulate our precoding problem as a standard power control problem, [15], [16],

$$\begin{array}{lll} \text{minimize} & \sum_s q_s & \qquad\text{P0} \\ \text{subject to} & \mathrm{SINR}_s^f \geq \gamma_s, \quad s = 1, ..., 2K & \end{array}$$

Note that this strategy guarantees a minimum level of required QoS to both information streams of all $K$ STs but only under feasibility conditions. If such conditions are not satisfied, an outage occurs.

The SINR for stream $s$, with $s = 1, \ldots, 2K$, in the reverse link is given by

$$\mathrm{SINR}_s^r = \frac{p_s|(\mathbf{F}^\dagger\tilde{\mathbf{D}}^\dagger)_{ss}|^2}{1 + \sum_{j\neq s} p_j|(\mathbf{F}^\dagger\tilde{\mathbf{D}}^\dagger)_{sj}|^2} \tag{12}$$

Let us define the vector $\mathbf{a}$ with the $s$-th component,

$$a_s = \frac{\mathrm{SINR}_s}{(1 + \mathrm{SINR}_s)|(\mathbf{F}^\dagger\tilde{\mathbf{D}}^\dagger)_{ss}|^2} \tag{13}$$

Substituting (12) into (13), the power allocation vector $\mathbf{p} = \mathrm{diag}(\boldsymbol{\mathcal{P}})$ is expressed as,

$$\mathbf{a} = (\mathbf{I} - \mathrm{diag}(a_1, a_2, ..., a_{2K})\mathbf{\Lambda}^\dagger)\mathbf{p} \tag{14}$$

where $\mathbf{\Lambda}$ is a square matrix with elements $(\mathbf{\Lambda})_{sj} = |(\tilde{\mathbf{D}}\mathbf{F})_{sj}|^2$. Similarly, for the dual forward channel, the SINR is given by

$$\mathrm{SINR}_s^f = \frac{q_s|(\tilde{\mathbf{D}}\mathbf{F})_{ss}|^2}{1 + \sum_{j\neq s} q_j|(\tilde{\mathbf{D}}\mathbf{F})_{sj}|^2} \tag{15}$$

Therefore, we can similarly express the vector $\mathbf{q} = \mathrm{diag}(\boldsymbol{\mathcal{Q}})$ as,

$$\mathbf{a} = (\mathbf{I} - diag(a_1, a_2, ..., a_{2K})\mathbf{\Lambda})\mathbf{q} \tag{16}$$

Note that (14) and (16) admit positive solution for $\mathbf{p}$ and $\mathbf{q}$, if and only if the matrix $\mathrm{diag}(a_1, a_2, ..., a_{2K})\mathbf{\Lambda}^\dagger$ and the matrix $\mathrm{diag}(a_1, a_2, ..., a_{2K})\mathbf{\Lambda}$ has the Perron-Frobenius eigenvalue lower than 1 respectively. Since both matrices have the same spectral radius, then if one of the power vectors, say $\mathbf{p}$ is positive and achieves a target SINR, then there exists another vector, say $\mathbf{q}$, for the same target SINR. It is worth to notice that these considerations hold for any linear detectors $\mathbf{F}^\dagger$, and any linear beamformer $\mathbf{F}$. The conditions of feasibility of the dual reverse beamforming problem P0 are summarized in the following theorem.

*Theorem 1:* [16] The feasibility of the target SINR vector is achieved in both reverse and forward link with linear processing matrices $\mathbf{F}^\dagger$, and $\mathbf{F}$, respectively, if and only if the non-negative matrix $\mathrm{diag}(\mathbf{a})\mathbf{\Lambda}$ has a Perron-Frobenius eigenvalue $\rho(\mathrm{diag}(\mathbf{a})\mathbf{\Lambda}) < 1$. In this case the allocation equations are given by,

$$\mathbf{q}^\star = (\mathbf{I} - \mathrm{diag}(\mathbf{a})\mathbf{\Lambda})^{-1}\mathbf{a} \tag{17}$$

and

$$\mathbf{p}^\star = (\mathbf{I} - \mathrm{diag}(\mathbf{a})\mathbf{\Lambda}^\dagger)^{-1}\mathbf{a} \tag{18}$$

The solutions $\mathbf{q}^\star$ and $\mathbf{p}^\star$ are the componentwise minimal powers that achieve the target SINR vector with equality, and $\sum_s q_s = \sum_s p_s$.

