Anyhow, anytime feedback in classical multiuser channels:

recent successes and many challenges

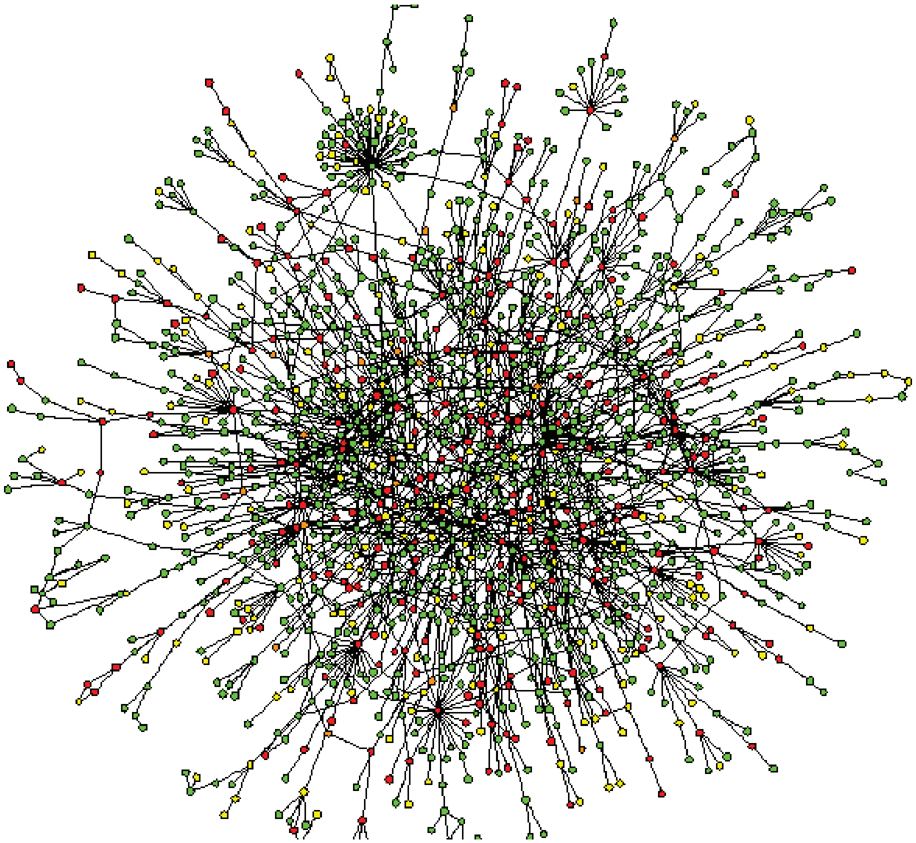
Petros Elia

EURECOM

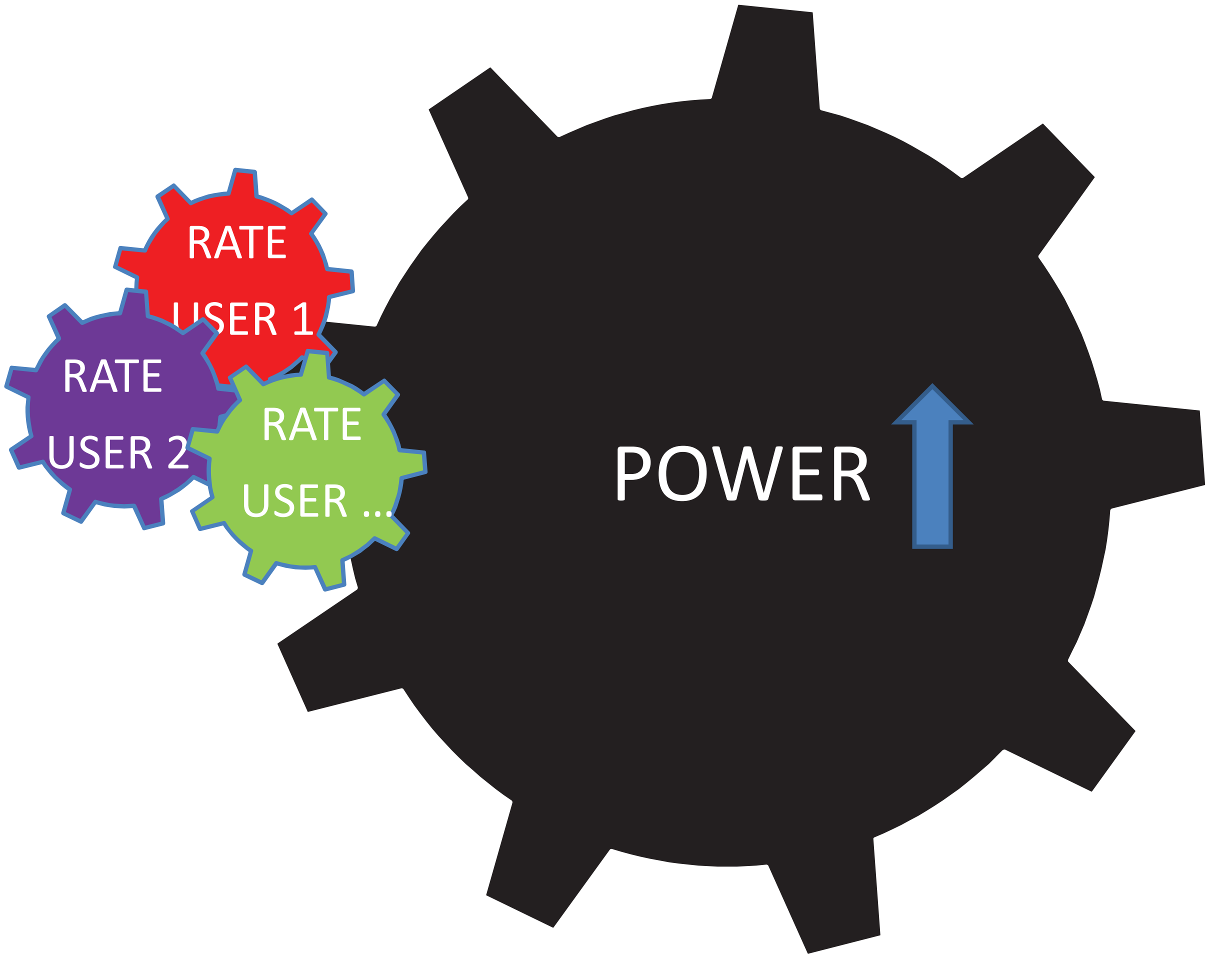
Sophia Antipolis - France

Wireless communications networks

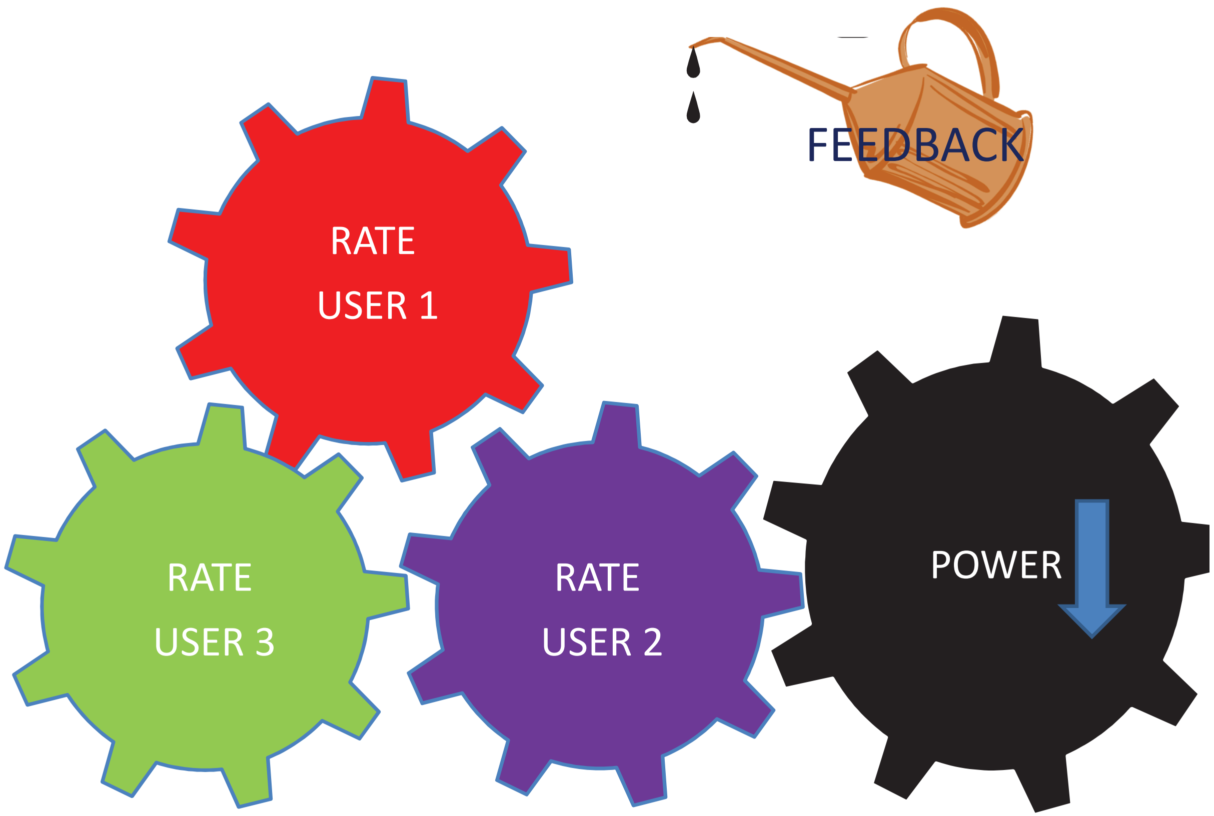
Big Challenge:
What is best way to communicate?



