DECOUPLED, RANK REDUCED, MASSIVE AND FREQUENCY-SELECTIVE ASPECTS IN MIMO INTERFERING BROADCAST CHANNELS

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ABSTRACT

The Interfering Broadcast Channel (IBC) applies to the downlink of cellular and heterogenous networks, which are limited by multi-user interference. The interference alignment (IA) concept has shown that interference does not need to be inevitable. In particular spatial IA in MIMO IBC allows for low latency and requires little diversity. However, IA requires perfect and typically global Channel State Information at the Transmitter(s) (CSIT), whose acquisition does not scale with network size. Hence, designs that are optimal in terms of Degrees of Freedom (DoF) may not be so in terms of more relevant net DoF, accounting for CSI acquistion. Also, the design of transmitters (Txs) and receivers (Rxs) is coupled and hence needs to be centralized or duplicated. Recently, a number of (usually suboptimal in terms of DoF) approaches with reduced, incomplete or local CSIT requirements have been introduced requiring less CSI acquisition overhead and allowing decoupled Tx/Rx designs. This network decomposition is aided by the for finite SNR more relevant topological IBC scenario, in which also reduced rank MIMO channels may appear. The transition to Massive MIMO furthermore introduces a reduced rank in the covariance CSIT and allows network decomposition. We will also highlight that any scenario allowing decomposition is favorable for the design of asynchronous frequency-selective networks.

Index Terms— Interfering Broadcast Channel (IBC), Interference Alignment, Channel State Information at the Transmitter (CSIT), Massive MIMO

1. INTRODUCTION

In this paper, Tx may denote transmit/transmitter/transmission and Rx may denote receive/receiver/reception. Interference is the main limiting factor in wireless transmission. Base stations (BS) disposing of multiple antennas are able to serve multiple users simultaneously, which is called Spatial Division Multiple Access (SDMA) or Multi-User (MU) MIMO. However, MU systems have precise requirements for Channel State Information at the Tx (CSIT) which is more difficult to acquire than CSI at the Rx (CSIR). Hence we focus here on the more challenging downlink (DL). In cellular systems, one can distinguish between the cell center where a single cell design is appropriate (due to high SIR) and the cell edge where a multi-cell approach is mandatory. The MU MIMO DL problem for the cell center users is called the (MIMO) Broadcast Channel (BC). For the cell edge users, the recent introduction of Interference Alignment (IA) has shown that approaching high system capacity through agressive frequency reuse should in principle be possible. Whereas precise capacities for cellular systems remain unknown, IA allows to reach the optimal high SNR rate prelog, called Degree of Freedom (DoF) (or spatial multiplexing factor), that is before accounting for CSI acquisition. Various IA flavors exist:

- *linear* IA [1], also called *signal space* IA, only uses the spatial dimensions introduced by multiple antennas.
- *asymptotic* IA [2] uses symbol extension (in time and/or frequency), leading to (infinite) symbol extension involving diagonal channel matrices, requiring infinite channel diversity in those dimensions. This leads to infinite latency also. The (sum) DoF of asymptotic MIMO IA are determined by the *decomposition* bound [3].
- *ergodic* IA [4] explains the factor 2 loss in DoF of SISO IA w.r.t. an interference-free Tx scenario by transmitting the same signal twice at two paired channel uses in which all cross channel links cancel out each other. Ergodic IA also suffers from uncontrolled latency but provides the factor 2 rate loss at any SNR. The DoF of ergodic MIMO IA are also determined by the decomposition bound [5].
- *real* IA [6], also called it signal scale IA, exploits discrete signal constellations and is based on the Diophantine equation. Although this approach appears still quite exploratory, some related work based on lattices appears promising.

We shall focus here on linear IA, in which the spatial Tx filters align their various interference terms at a given user in a common subspace so that a Rx filter can zero force (ZF) it. Since linear IA only uses spatial filtering, it leads to low latency. The DoF of linear IA are upper bounded by the so-called *proper bound* [7], [8], [9], which simply counts the number of filter variables vs. the number of ZF constraints. The proper bound is not always attained though because to make interference subspaces align, the channel subspaces in which they live have to sufficiently overlap to begin with, which is not always the case, as captured by the so-called *quantity bound* [10] and first elucidated in [11], [12], [3]. The transmitter coordination required for DL IA in a multi-cell setting corresponds to the Interfering Broadcast Channel (IBC). Depending on the number of interfering cells, the BS may run out of antennas to serve more than one user, which then leads to the Interference Channel (IC).