Algorithm 1 provides the iterative solution of the power control and precoding problem.

**Algorithm 1: Adaptive Beamforming Algorithm**

**Input:** For each ST $k$:

**step 1:** Estimate the directivity row matrices $\mathbf{D}_k$, $k = 1, \ldots, K$ using PLSE algorithm in [9] and the corresponding directivity matrix $\mathbf{D}$.

**step 2:** Determine matrix $\tilde{\mathbf{D}}$ as defined in Section III-A and denote by $\tilde{\mathbf{d}}_s$ the $s$-th row vector.

**step 3:** Determine the vector $\mathbf{p}^\star$ by the fixed point equation,

$$p_s^{(\mu)} = \frac{\gamma}{\tilde{\mathbf{d}}_s(\mathbf{I} + \sum_{j\neq s} p_j{}^{(\mu-1)}\tilde{\mathbf{d}}_j^\dagger\tilde{\mathbf{d}}_j)^{-1}\tilde{\mathbf{d}}_s^\dagger} \tag{19}$$

for $\mu = 1, 2, ..$, with initial condition $\mathbf{p}^0 = 0$, and set $\mathbf{p}^\star = \lim_{\mu\to\infty}\mathbf{p}^\mu$.

**step 4:** Determine the beamforming matrix $\overline{\mathbf{F}}^\star$

$$\overline{\mathbf{F}}^\star = (\mathbf{I} + \tilde{\mathbf{D}}^\dagger\mathrm{diag}(\mathbf{p}^\star)\tilde{\mathbf{D}})^{-1}\tilde{\mathbf{D}}^\dagger\mathrm{diag}\mathbf{u} \tag{20}$$

By choosing $\mathbf{u}$ such that $\mathrm{diag}\overline{\mathbf{F}}^\dagger\overline{\mathbf{F}} = \mathbf{I}$.

**step 5:** Determine the vector $\mathbf{q}^\star$ by applying (16).

### B. Estimation of the Directivity Components at the Gateway

CSI acquisition is a crucial problem in the design of adaptive beamforming systems and strongly depends on channel characteristics. The acquisition of slow-varying partial CSI at the gateway, for a satellite system with MSTs, equipped eventually with multiple antennas and transmitting in left and right polarization, presents completely new challenges compared to the thoroughly studied field of satellite channel estimation. By assuming that the statistics of the propagation coefficients are available at the gateway and follow the model in [13], the partial CSI estimation reduces to the estimation of the slow varying components, i.e, the directivity vectors. From a signal processing perspective, this implies the challenging task of estimating parameters observed through multiplicative nuisance.

The directivity component could be estimated based on two fundamental assumptions proposed: (a) the reciprocity principle holds for the directivity component and (b) the directivity component is substantially independent of the the frequency band. This implies that observations of signals at the gateway received on a RACH or on a reverse link of a channel supporting connection-oriented communications can be conveniently exploited for directivity estimation of the forward link although the observations are affected by both multiplicative nuisance (propagation component) and additive noise (intermodulation noise at the satellite and thermal noise at the gateway).

The key tool for the estimation of the directivity vectors is the PLSE algorithm in [9]. The PLSE algorithm does not require the estimation of nuisance parameters. This enables a considerable complexity reduction, where the estimation problem reduces to an eigenvalue complementary problem. A detailed discussion about the pros and contra of each of the classes of the PLSE algorithm are discussed in [9].