Distilled point-of-view in this presentation



Distilled point-of-view in this presentation[1]

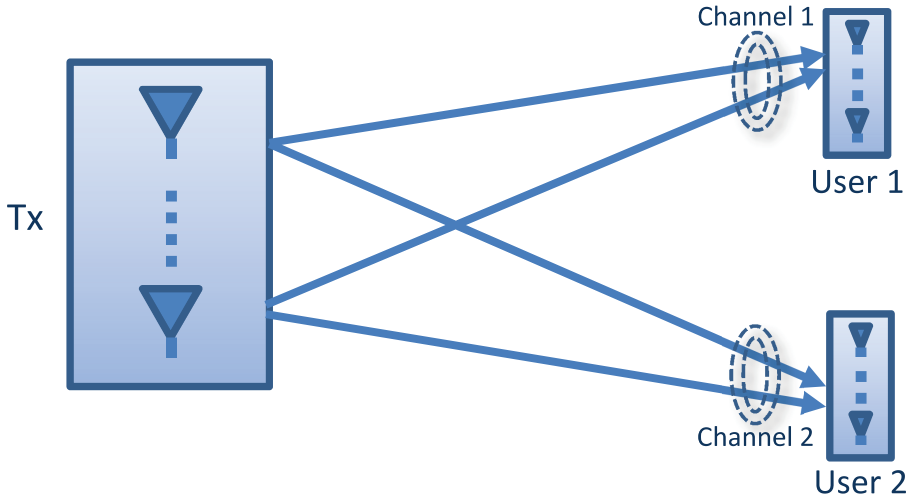


Distilled point-of-view in this presentation$_2$

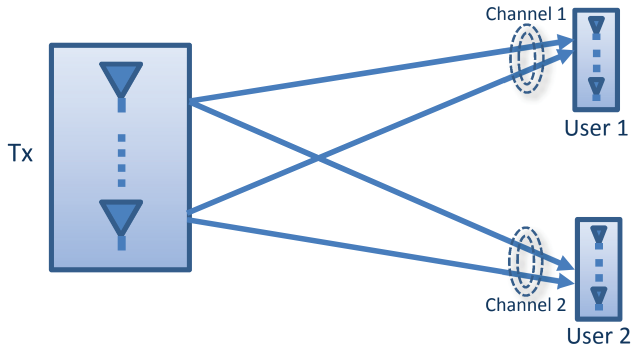
Summary of slides

- Part 1
  - ⋆ Motivation (Why feedback is important)
  - ⋆ Quick summary of basics (Capacity, Degrees-of-Freedom)
  - ⋆ Early results and basic encoding/decoding/feedback tools
- Part 2
  - ⋆ A unified exposition and a general framework
  - ⋆ Insight and answers to fundamental questions
  - ⋆ Open problems

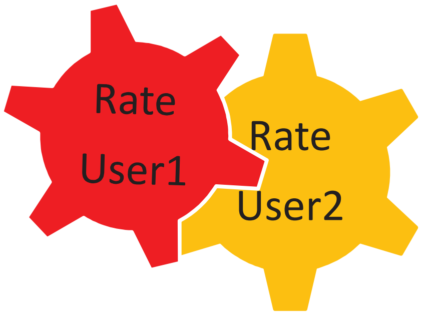
Typical multiuser scenario



Typical multiuser scenario: Interference



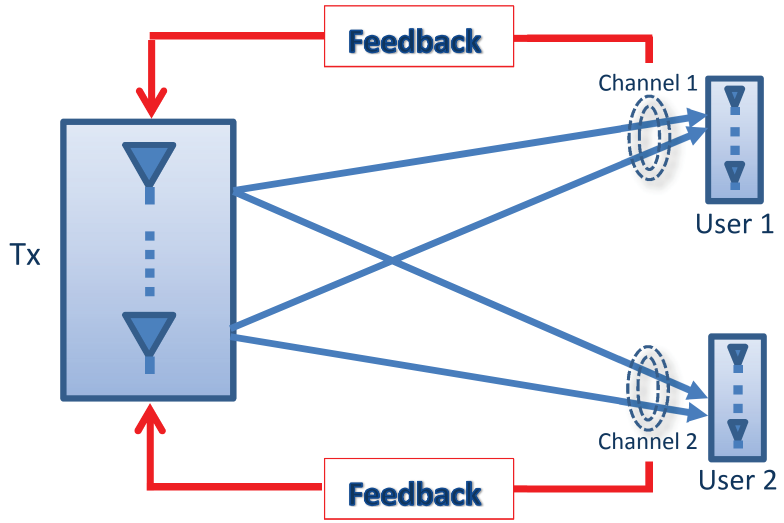
Users interfere, and must share the pie



Communications with feedback

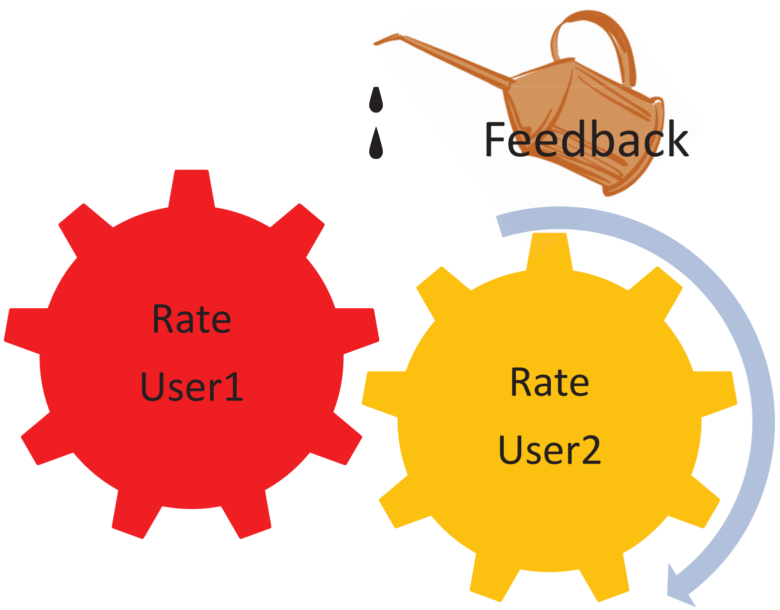
Feedback: notify transmitter of the channel state

Channel State Information at Transmitter (CSIT)



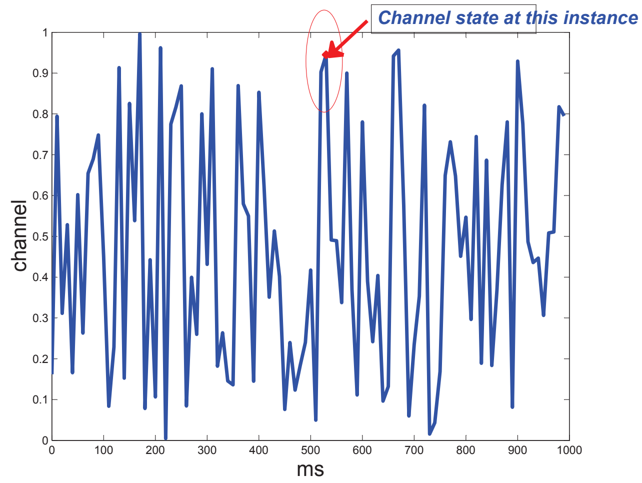
Communications with feedback$_1$

Feedback is crucial: Interference $\downarrow$ Rates $\uparrow$



Communications with feedback$_2$

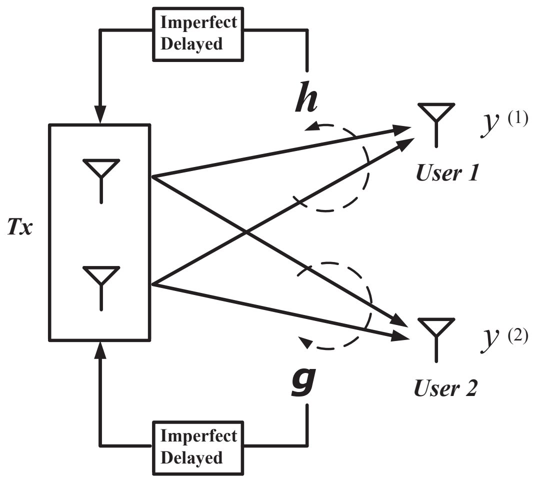
BUT! Feedback is hard to get

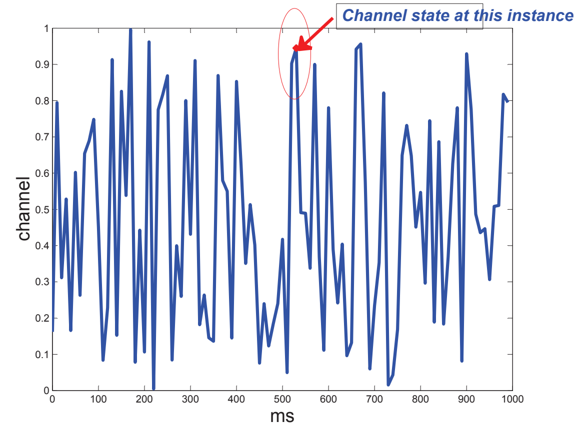


Long-standing challenge:
How to use imperfect feedback?

optimize (SNR Rate1 Rate2)

What is the source of this challenge?