The main difficulty in realizing linear IA for MIMO I(B)C is that the design of any BS Tx filter depends on all Rx filters whereas in turn each Rx filter depends on all Tx filters [13]. As a result, all Tx/Rx filters are globally coupled and their design requires global CSIT. To carry out this Tx/Rx design in a distributed fashion, global CSIT is required at all BS [14]. The overhead required for this global distributed CSIT is substantial, even if done optimally, leading to substantially reduced Net DoF [15]. In [16] the simplified SIMO uplink (UL) problem with only CSIR acquisition is considered and it is shown that it is impossible to maintain any positive Net DoF if one wants to design a cellular network that extends infinitely far (the problem arises already at a finite network size that depends on the Doppler bandwidth). However, all these DoF considerations may be of limited relevance for operation at any finite SNR, in which case interfering Tx that are sufficiently far away can be ignored, leading to the topic of *topological* IA [17]. This is one of the angles we shall consider here to allow to decompose the global Tx design problem.

2. IBC SIGNAL MODEL

In the general IBC setting, with C cells and K_c users in cell c, the $N_{c,k} \times 1$ received signal at user k in cell c is

$$\mathbf{y}_{c,k} = \mathbf{H}_{c,k,c}\,\mathbf{G}_{c,k}\,\mathbf{x}_{c,k} + \sum_{(j,i)=(1,1),\neq(c,k)}^{(C,K_j)} \mathbf{H}_{c,k,j}\,\mathbf{G}_{j,i}\,\mathbf{x}_{j,i} + \mathbf{v}_{c,k} \tag{1}$$

where $\mathbf{x}_{c,k}$ are the $d_{c,k} \times 1$ intended (white, unit variance) signal streams for that user, $\mathbf{H}_{c,k,j}$ is the $N_{c,k} \times M_j$ channel from BS j to user k in cell c. We assume that we are considering a noise whitened signal representation so that we get for the noise $\mathbf{v}_{c,k} \sim \mathcal{CN}(0, I_{N_{c,k}})$. The $M_c \times d_{c,k}$ matrix spatial Tx filter of beamformer (BF) is $\mathbf{G}_{c,k}$. In the multiple user per cell setting, the most typical configuration will be that of $d_{c,k} \equiv 1$ stream per user since some user selection can make this normally preferable over multiple streams/user. In that case we get scalar signals and BF vectors

$$\mathbf{y}_{c,k} = \mathbf{H}_{c,k,c}\,\mathbf{g}_{c,k}\,x_{c,k} + \sum_{(j,i)=(1,1),\neq(c,k)}^{(C,K_j)} \mathbf{H}_{c,k,j}\,\mathbf{g}_{j,i}\,x_{j,i} + \mathbf{v}_{c,k}\,. \tag{2}$$

Below we shall often focus on special cases for clarity. The single cell MU downlink or BC is obtained when $C = 1$ and the IC case corresponds to $K_c \equiv 1$, $c = 1, ..., C$. Also, the analysis simplifies significantly for the so-called symmetric case in which $K_c \equiv K$, $M_c \equiv M$, $N_{c,k} \equiv N$, $d_{c,k} \equiv d$ leading to the symmetric IBC configuration (M, N, C, K, d).

There are a number of cases in which the DoF of linear IA are captured completely by the proper bound [18]. For the symmetric MIMO IC ($K = 1$), when $\min(M, N) >\geq 2d$, alignment is feasible iff $M + N \geq (C + 1)d$ (proper bound). This is a generalization of [12] which only considered the square case $M = N$. Then there is the "divisible case": if $d_{c,1} \equiv d$ and $d|N_{c,1}, \forall c$ OR $d|M_c, \forall c$, then alignment is feasible iff condition (11) in [18] is satisfied. This is again a bit more general than similar results by [9] where they had $d_{c,1} = 1$ which of course divides everything, or [19] where the $d_{c,1}$ needed to divide both the $N_{c,1}$ and the M_c. For the IBC, [10] finds that the proper bound $M + N \geq (CK + 1)d$ is sufficient in the symmetric IBC when either M or N is divisible by d.