### C. Adaptive Beamforming and Carrier Allocation

In conventional fixed beamforming with frequency reuse, frequencies are preassigned to beams and a frequency band is allocated to a ST depending on its position and the corresponding coverage beam. However, this does not hold for an adaptive beamforming system. Frequency reuse becomes a NP-hard problem with high complexity. From a system's point of view, it is crucial to reduce the computational complexity while attaining the QoS promised by SDMA. Therefore, we propose a low complexity algorithm for frequency band allocation based on the directivity components.

The algorithm assumes that the system is initially empty with a total number of $U$ available frequency bands. Thus, $U$ STs are randomly allocated to $U$ distinct carriers and the directivity matrix $\mathbf{D}(u)$, $u = 1, \ldots, U$ for carrier $u$ is generated. Afterwards, the remaining MSTs are processed sequentially and the MST with directivity vector $\mathbf{D}_k$ is assigned to the carrier $u^\star$ for which the maximum of the crosscorrelations $\text{diag}(\mathbf{D}_k \mathbf{D}(u)^\dagger)$ is minimized. Once carrier $u^\star$ is allocated to MST $k$, the directivity matrix $\mathbf{D}(u^\star)$ is updated to include the directivity block row of the incoming MST. Finally, an updated beamformer is designed. The ST is refused by the system if the power required by the updated beamformer exceeds the carrier power constraint. Otherwise the MST is successfully included in the system.

Algorithm 2 summarizes the steps of allocating STs to beams in a system with adaptive beamforming.

**Algorithm 2: Allocation of STs to Beams in an Adaptive Beamforming System**

**Input:** $\mathcal{K}$: is the set of STs active in the system
**step 1:** for $k = 1, ..., U$
  Carrier k is allocated randomly to ST $k$
 end
**step 2:** for $k = U + 1, ..., |\mathcal{K}|$
  for $m = 1, .., U$
  Find:
$$\Psi_m = max \ \ \text{diag}(\mathbf{D}_k \mathbf{D}(m)^\dagger) \quad (21)$$
 end
**step 3:** Select the carrier $u^\star$ that satisfies,
$$u^\star = arg \ min_{m=1,..,U} \ \ \Psi_m \quad (22)$$
**step 4:** Update the beamforming matrix $\mathbf{F} = \mathbf{F}_m$, for the new carrier $u^\star$, based on (20), Algorithm 1.
**step 5:** If $\mathbf{q}_m^\star$, the power required by updated beamformer exceeds carrier power constraint $P_{max}$, **then**
**step 6:** ST $k$ is excluded from the system; **else**
**step 7:** Allocate ST $k$ to carrier $u^\star$.
 end
 end
end

An extensive set of carrier allocation algorithms have been proposed in [17] and compared to the conventional carrier allocation with fixed beamforming.

### D. Contention Resolution and Directivity Estimation at the RACh

As already mentioned, in a MSS based on adaptive beamforming the RACh has the function not only to detect service requests of incoming MSTs but also to estimate their directivity vectors. In [11], we propose two algorithms able of estimate the directivity vectors of MSTs transmitting on the RACh and, possibly resolve contentions thanks to the estimated directivity vectors: Grid Reduction (GR) and Successive Channel Cancellation (SCC) algorithms. Due to the superior performance shown by the SCC algorithm we adopted it for further study and global performance simulations. It is utilized in the global system simulator presented here. The interested reader is referred to [11] for a detailed description and analysis of the algorithm.

## IV. Benchmark System

In a conventional satellite system, fixed beams cover a certain area and guarantee a minimum QoS to each active MST. In the conventional beamformer used here as benchmark, the weights of the BFN at the satellite antennas are kept constant. As for the adaptive beamforming, we determine the beamforming matrix in order to achieve a target SINR in each point of the coverage area. More specifically, each beam will point at a specific location of the fixed grid shown in 2. The reference points are regularly distributed on the map. The distance between adjacent points are equal on latitude and longitude. In order to keep limited interference from adjacent beams, frequency reuse is adopted. For fair comparison, the weights of the BFN are designed applying the same approach adopted to design adaptive beamformers, but the centers of beams are assigned in given fixed positions independent of the positions of the MSTs. The target SINR is guaranteed by assuring it at the edge of the beams. Each MST receives