- Transmit: $(\overbrace{\text{Inverse-channel}}^{\text{Feedback}} \times \text{Message}) \Rightarrow$ separates users' messages
  - ⋆ Channel $\times$ Inverse-channel $\times$ Message $\longrightarrow$ Message OK
- BUT, channel changes: Feedback can be imperfect, limited and delayed
  - ⋆ Channel $\times$ Approximately-inverse-channel $\times$ Message $\longrightarrow$ Ⓡ‡♠H∅ℷ ≗

Massive gains from resolving challenge

- No feedback: one user served at a time
- Perfect and immediate feedback: many users at a time

- Challenge: new algorithms that bridge gap

- Recent tools brought unprecedented excitement
  - ⋆ New insight sparked worldwide race to resolve challenge
  - ⋆ Much of work done after 2012

Quick summary of basics

Single link (SISO) capacity

- Flat fading (single-input single-output) channel model

$$y_t = h_t x_t + z_t$$

- Ergodic (average) capacity $\mathbb{E}_h[C(h_t)]$:

$$C_{\text{erg}} = \mathbb{E}_h \log\big(1 + P|h_t|^2\big) \approx \log P$$

Degrees of Freedom (DoF)

Degrees of freedom d

- Capacity $\approx d \cdot$ SNR
- Number of dimensions available per user

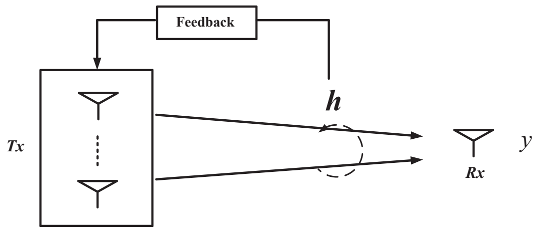
Single link (SISO) degrees of freedom

$$\text{DoF} = d \triangleq \lim_{P\to\infty} \frac{\text{Capacity}}{\log P} = \lim_{P\to\infty} \frac{\approx \log P}{\log P} = 1$$

$$\Rightarrow \text{ SISO: DoF } = 1$$

Single receiving antenna: DoF = 1

- Same holds for $n \times 1$ MISO (multiple input single output):



⋆ Instantaneous Capacity $C(\boldsymbol{h}_t)$:

$$C(\boldsymbol{h}_t) = \log\big(1 + P\|\boldsymbol{h}_t\|^2\big)$$

⋆ Ergodic capacity (MISO)

$$C_{\text{erg}} = \mathbb{E}_{\boldsymbol{h}} \log\big(1 + P\|\boldsymbol{h}_t\|^2\big) = \log P + o(\log P)$$

⋆ DoF MISO Fading

$$d = \lim_{P \to \infty} \frac{\log P + o(\log P)}{\log P} = 1$$

Importance of DoF

DoF increase means exponential power reductions

- Want to communicate at rate R
- Over 'system' with d DoF:

$$C \approx d \log_2 P$$

- Thus minimum power $P_{\min}$ so that

$$R \approx C \approx d \log_2 P_{\min}$$

$$\Rightarrow P_{\min} \approx 2^{R/d}$$

Importance of DoF$_1$

$$\Rightarrow P_{\min} \approx 2^{R/d}$$

Example ($R = 5$):

- If normal MISO ($d = 1$)

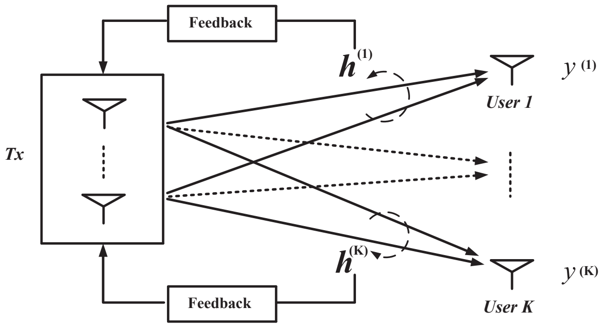
$$d = 1 \Rightarrow R \approx C \approx 1 \cdot \log_2 P \Rightarrow P_{\min} \approx 2^C \approx 2^R \approx 2^5 \approx 30$$

- If reduced MISO ($d = 1/2$)

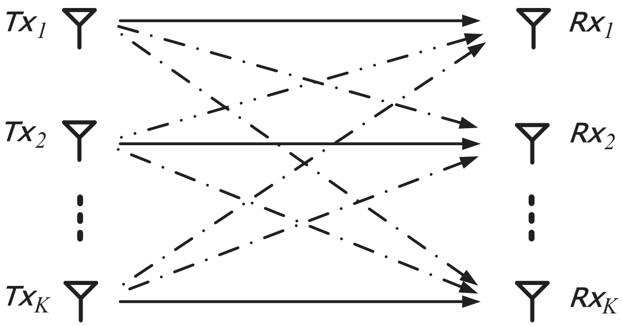
$$d = 1/2 \Rightarrow R \approx C \approx \frac{1}{2} \cdot \log_2 P \Rightarrow P_{\min} \approx 2^{10} \approx 1000$$

Multiuser Channels suffer from interference

- Interference: users must share signal dimensions
  - ⋆ DoF reduction $\Rightarrow$ Rates $\downarrow$, Power $\uparrow$,

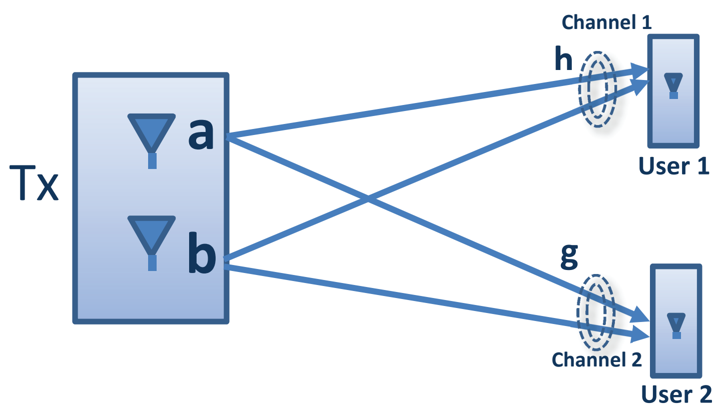


Multiuser Broadcast Channel



Multiuser Interference Channel
Multiuser X Channel

Example: interference in two-user MISO BC



- Let information symbol "a" for user 1 $\qquad \mathbb{E}|a|^2 = P$
- Let information symbol "b" for user 2 $\qquad \mathbb{E}|b|^2 = P$

Example: interference in two-user MISO BC[1]

- No feedback $\Rightarrow$ transmit $\boldsymbol{x} = \begin{bmatrix} a \\ b \end{bmatrix}$
- User 1 receives:

$$y^{(1)} = \boldsymbol{h}^T \boldsymbol{x} + w = [\ h_1 \ \ h_2\ ] \begin{bmatrix} a \\ b \end{bmatrix} = h_1 a + \underbrace{h_2 b + w}_{\text{NOISE POWER } \approx P+1}$$

- User 1 treats $h_2 b$ as noise:

$$\text{average effective SNR} = \frac{\text{'signal' power}}{\text{'noise' power}} \approx \frac{P}{P+1} \approx \text{Constant}$$

- Received SNR does not increase with transmit power!

Example: interference in two-user MISO BC$_2$

- Thus maximum rate R_{max} does not increase with increasing transmit power

$$R_{\text{max}} \approx \log\left(1 + \frac{P}{P+1}\right) = \text{constant}$$

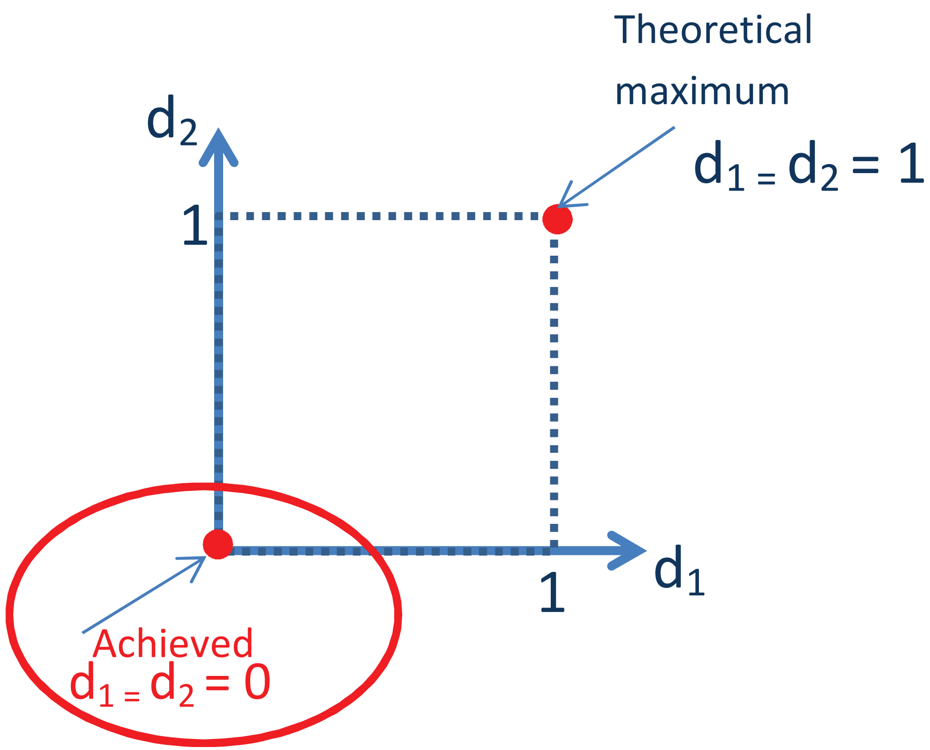
- Which means, zero DoF

$$d = \lim_{P\to\infty} \frac{R_{\text{max}}}{\log P} = \lim_{P\to\infty} \frac{\text{constant}}{\log P} = 0$$