3. REDUCED CSIT AND DECOUPLED TX/RX DESIGN

In order for IA to become applicable to cellular networks, the overall Tx/Rx design has to decompose so that the CSIT required is no longer global and remains bounded everywhere in the network, regardless of the network size growing unboundedly.

The simplest case is that of *local* CSIT. Local CSIT means that a BS only needs to know the channels from itself to all terminals. In the TDD case this could even be obtained by reciprocity from the UL training, without feedback. The local CSIT case arises when all ZF work needs to be done by the Tx: $d_{c,k} = N_{c,k}, \forall c, k$. The most straightforward such case is of course the MISO case: $d_{c,k} = N_{c,k} = 1$. It extends to cases of $N_{c,k} > d_{c,k}$ if less than optimal DoF are accepted. One of these cases is that of reduced rank MIMO channels, as will be discussed below.

Another case is that of *reduced* CSIT as discussed in [20] where a variety of approaches are explored with reduced CSIT feedback requirements in exchange for associated variable DoF reductions.

In [21], the concept of *incomplete* CSIT is introduced. It turns out that in some MIMO IC configurations de optimal DoF can be attained with less than global CSIT. This only occurs when the numbers of antennas M and/or N vary substantially so that subnetworks of a subgroup of BS and another subgroup of terminals arise in which the numbers of antennas available are just enough to handle the interference within the subnetwork. This means that this subnetwork does not need CSI from nodes outside the network. We shall explore this direction further below.

The introduction of Massive MIMO leads to an increased interest in exploiting *covariance* CSIT, which will tend to have reduced rank and allows decoupled approaches.

4. CLUSTERED TOPOLOGICAL MIMO IBC

We propose here an approach to infinite IBC network by exploiting topology, enforcing CSI to be local to clusters, and reverse engineering the numbers of antennas required. Consider partitioning an infinite IBC into finite IBC clusters. Within a finite IBC cluster, CSI acquistion can be performed in a distributed fashion as in [15]. Then antennas get added to the BS in order to perform ZF of the finite inter-cluster links (due to topology, longer links can be neglected). So consider building a Tx filter as the cascade

$$\underbrace{\mathbf{G}}_{M\times d} = \underbrace{\mathbf{S}}_{M\times M'}\ \underbrace{\mathbf{T}}_{M'\times d} \tag{3}$$

where we omit indices for simplicity. $\mathbf{T}$ is the normal Tx filter assuming that only the cluster of C BS would exist, using a fictitious number $M^{'}$ of Tx antennas. $\mathbf{S}$ is the prefilter that does ZF to all Rx antennas outside of this cluster that receive signal from this BS. Hence M needs to be augmented w.r.t. $M^{'}$ by this number. Even if the nominal IBC clusters would be symmetric, the actual M may vary by BS depending on the topology. This approach is suboptimal from a DoF point of view (an optimal approach would only ZF to Rx outputs) but it is a decoupled approach requiring only local CSI. Reductions in M may be obtained by accounting for reduced rank MIMO channels. The inter-cluster ZF may also be shared with MTs in a similar fashion.

5. SPECULAR WIRELESS MIMO CHANNEL MODEL

We get for the matrix impulse response of a time-varying MIMO channel $\mathbf{H}(t, \tau)$ [22]

$$\mathbf{H}(t,\tau) = \sum_{i=1}^{N_p} A_i(t)\, e^{j2\pi\, f_i\, t}\, \mathbf{h}_r(\phi_i)\, \mathbf{h}_t^T(\theta_i)\, p(\tau - \tau_i)\quad . \tag{4}$$

The channel impulse response $\mathbf{H}$ has per path a rank 1 contribution in 4 dimensions (Tx and Rx spatial multi-antenna dimensions, delay spread and Doppler spread); there are N_p (specular) pathwise contributions where

- A_i: complex attenuation
- f_i: Doppler shift

- θ_i: direction of departure (AoD)
- ϕ_i: direction of arrival (AoA)
- τ_i: path delay (ToA)
- $\mathbf{h}_t(.)$, $\mathbf{h}_r(.)$: $M/N \times 1$ Tx/Rx antenna array response
- $p(.)$: pulse shape (Tx filter)