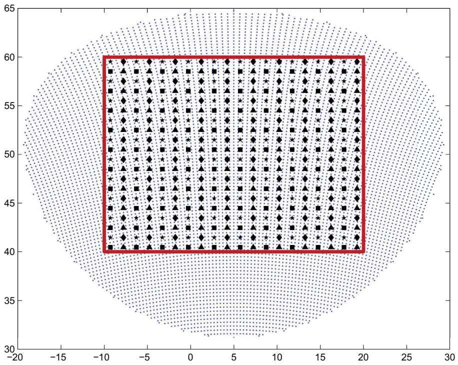


Figure 2. Coverage area of the satellite antenna arrays. The MSTs are generated in the rectangular area. Markers correspond to locations for the design of the benchmark SS. Equal markers are centers of beams served by the same frequency band.

information from the stronger beam that points to it. One beam can serve a single ST at one time. Each active MST designs its own multistream detector according to its channel estimate.

The RACh plays a role substantially different in a system based on fixed beamforming compared to a MSS based on adaptive beamforming. In fact, in the former case, the BFN in the reverse link is designed to convey information signals coming from a certain known coverage area associated to a certain beam. Then, the detection of a ST in a certain coverage area determines immediately the association of the MST to a certain cluster of frequency bands and beams covering such an area. No additional pieces of information are required on the CSI or on the directivity vector of the MST requiring service.

## V. Performance Analysis

In order to keep the complexity of the MSS implementation as low as possible and assuming a frequency reuse factor four for the conventional system, only four frequency bands are simulated. The simulator was calibrated and validated against a reference simulator provided by CNES. The directivity vector were generated according to the radiation pattern of a satellite equipped with 163 antennas, covering the most of Europe, and provided also by CNES. The positions of the MSTs are generated randomly and uniformly in a rectangular region shown in Figure 2. The frame duration is 60 seconds. Thus, each 60 seconds the directivity vectors are estimated, the frequency band allocation and the beamformers are updated. The beamformer is designed based on the initial perturbed knowledge of the directivity matrix or on the following estimates of the directivity vectors. The actual directivity vectors are updated each 10 seconds according to random movements of the MSTs. The speeds of the MSTs varies in the range between 50 and 130 km/h. The MSTs perform channel estimation and multistream detection to detect the two information streams sent to them. The conventional beamformer is designed applying a similar approach, with the centers of beams are assigned in given fixed positions, as shown in Figure 2. The following will provide a set of insightful results.

### *A. Beamforming Based on Estimated Directivity Vectors*

To simulate the connection oriented communications, we perform a set of simulations for $K$ randomly generated MSTs. The STs are mobile but during the experiment neither new MSTs enter in the system nor MSTs leave it. The experiments are performed fixing a target SINR and designing a beamformer that achieves such a target based on the estimation of the directivity vectors in the return link.

The performance presented here are obtained by averaging over several simulations. In each simulation $K = 70$ STs are randomly generated in the coverage area. Their directivity vectors are estimated at the gateway based on the observation of $Q = 30$ independent coherence intervals during which QPSK randomly generated training sequences of length $300$ are transmitted. The estimation is based on the PLSE algorithm [9]. Note that setting $L = 300$ enables to consider negligible the effect of estimation failure probability for a large range of $K$; while the estimation error of STs positions is still varying with the number of STs. On the base of the estimated directivity matrix of the $K$ STs the gateway designs the adaptive beamforming according to Algorithm 1. The beamforming is designed to provide a certain target SINR to all the STs in the system with a total transmit power not higher than 50 dBW. Due to the heuristic nature of the approach proposed for the beamforming design there is a mismatch between the target SINR and the actually achieved SINR. This mismatch depends substantially on the number of STs in the system and varies only slightly with the target SINR.