$\star \Rightarrow$ Massive damage from inter-user interference

Example: interference in two-user MISO BC$_3$

Treating interference as noise



Time division for interference avoidance

Still No Feedback
Simple but inefficient solution: Time division (TDMA)

- First send "a" to user 1 ($t = 1$)
- Then send "b" to user 2 ($t = 2$)

- Send

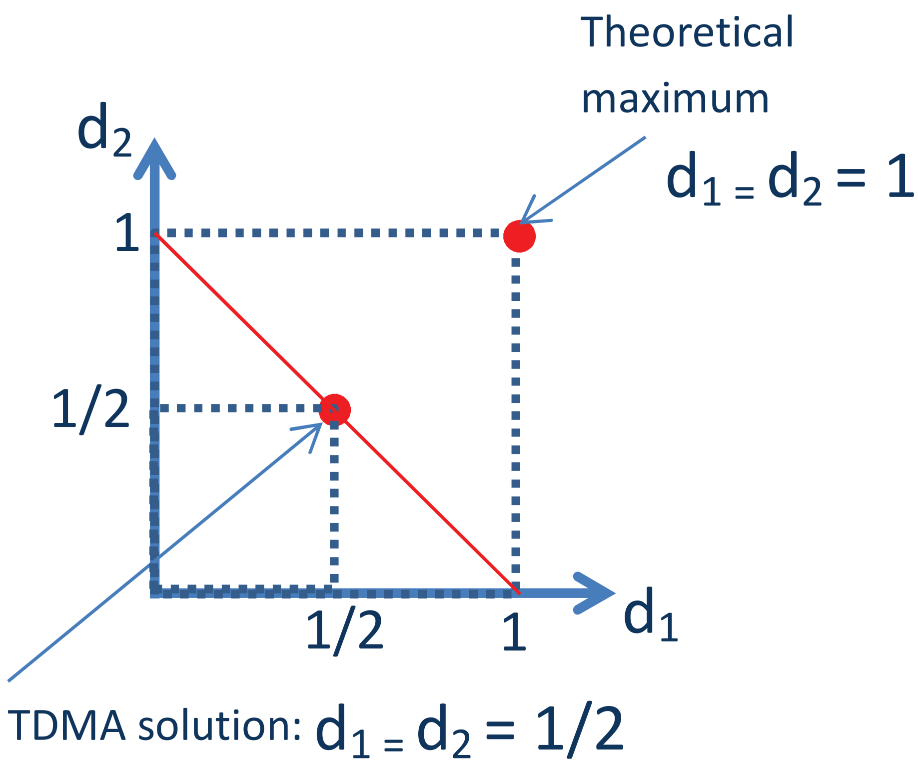
$$x(t=1) = \begin{bmatrix} a \\ 0 \end{bmatrix}, \quad x(t=2) = \begin{bmatrix} b \\ 0 \end{bmatrix}$$

- A SISO channel per user (no interference) - but double the time

$$d = \lim_{P\to\infty} \frac{1}{2}\frac{C_{\text{siso}}}{\log P} = \frac{1}{2}$$

Time division for interference avoidance$_1$

TDMA SOLUTION

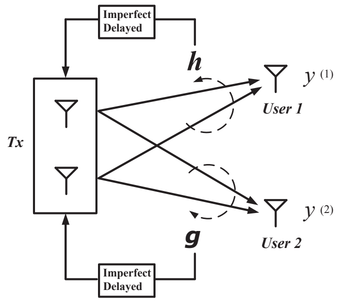


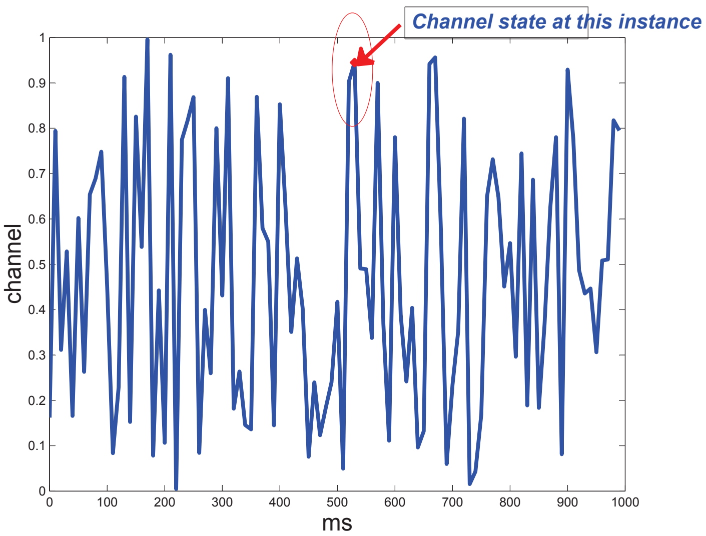
Precoding with perfect feedback

- But what if we could feedback the channel state?

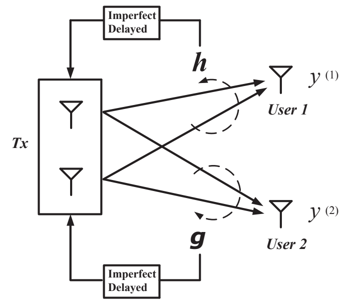
$$\mathbf{H} = \begin{bmatrix} \boldsymbol{h}^T \\ \boldsymbol{g}^T \end{bmatrix}$$

- Send $\mathbf{H}$ to the transmitter





Precoding with perfect feedback



PRECODING

- Instead of sending $\begin{bmatrix} a \\ b \end{bmatrix}$, now could send

$$\boldsymbol{x} = \mathbf{H}^{-1} \begin{bmatrix} a \\ b \end{bmatrix}$$

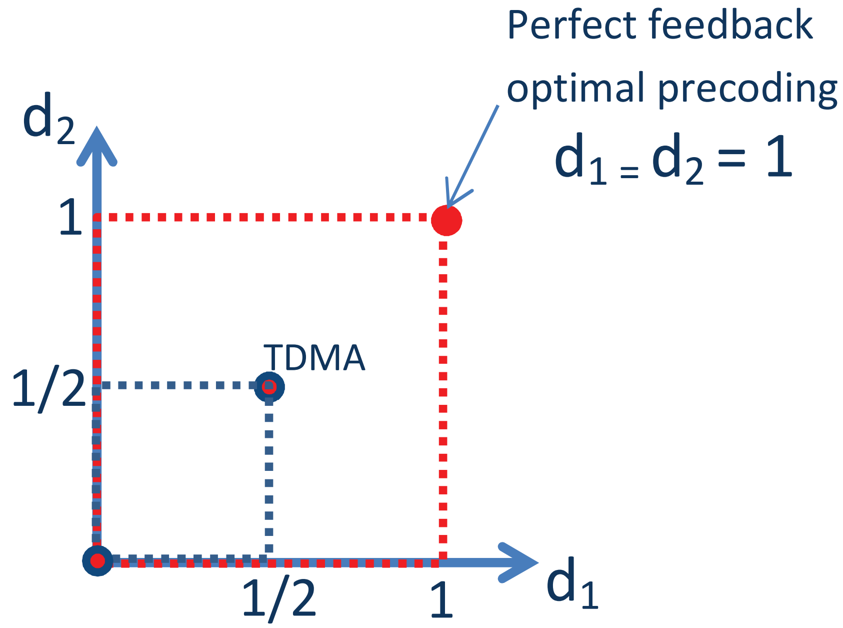
Precoding with perfect feedback₁

$$\begin{bmatrix} y^{(1)} \\ y^{(2)} \end{bmatrix} = \mathbf{H}\boldsymbol{x} + \boldsymbol{z} = \mathbf{H}\,\overbrace{\mathbf{H}^{-1}\begin{bmatrix} a \\ b \end{bmatrix}}^{\boldsymbol{x}} + \boldsymbol{z} = \begin{bmatrix} a \\ b \end{bmatrix} + \boldsymbol{z}$$

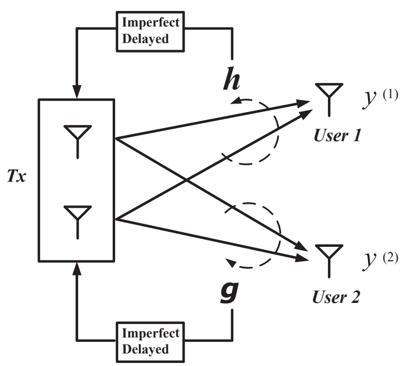
$$\begin{aligned} y^{(1)} &= a + z^{(1)} \qquad \text{user 1: DoF} = d_1 = 1 \\ y^{(2)} &= b + z^{(2)} \qquad \text{user 2: DoF} = d_2 = 1 \end{aligned}$$