The antenna array responses are just functions of angles AoD, AoA in the case of standard antenna arrays with scatterers in the far field. In the case of distributed antenna systems, the array responses become a function of all position parameters of the path scatterers. The fast variation of the phase in $e^{j2\pi f_i t}$ and possibly the variation of the A_i (when the nominal path represents in fact a superposition of paths with similar parameters) correspond to the fast fading. All the other parameters (including the Doppler frequency) vary on a slower time scale and correspond to slow fading. We shall assume here OFDM transmission, as is typical for 4G systems, with the Doppler variation over the OFDM symbol duration being negligible. We then get for the channel transfer matrix at any particular subcarrier of a given OFDM symbol

$$\mathbf{H} = \sum_{i=1}^{N_p} A_i\, e^{j\psi_i}\, \mathbf{h}_r(\phi_i)\, \mathbf{h}_t^T(\theta_i) \tag{5}$$

where with some abuse of notation we use the same A_i to denote this time the path amplitude $A_i \geq 0$ and introduced also the path phase ψ_i, in both of which we ignored the dependence on time (particular OFDM symbol), through at least the Doppler shift, and on frequency (subcarrier), through the Tx (and Rx) filter(s).

6. REDUCED RANK MIMO IC

6.1. IA feasibility singular MIMO IBC

This subject was treated by [23] for IC the general $C = 2$ cell case and for certain symmetric cases with $C = 3$. Related work also appears in [24] where for the case of no relay (as considered here) only some bounds were provided.

For $d_{i,k}$ streams of user k in cell i, a $M_i \times d_{i,k}$ Tx filter $\mathbf{G}_{i,k}$ and a $N_{i,k} \times d_{i,k}$ Rx filter $\mathbf{F}_{i,k}$ is used. In the rank deficient case, let $0 \leq r_{i,k,j} \leq \min(N_{i,k}, M_j)$ denote the rank of MIMO channel $\mathbf{H}_{i,k,j}$. This means essentially that only $r_{i,k,j}$ distinguishable significant paths contribute to $\mathbf{H}_{i,k,j}$, where distinguishable means with linearly independent antenna array responses from other paths, at both the Tx side and the Rx side. Then we can factor $\mathbf{H}_{i,k,j} = \mathbf{B}_{i,k,j}\mathbf{A}^H_{i,k,j}$ for some full rank $N_{i,k} \times r_{i,k,j}$ factor $\mathbf{B}_{i,k,j}$ and $M_j \times r_{i,k,j}$ factor $\mathbf{A}_{i,k,j}$. The ZF from BS j to MT (i, k) requires

$$\mathbf{F}^H_{i,k}\mathbf{H}_{i,k,j}\mathbf{G}_{j,n} = \mathbf{F}^H_{i,k}\mathbf{B}_{i,k,j}\, \mathbf{A}^H_{i,k,j}\mathbf{G}_{j,n} = 0 \tag{6}$$

which involves $\min(d_{i,k}d_{j,n}, d_{i,k}r_{i,k,j}, r_{i,k}, d_{j,n})$ constraints to be satisfied by the $(N_{i,k} - d_{i,k})d_{i,k}$/$(M_j - d_{j,n})d_{j,n}$ variables parameterizing the column subspaces of $\mathbf{F}_{i,k}$/$\mathbf{G}_{j,n}$. The overall IA feasibility gets determined by verifying whether the system is proper [25]: for each subset of MTs and subset of BSs, the total number of Tx/Rx variables involved needs to be at least equal to the total number of constraints in the corresponding conditions (6). When the rank constraints are active (number of constraints involves $r_{i,k,j}$), counting variables vs. ZF constraints gives the complete answer since we have traditional (one-sided) Tx or Rx ZF ($\mathbf{F}^H_{i,k}\mathbf{B}_{i,k,j} = 0$ or $\mathbf{A}^H_{i,k,j}\mathbf{G}_{j,n} = 0$). When the rank constraints are not active (min attained for $d_{i,k}d_{j,n}$) then counting arguments may not be sufficient in very rectangular (non-square) MIMO channel cases [11], [12]. Note also that the full rank requirement on $\mathbf{F}^H_{i,k}\mathbf{H}_{i,k,i}\mathbf{G}_{i,k}$ leads to $1 \leq d_{i,k} \leq r_{i,k,i} \leq \min(N_{i,k}, M_i)$ (the first inequality reflects that we consider only active links), whereas their joint consideration for all users of a BS i leads to $\sum_{k=1}^{K_i} d_{i,k} \leq \sum_{k=1}^{K_i} r_{i,k,i} \leq \min(\sum_{k=1}^{K_i} N_{i,k}, M_i)$.