Figure 3 shows the achieved SINR versus target SINR for $K = 70$. The solid line corresponds to the ideal match between target and achieved SINR, the solid line with circle markers corresponds to the achieved SINR assuming perfect knowledge of the directivity vectors. The dashed line with square markers corresponds to the achieved SINR with directivity vectors estimated by the PLSE. Note that the heuristic algorithm leads to a beamformer whose performance is greater than the required one. Thus, the transmit power could be decreased without affecting the QoS required by STs. Interestingly, the beamformer design is not substantially affected by the degraded knowledge on the directivity matrix and the curves corresponding to perfect and estimated knowledge of the directivity match each other very well.

A similar behaviour is shown in Figure 4, the outage probability versus the achieved SINR is presented. Here, the outage probability is defined as the probability that a ST is not served. This kind of event occurs mainly due to the fact that, with the maximum total transmit power available at the satellite it is not possible to design a beamformer which achieves the target SINR. Seldom, a ST is not served because its directivity vector was erroneously estimated. The outage probability is computed in the simulations by averaging over the percentage of the STs that cannot be served when $K$ STs are randomly generated. Then, the number of STs actually served by the system in average is $K(1-p_{outage})$. We conclude this analysis by comparing the transmit power required in the case of perfect knowledge and estimation of the directivity vectors. Figure 5 shows the total transmit power versus the achieved SINR. Also in this case the mismatch between the case of perfect knowledge of the directivity vectors (solid line with circle markers), and the case of estimated directivity vectors (dashed line with square markers) is negligible.

### *B. Joint Carrier Allocation and Adaptive Beamforming*

In this section, we assess the gain offered by user diversity by comparing the performance of joint carrier allocation and adaptive beamforming to the performance of adaptive beamforming for a random generated set of $K = 75$ MTS.

For the former scenario, we assume that 4 orthogonal frequency bands are available and $4K$ STs are allocated to different frequency bands according to Algorithm 2.

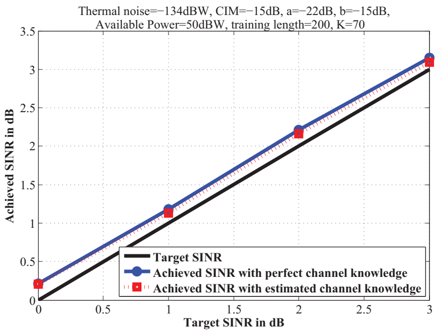


Figure 3. Achieved SINR vs target SINR.

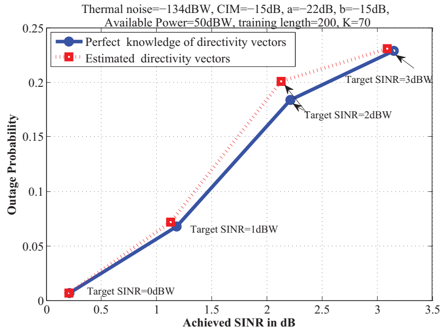


Figure 4. Outage Probability vs the achieved SINR

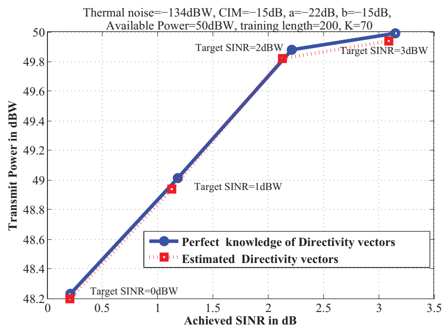


Figure 5. Total transmit power vs the achieved SINR

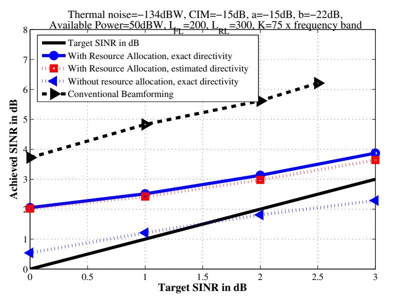


Figure 6. Achieved SINR vs target SINR.