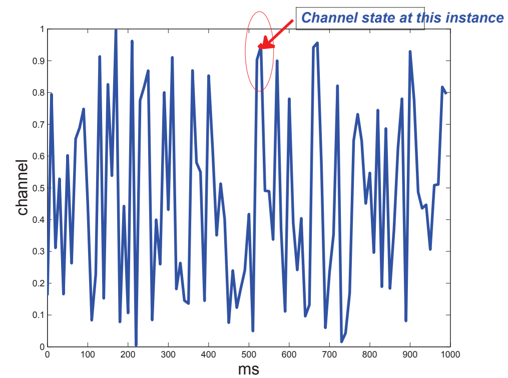
- Knowledge of *channel state information at the transmitter* (CSIT) is important (knowledge of $\mathbf{H}$)
  - ⋆ Precoding allows for separation of signals

Perfect feedback allows for optimal DoF



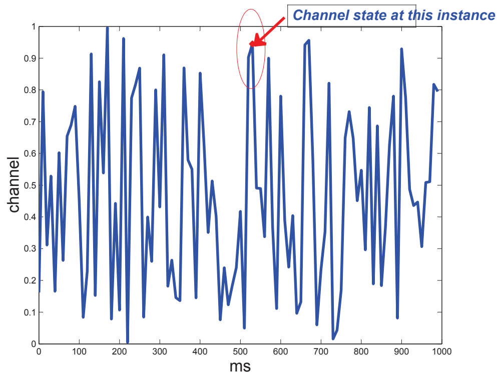
But remember our problem





- Feedback can be imperfect, limited and delayed
  - ⋆ Channel $\times$ Approximately-inverse-channel $\times$ Message $\longrightarrow$ $\circledR\ddagger\spadesuit_{H}\emptyset\daleth\ \circeq$

Pertinent questions



- How to exploit predicted CSIT
- How to exploit delayed CSIT
- How to exploit imperfect CSIT
- How to minimize total amount of (delayed + current) feedback?
- How to achieve optimality even with feedback delays?
- How to utilize gradually arriving feedback?
- How much feedback quality and when?

Toy examples for insight

Of course, the problem has randomness

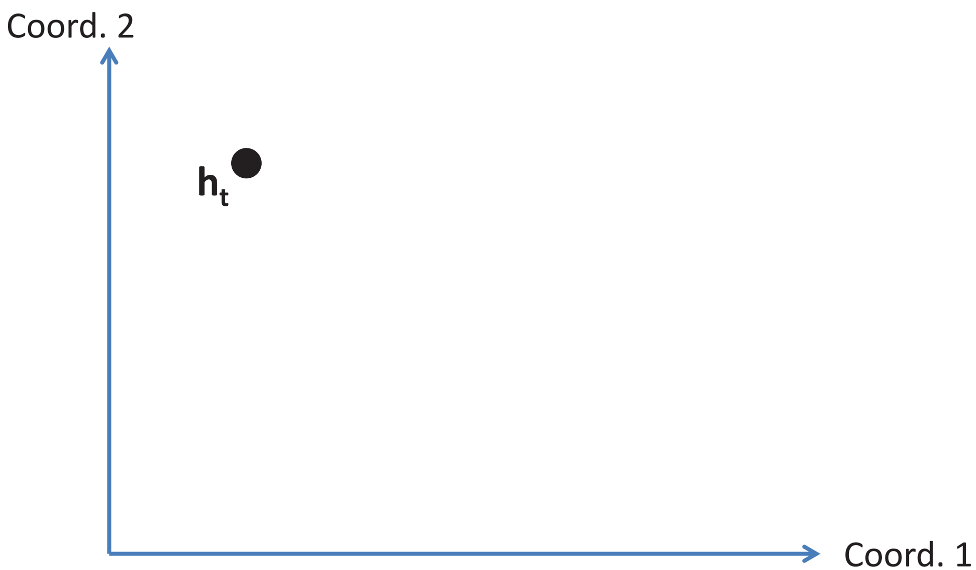
Let us get some insight on the involved randomness

Let us look at some (simplistic) toy examples

An instance of channel at time t

Fix channel time **t,** and estimation time **t'**

$\mathbf{h}_t$ and $\hat{\mathbf{h}}_{t,t'}$ (random vector)



Instance of channel and its estimate at time t'

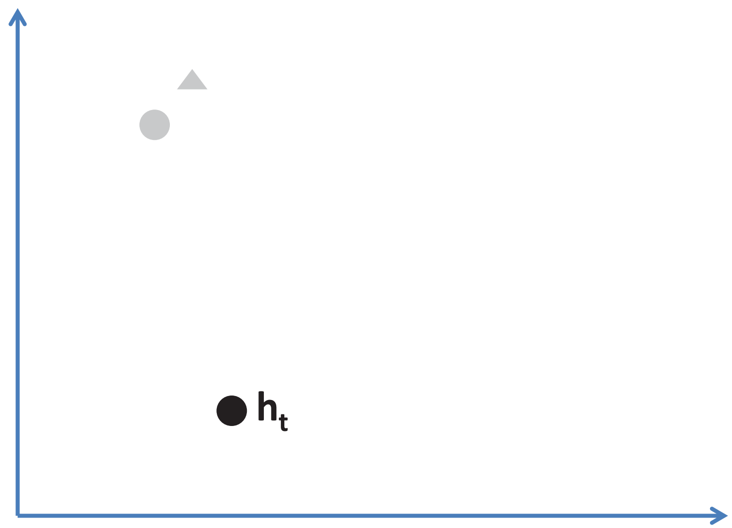
Fix channel time **t,** and estimation time **t'**

$\mathbf{h}_t$ and $\hat{\mathbf{h}}_{t,t'}$ (random vector)

Another instance of channel at time t

Fix channel time **t,** and estimation time **t'**

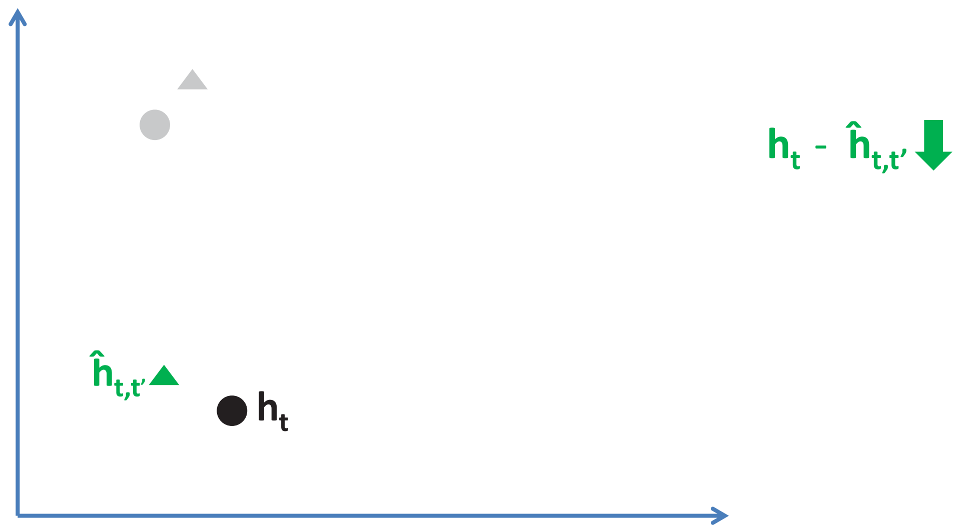
$\mathbf{h}_t$ and $\hat{\mathbf{h}}_{t,t'}$ (random vector)



Again estimate was good

Fix channel time **t,** and estimation time **t'**

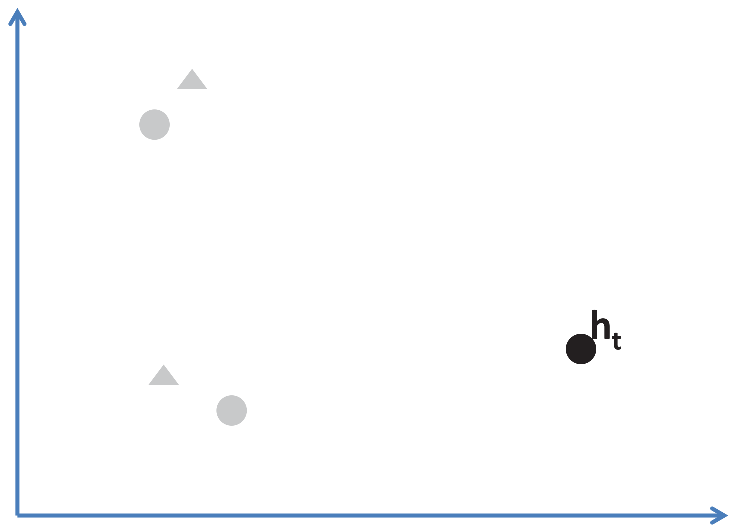
$\mathbf{h}_t$ and $\hat{\mathbf{h}}_{t,t'}$ (random vector)