6.2. IA feasibility singular MIMO IC with Tx/Rx decoupling

In this case we shall insist that (6) be satisfied by

$$\mathbf{F}^H_{i,k}\mathbf{B}_{i,k,j} = 0 \text{ or } \mathbf{A}^H_{i,k,j}\mathbf{G}_{j,n} = 0\ . \tag{7}$$

This leads to a possibly increased number of ZF constraints $r_{i,k,j}\min(d_{i,k}, d_{j,n})$ and hence to possibly reduced IA feasibility. When $d_{i,k} = d_{j,n}$, the constraints could be shared between the two factors with $d_{i,k}t_{i,k,j}$ constraints on $\mathbf{F}^H_{i,k}\mathbf{B}_{i,k,j}$ and $d_{i,k}(r_{i,k,j} - t_{i,k,j})$ constraints on $\mathbf{A}^H_{i,k,j}\mathbf{G}_{j,n}$ with $t_{i,k,j} \in \{0, 1, \ldots, r_{i,k,j}\}$. Of course, the task of ZF of every cross link now needs to be partitioned between all Txs and Rxs, taking into account the limited number of variables each Tx or Rx has. The main goal of this approach however is that it leads to Tx/Rx decoupling. Whereas in the general case (6) the design of the Txs depends on the Rxs and vice versa, in (7) the ZF constraints are linear and involve Tx or Rx but not both. The ZF constraints for a Tx (or a Rx) only require local channel knowledge (of the channel connected to it). Of course, the gobal ZF task partitioning needs to be known. This leads to a category of IA feasibility with incomplete CSIT, different from the one appearing in e.g. [25] as described earlier.

In the uniform case (M, N, K, C, d) with $d \leq r$ per user, (7) leads to

$$d \leq \frac{1}{2}(M + N - (KC - 1)r) \tag{8}$$

whereas the general coupled case (6) would have led to $d \leq \frac{1}{2}(M + N - (KC - 1)d)$. There is no loss if $d = r$, in which case $d = r \leq \frac{M+N}{KC+1}$.

In the case of general rank distribution but with a single stream per user ($d_{i,k} \equiv 1$) (which is the most likely scenario in an IBC context), we get

$$\sum_{i=1}^{C}\{K_iM_i + \sum_{k=1}^{K_i} N_{i,k}\} \geq 2K_{tot} + \sum_{i=1}^{C}\sum_{j=1}^{C}\sum_{k=1}^{K_i}\sum_{n=1,\neq k}^{K_j} r_{i,k,j} \tag{9}$$

where $K_{tot} = \sum_{i=1}^{C} K_i$ is the total number of users in the IBC. The non-decoupled case would correspond to replacing all the $r_{i,k,j}$ in (9) by 1. Now consider again a uniform case with $d = 1$, $(M, N, K, C, 1)$, where now all the cross links between cells are assumed to be of rank 1 (essentially LoS), whereas the BC channels within a cell are assumed to be of rank r. This leads to

$$M + N \geq KC + K(r - 1) + 2 - r\ . \tag{10}$$

7. COVARIANCE CSIT

In this section we drop the channel index (i, k, j) for simplicity. Mean information about the channel can come from channel feedback or reciprocity, and prediction, or it may correspond to the non fading (e.g. LoS) part of the channel (note that an unknown phase factor $e^{j\psi}$ in the overall channel mean does not affect the BF design). When (as good as) perfect CSIT is unavailable, we shall focus on the case of zero mean. Covariance information may correspond to channel estimation (feedback, prediction) errors and/or to information about spatial correlations. The separable (or Kronecker) correlation model (for the channel itself, as opposed to its estimation error or knowledge) which is often assumed is not applicable when the antenna dimensions get large. Indeed, averaging over the (uniform) path phases ψ_i in (5) leads to