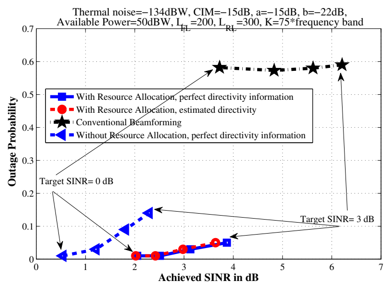


Figure 7. Outage Probability vs the achieved SINR

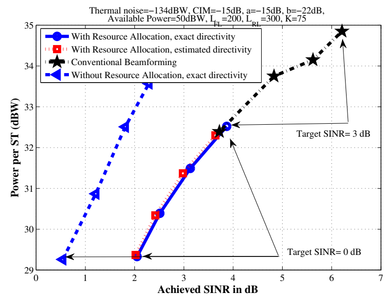


Figure 8. Total transmit power vs the achieved SINR

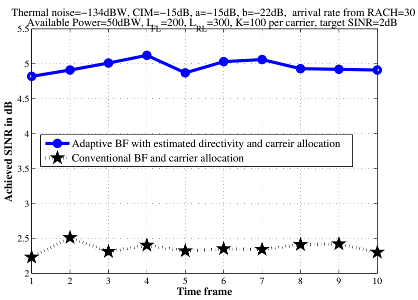


Figure 9. Achieved SINR vs target SINR.

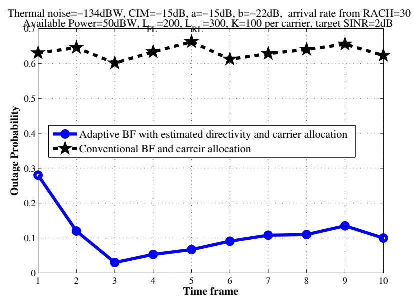


Figure 10. Outage Probability vs the achieved SINR

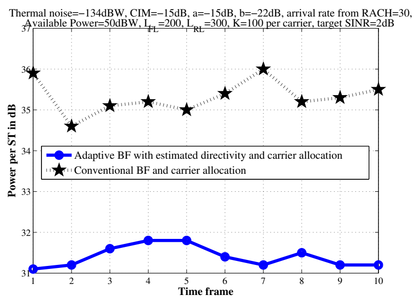


Figure 11. Total transmit power vs the achieved SINR

In Figure 6, we adopt as figure of merit the achieved SINR versus the target SINR to compare the two scenarios. The plots show that, for a given target SINR, in the case of carrier allocation, we achieve approximately 1.4 dB higher SINR than in the case when no carrier allocation is performed. The higher mismatch between achieved and target SINR in the case of resource allocation implies that, for this scenario, we have a higher margin in performance that could be used to reduce outage probability by finer refinement of the adaptive beamforming.

Figure 7 shows the outage probability versus the achieved SINR for a system with frequency band allocation and without carrier allocation. Interestingly, joint carrier allocation and adaptive beamforming improves considerably the system performance in terms of outage compared to solely adaptive beamforming. In fact, the system with joint frequency allocation and beamforming can serve in average 297 STs with the required $SINR = 2.5$dB and outage probability of $1\%$. On the contrary, the system without carrier allocation can serve 258 with an outage probability of $14\%$.

It is worth to notice that if we could refine the adaptive beamforming by reducing the mismatch between target SINR and achieved SINR, we could reduce dramatically the transmit power of the MST. On the contrary, the benchmark system designed to guarantee QoS on the worst case does not leave room for such improvements.