...yet another instance

Fix channel time $\mathbf{t}$, and estimation time $\mathbf{t'}$

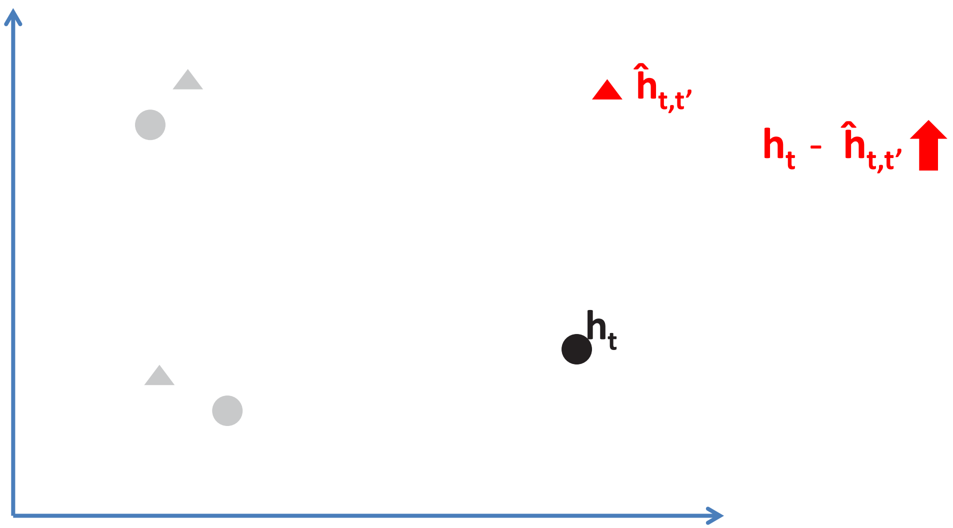
$\mathbf{h_t}$ and $\hat{\mathbf{h}}_{\mathbf{t,t'}}$ (random vector)



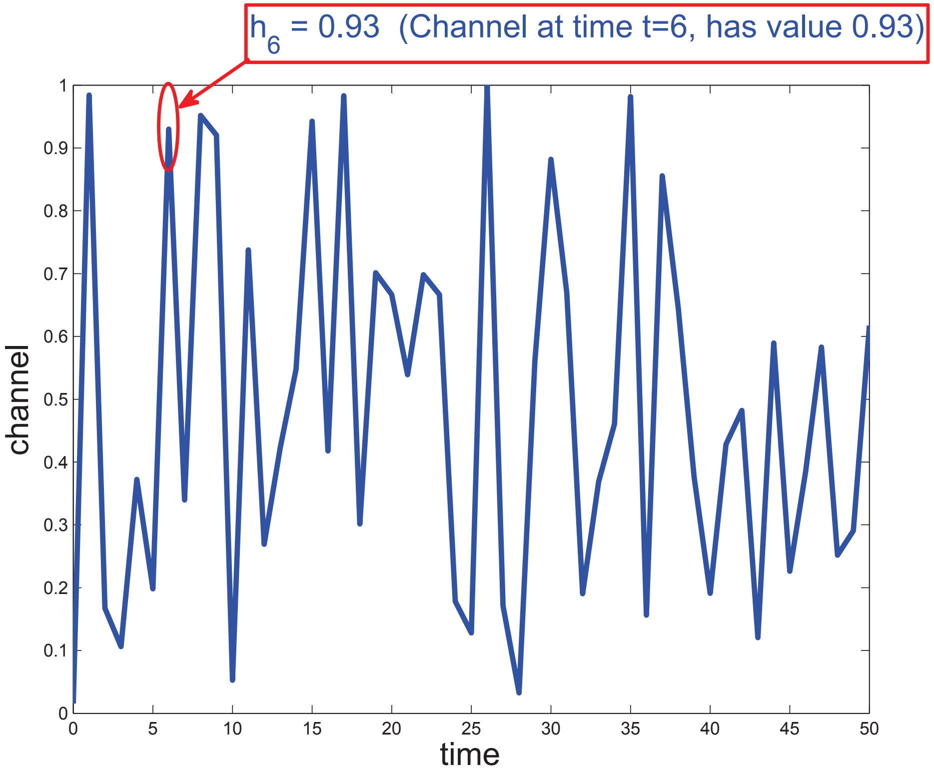
Now, estimation was bad

Fix channel-time **t,** and estimation-time **t'**

$\mathbf{h}_t$ and $\hat{\mathbf{h}}_{t,t'}$ (random vector)



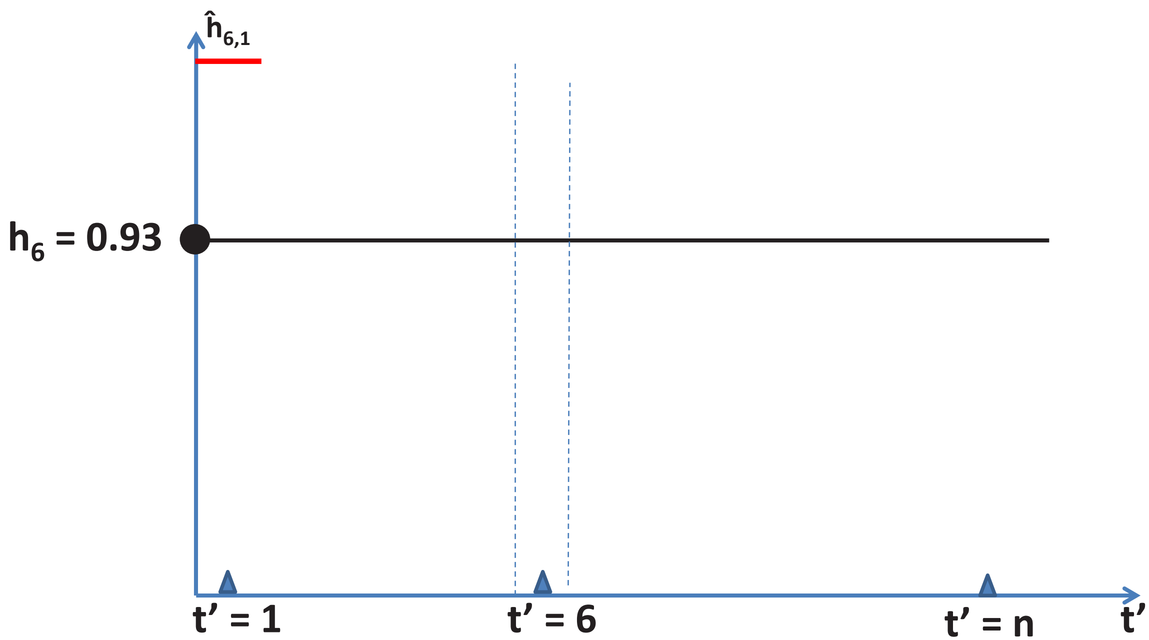
Another point of view: progressive knowledge of channel



Progressive knowledge of channel

What do we know - at any point in time t' - about channel $\mathbf{h}_t$ (e.g **t=6**)?

$\hat{\mathbf{h}}_{6,t'}$ **t' = 1,2,3.....**

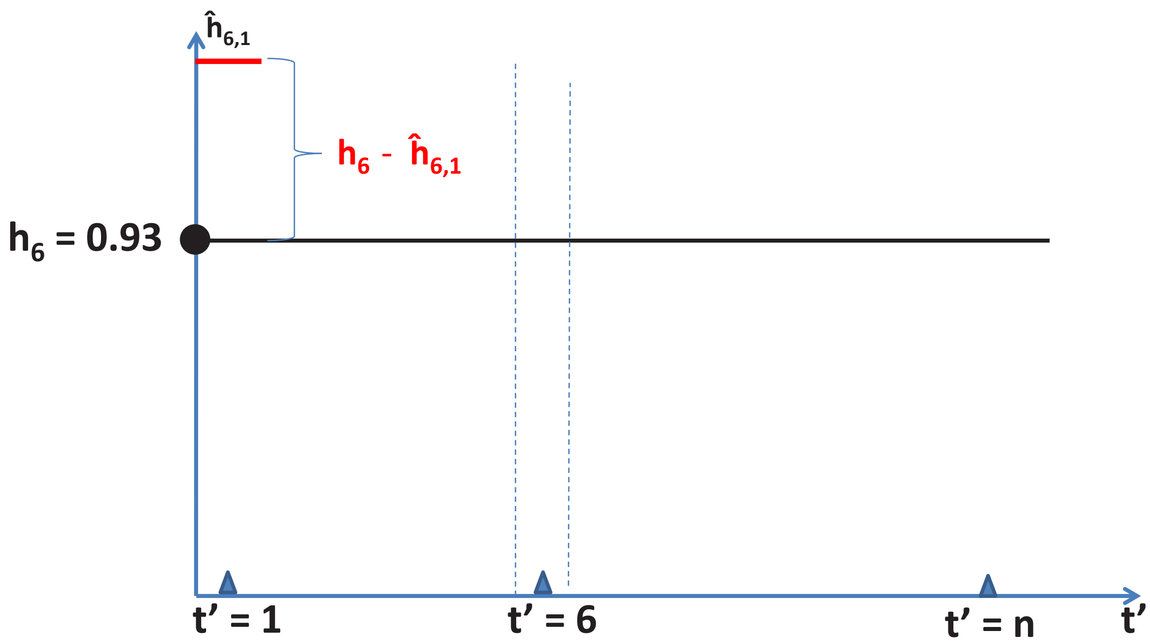


Knowledge at time t' = 1,2,3.....

No prediction at $t' = 1$ of $\boldsymbol{h}_6$

What do we know - at any point in time t' - about channel $\mathbf{h_6}$?