$$\mathbf{C}_{\mathbf{hh}} = \sum_{i=1}^{N_p} A_i^2 \, \mathbf{h}_i \mathbf{h}_i^H = \sum_{i=1}^{N_p} A_i^2 \, (\mathbf{h}_r(\phi_i)\mathbf{h}_r^H(\phi_i)) \otimes (\mathbf{h}_t(\theta_i)\mathbf{h}_t^H(\theta_i)) \tag{11}$$

where $\mathbf{C}_{\mathbf{hh}} = \mathrm{E}\, \mathbf{h}\mathbf{h}^H$, $\mathbf{h} = \mathrm{vec}(\mathbf{H})$ and $\mathbf{h}_i = \mathbf{h}_t(\theta_i) \otimes \mathbf{h}_r(\phi_i)$. Note that the rank of $\mathbf{C}_{\mathbf{hh}}$ can be substantially less than the number of paths. Consider e.g. a cluster of paths with narrow AoD spread, then we have

$$\theta_i = \theta + \Delta\theta_i \tag{12}$$

where θ is the nominal AoD and $\Delta\theta_i$ is small. Hence

$$\mathbf{h}_t(\theta_i) \approx \mathbf{h}_t(\theta) + \Delta\theta_i \; \dot{\mathbf{h}}_t(\theta) \; . \tag{13}$$

Such a cluster of paths only adds a rank 2 contribution to $\mathbf{C}_{\mathbf{hh}}$.

The sum in (11) is not of Kronecker form. Nevertheless, gearing up towards massive MIMO, we shall consider exploiting the Tx side covariance matrix $\mathbf{C}_t$, which only explores the channel correlations as they can be seen from the BS side

$$\mathbf{C}^t = \mathrm{E}\, \mathbf{H}^H \mathbf{H} \tag{14}$$

Note that from (5), regardless of the number of paths we can factor the channel response as

$$\mathbf{H} = \mathbf{B}\; \mathbf{A}^H \; , \;\; \mathbf{B} = [\mathbf{h}_r(\phi_1)\, \mathbf{h}_r(\phi_1) \cdots] \begin{bmatrix} e^{j\psi_1} & & \\ & e^{j\psi_2} & \\ & & \ddots \end{bmatrix} ,$$
$$\mathbf{A}^H = \begin{bmatrix} A_1 & & \\ & A_2 & \\ & & \ddots \end{bmatrix} \begin{bmatrix} \mathbf{h}_t^T(\theta_1) \\ \mathbf{h}_t^T(\theta_2) \\ \vdots \end{bmatrix} \tag{15}$$

Averaging of the path phases ψ_i, we get for the Tx side covariance matrix

$$\mathbf{C}^t = \mathbf{A}\mathbf{A}^H \tag{16}$$

since due to the normalization of the antenna array responses, $\mathrm{E}\, \mathbf{B}^H\mathbf{B} = \mathrm{diag}\{[\mathbf{h}_r(\phi_1)\, \mathbf{h}_r(\phi_1) \cdots]^H \, [\mathbf{h}_r(\phi_1)\, \mathbf{h}_r(\phi_1) \cdots]\} = \mathbf{I}$.

8. MASSIVE MIMO ASPECTS

One of the ideas behind massive MIMO is to have a certain excess of BS antennas, in order to compensate for RF chains of limited quality, but also to simplifiy processsing. The parsimonious use of antennas dictates to use the precise spatiotemporal channel response structure but the exploitation of excess BS antennas allows us to go back from spatiotemporal (user-wise) processing to spatial (path-wise) processing. Path-wise processing was introduced in CDMA systems. E.g. the RAKE Rx is a path-wise processing version of a channel matched filter decorrelator cascade. In [26], more sophisticated MU detectors such as Polynomial Expansion (PE) approximations of LMMSE Rxs were considered to resolve the interference, not just between individual user signals, but between path-wise contributions of user signals because it was considered that (at least for the intering users' signals) the complex path gains (or at least their phases) that vary at the fast fading rate would be impossible to track sufficiently rapidly whereas the correlation between paths only varies at slow fading speeds. In CDMA, a typical excess spreading factor introduces the room to unravel such higher-dimensional signal mixtures.