Figure 8 shows the transmit power per ST versus the achieved SINR. The transmit power reduction per ST is of the order of 3.5 dBW for $K = 75$. The trend of the outage probability versus the achieved SINR makes clear the source of the benchmark system inefficiency: since the benchmark system is designed to satisfy QoS constraint in the worst case conditions, resources are spent to provide service with SINR levels far above the ones required by the STs. In terms of number of STs actually served, the gap between the two systems is shown on Table 1. Table 1 provides a comparison between the system based on joint frequency band allocation and adaptive beamforming (proposed sys.) and the system based on fixed beamforming (benchmark sys.) in terms of actually served STs and satisfaction of the QoS constraints.

| STs requiring service | Target SINR | Achieved SINR benchmark/proposed | STs actually served (proposed sys.) | STs actually served (benchmark sys.) | Outage probability constraint ($\leq$ 0.1) benchmark/proposed |
|---|---|---|---|---|---|
| 200 | 0 dB | 3.89/2.35 dB | 200 | 111 | no/yes |
| 200 | 1 dB | 4.90/2.92 dB | 200 | 111 | no/yes |
| 200 | 2 dB | 5.94/3.95 dB | 200 | 111 | no/yes |
| 200 | 3 dB | —-/4.93 dB | 196 | 111 | no/yes |
| 300 | 0 dB | 3.72/2.02 dB | 297 | 126 | no/yes |
| 300 | 1 dB | 4.83/2.42 dB | 297 | 126 | no/yes |
| 300 | 2 dB | 5.26/2.98 dB | 291 | 126 | no/yes |
| 300 | 0 dB | —-/3.64 dB | 273 | 126 | no/yes |
| 360 | 0 dB | 3.47/1.84 dB | 356.4 | 140.4 | no/yes |
| 360 | 1 dB | 4.44/2.23 dB | 349.2 | 140.4 | no/yes |
| 360 | 2 dB | 5.33/2.79 dB | 327.6 | 140.4 | no/yes |
| 360 | 3 dB | —-/2.79 dB | 291.6 | 140.4 | no/no |

Table I. COMPARISON BETWEEN PROPOSED SYS. AND BENCHMARK SYS.

### C. System Dynamics

Here we analyze the behavior of the whole satellite system with interactions betweens RACh and transmissions on a data transmit channel supporting connection oriented services. The number of MSTs in the system is stationary, i.e. the arrival rate of new requests of service on the RACh equals the departure rate of MSTs from the system. Initially, $K = 100$ MSTs per frequency band and randomly generated. Then, the system dynamics are determined by arrival and departure rates equal 30 MSTs per minute. The simulations last 11 minutes such that the most of the MSTs are renewed in the system. Objective of these simulations is to verify the stability of the system in dynamic conditions.

As from the analysis in the previous sections, with the setting utilized in this work and outage probability not greater than $10\%$, the satellite system is overloaded and it is not possible to serve STs with the required QoS. We consider a MSS designed for a target SINR of 2 dB. As shown in Figure 9, the outage probability in the first time interval is very high and a considerable number of MSTs is rejected. In the following time intervals, the outage probability becomes rather stable around a lower value. Despite the heavy load that does not allow the system to guarantee the required outage probability, once the system has rejected the STs that cannot be supported, it is able to guarantee a stability of the other metrics, such as the achieved SINR (see Figure 10) and the transmit power per ST (see Figure 11).

## VI. CONCLUSIONS

In this paper, we show that adaptive beamforming is an excellent candidate for next generation satellite systems. The implication of such proposed system dictates a redesign of the RACh protocol and the data transmission framework, aiming to adapt the satellite terminals access and data transmission to the adapted beams and frequency carriers. In fact, when compared to conventional beamformers, it provides high gain in capacity, i.e., number of STs that can be supported by the system at a given guaranteed QoS. Moreover, the mismatch or the variance between the target and achieved SINR as a function of the number of STs, when adaptive beamforming is used, provides the potential for better utilization of the resources by tuning the proposed approaches. In particular, additional QoS and capacity gains are obtained by joint adaptive beamforming and carrier allocation. Finally, its of particular relevance to highlight that although higher number of STs are served due to less outage probability with joint adaptive beamforming and carrier allocation, the reduction in the transmit power per ST is relevant between 1.5dBW to 3dBW.


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