$\hat{\mathbf{h}}_{6,t'}$ t' = 1,2,3.....

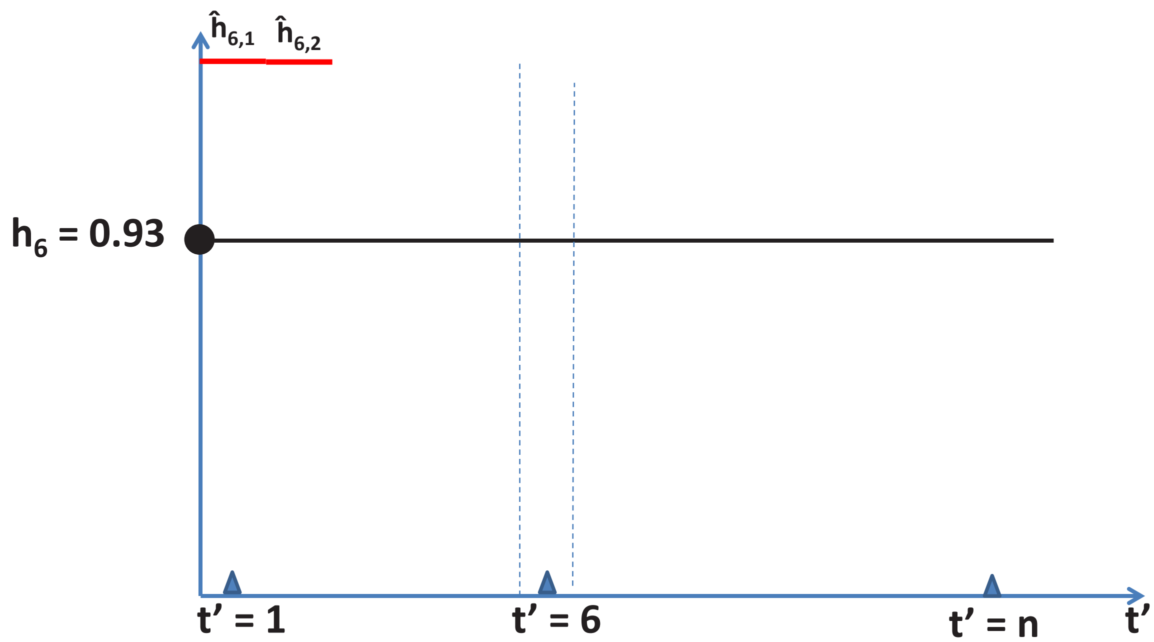


Knowledge at time t' = 1,2,3.....

Still no (of $\boldsymbol{h}_6$) prediction at $t' = 2$

What do we know - at any point in time t' - about channel $\mathbf{h}_6$?

$\hat{\mathbf{h}}_{6,t'}$ **t' = 1,2,3.....**

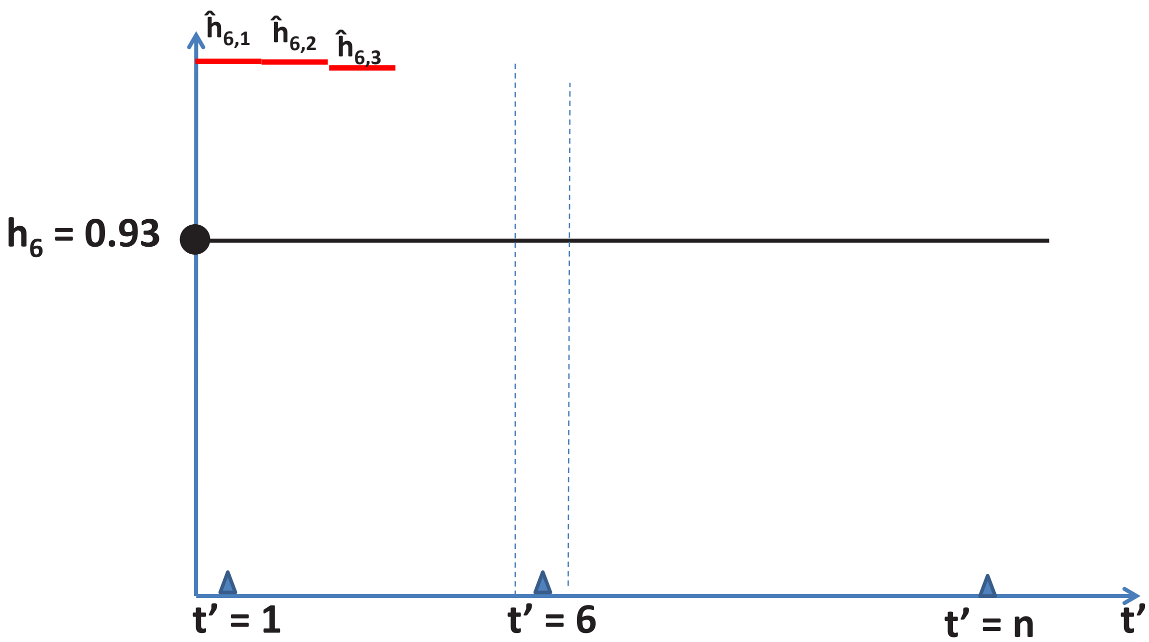


Knowledge at time t' = 1,2,3.....

Vague prediction (of $\boldsymbol{h}_6$) at time $t' = 3$ - high error

What do we know - at any point in time t' - about channel $\mathbf{h_6}$?

$\hat{\mathbf{h}}_{6,t'}$ **t' = 1,2,3.....**

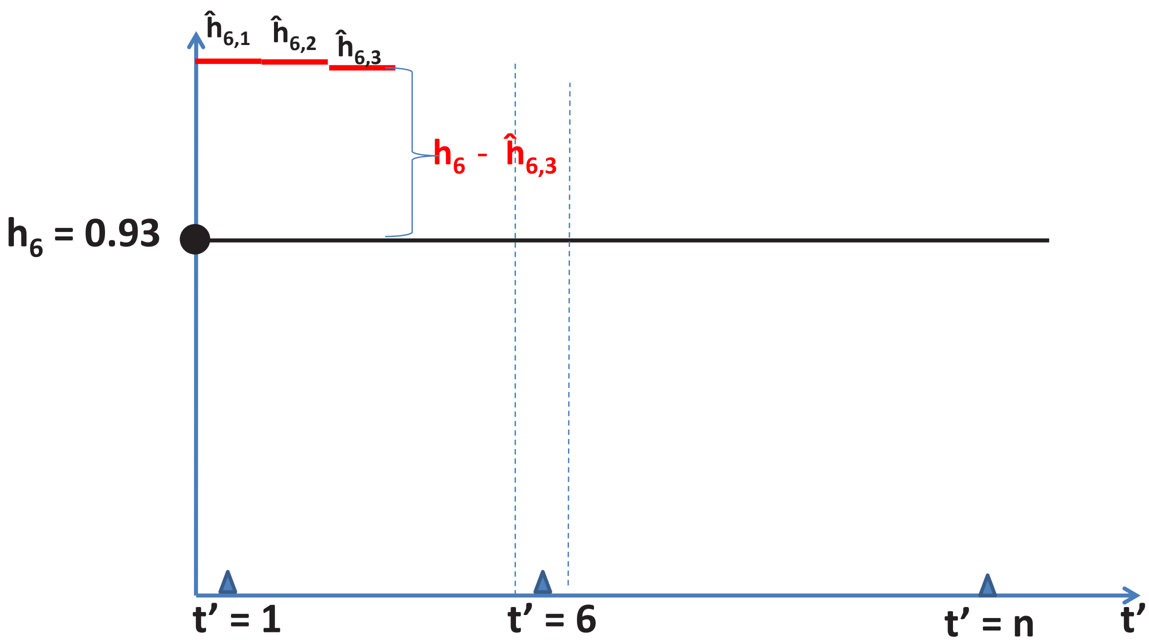


Knowledge at time t' = 1,2,3.....

Vague prediction (of $\boldsymbol{h}_6$) at time $t' = 3$ - high error[1]

What do we know - at any point in time t' - about channel $\mathbf{h_6}$?

$\hat{\mathbf{h}}_{6,t'}$ **t' = 1,2,3.....**

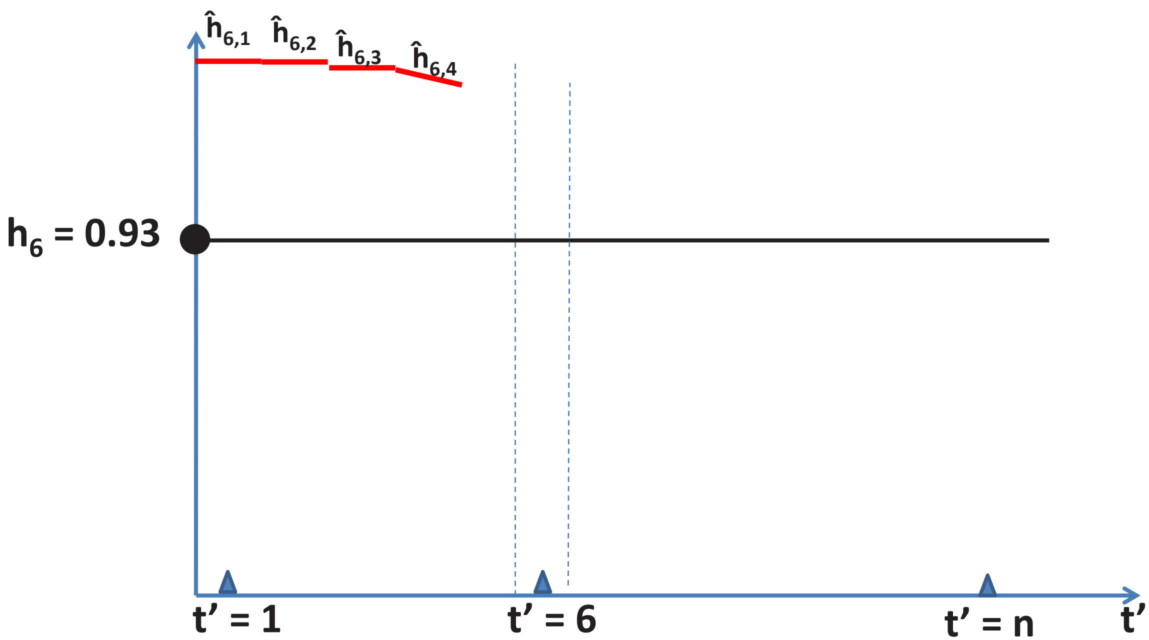


Knowledge at time t' = 1,2,3.....