In massive MIMO, the Tx side channel covariance matrix (16) is very likely to be (very) singular even though the channel response $\mathbf{H}$ may not be singular. So consider now singular covariance CSIT with

$$\mathrm{rank}(\mathbf{C}^t_{i,k,j} = \mathbf{A}_{i,k,j}\mathbf{A}_{i,k,j}^H) = r_{i,k,j} \; , \; \mathbf{A}_{i,k,j} : M_j \times r_{i,k,j} \tag{17}$$

Let $\mathbf{P}_{\mathbf{X}} = \mathbf{X}(\mathbf{X}^H\mathbf{X})^{\#}\mathbf{X}^H$ and $\mathbf{P}_{\mathbf{X}}^{\perp}$ be the projection matrices on the column space of $\mathbf{X}$ and its orthogonal complement resp., and $(.)^{\#}$ denotes Moore-Penrose pseudo-inverse. Consider now a massive MIMO IBC with C cells containing K_i users each to be served by a single stream. The following result states when this will be possible.

Theorem 1 Sufficiency of Covariance CSIT for Massive MIMO IBC *In the MIMO IBC with (local) covariance CSIT, all BS will be able to perform ZF BF if the following holds*

$$||\mathbf{P}^{\perp}_{\mathbf{A}_{\overline{i,k},j}} \mathbf{A}_{i,k,j}|| > 0 \; , \; \forall i,k,j \tag{18}$$

where $\mathbf{A}_{\overline{i,k},j} = \{\mathbf{A}_{n,m,j}, \, (n,m) \neq (i,k)\}$.

These conditions will be satisfied w.p. 1 if $\sum_{i=1}^{C}\sum_{k=1}^{K_i} r_{i,k,j} \leq M_j \; , \; j = 1,\ldots,C$. In that case all the column spaces of the $\mathbf{A}_{i,k,j}$ will tend to be non-overlapping. However, the conditions could very well be satisfied even if these column spaces are overlapping, in contrast to what [27], [28] appear to require.

In Theorem 1, we assume that all ZF work is done by the BS. However, if the MT have multiple antennas, they can help to a certain extent.

Theorem 2 Role of Receive Antennas in Massive MIMO IBC *If MT (i,k) disposes of $N_{i,k}$ antennas to receive a stream, it can perform rank reduction of a total amount of $N_{i,k} - 1$ to be distributed over $\{r_{i,k,j} \; , \; j = 1,\ldots,C\}$.*

Such rank reduction (by ZF of certain path contributions) facilitates the satisfaction of the conditions in Theorem 1. Note that the idea of working with singular covariance CSIT is pretty much as old as SDMA itself as in e.g. [29] where the instantaneous (rank 1, MISO) $\mathbf{H}^H\mathbf{H}$ was replaced by its excpected value $\mathbf{C}^t$, hoping that it would be quite singular.

9. FIR IA FOR ASYNCHRONOUS FIR FREQUENCY-SELECTIVE IBC

FIR frequency-selective channels can be handled by OFDM. However, this assumes that the same OFDM is used by synchronized BS. In HetNets, this may not be the case. Then FIR Tx/Rx filters may be considered. We get in the z-domain:

$$\mathbf{F}_{i,k}(z)\mathbf{H}_{i,k,j}(z)\mathbf{G}_{j,n}(z) = 0 \; , \; (i,k) \neq (j,n) \; , \tag{19}$$

If we denote by L_F, L_H, L_G the length of the 3 types of filters, then in a symmetric configuration, the proper conditions for (19) become

$$\begin{aligned} & KC\,[d(ML_G - d) + d(NL_F - d)] \geq \\ & \qquad KC(KC-1)d^2(L_H + L_G + L_F - 2) \\ & \Rightarrow d \leq \frac{ML_G + NL_F}{(KC-1)(L_H + L_G + L_F - 2) + 2} \leq \frac{\max\{M,N\}}{KC-1} \end{aligned} \tag{20}$$

where the last inequality can be attained by letting L_G or L_F tend to infinity. Unless $M \gg N$, this represents reduced DoF compared to the frequency-flat case ($d \leq (M+N)/(KC+1)$). Alternatively, the double convolution by both Tx and Rx filters can be avoided by considering most of the decoupled approaches above, leading to more traditional equalization configurations, with equal DoF possibilities for frequency-selective as for frequency-flat cases.

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