..getting better ($t' = 4$)

What do we know - at any point in time t' - about channel $\mathbf{h_6}$?

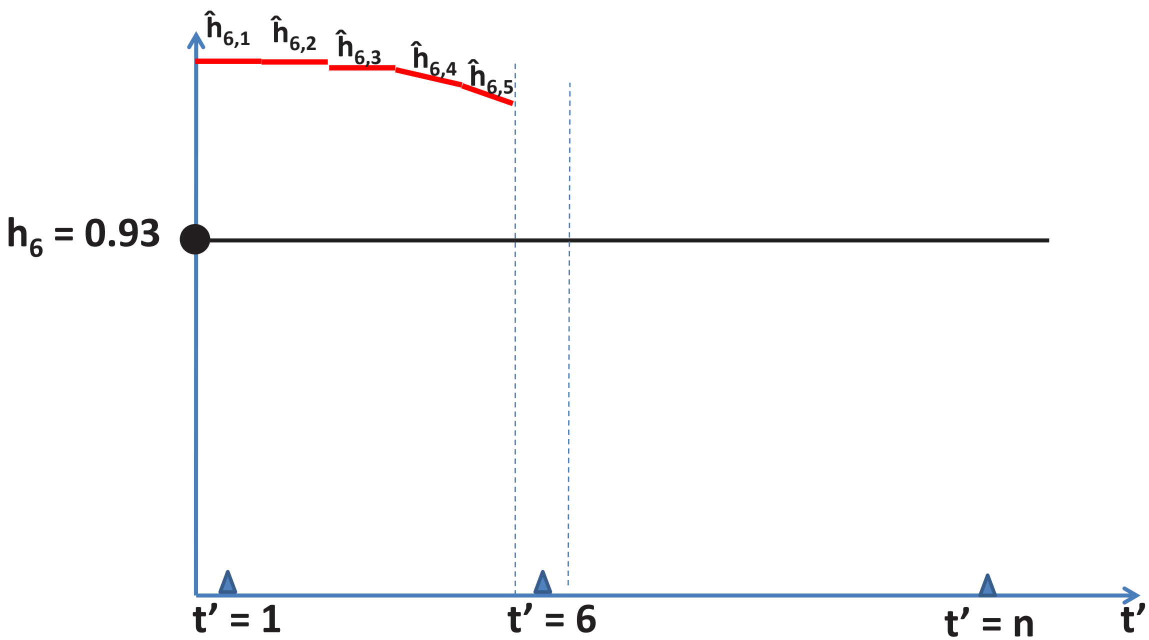
$\hat{\mathbf{h}}_{6,t'}$ **t' = 1,2,3.....**



Knowledge at time t' = 1,2,3.....

What do we know - at any point in time t' - about channel $\mathbf{h_6}$?

$\hat{\mathbf{h}}_{6,t'}$ **t' = 1,2,3.....**

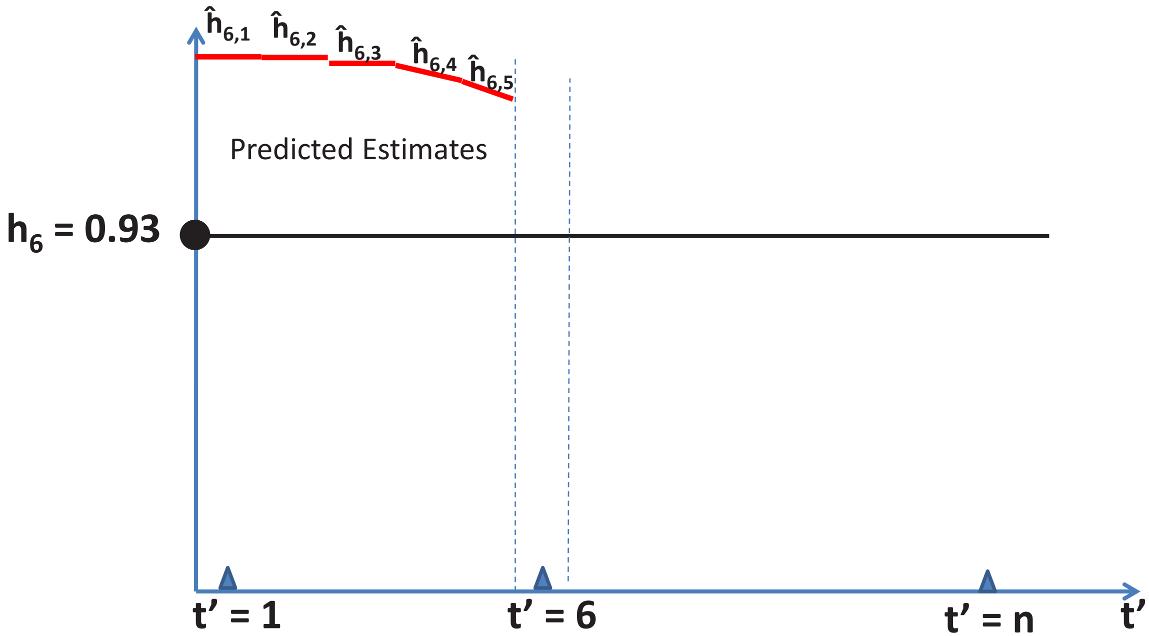


Knowledge at time t' = 1,2,3.....

These are the predicted estimates of $\boldsymbol{h}_6$

What do we know - at any point in time t' - about channel $\mathbf{h_6}$?

$\hat{\mathbf{h}}_{6,t'}$ **t' = 1,2,3.....**

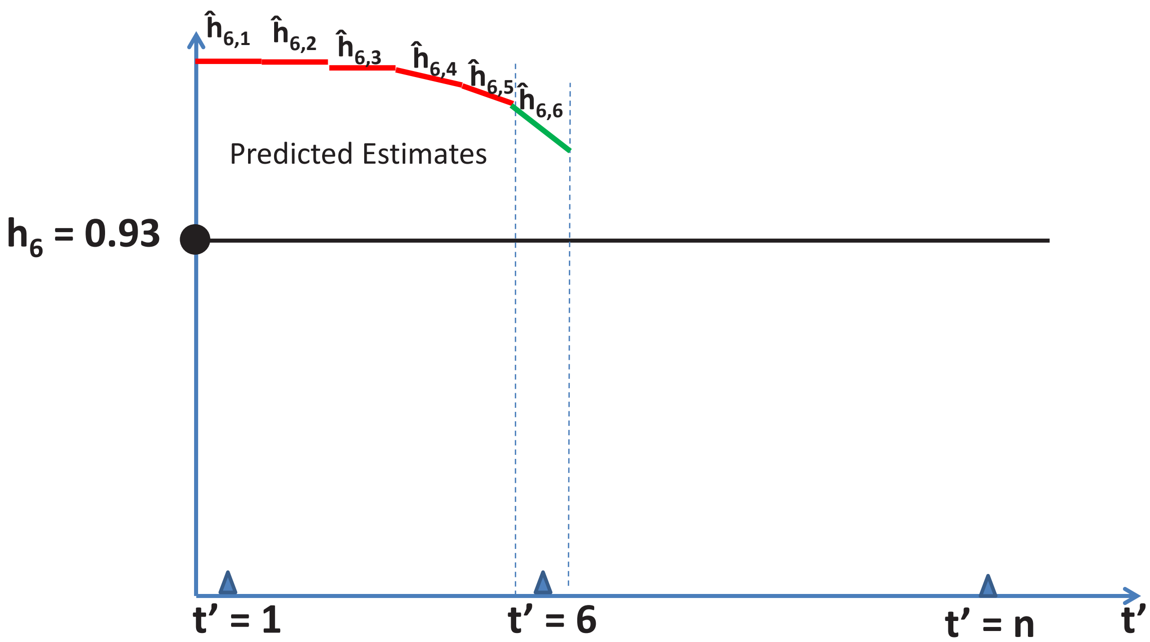


Knowledge at time t' = 1,2,3.....

‘Current estimate’ of $\boldsymbol{h}_6$ at $t' = t = 6$

What do we know - at any point in time t’ - about channel $\mathbf{h_6}$?

$\hat{\mathbf{h}}_{6,t'}$ **t’ = 1,2,3.....**

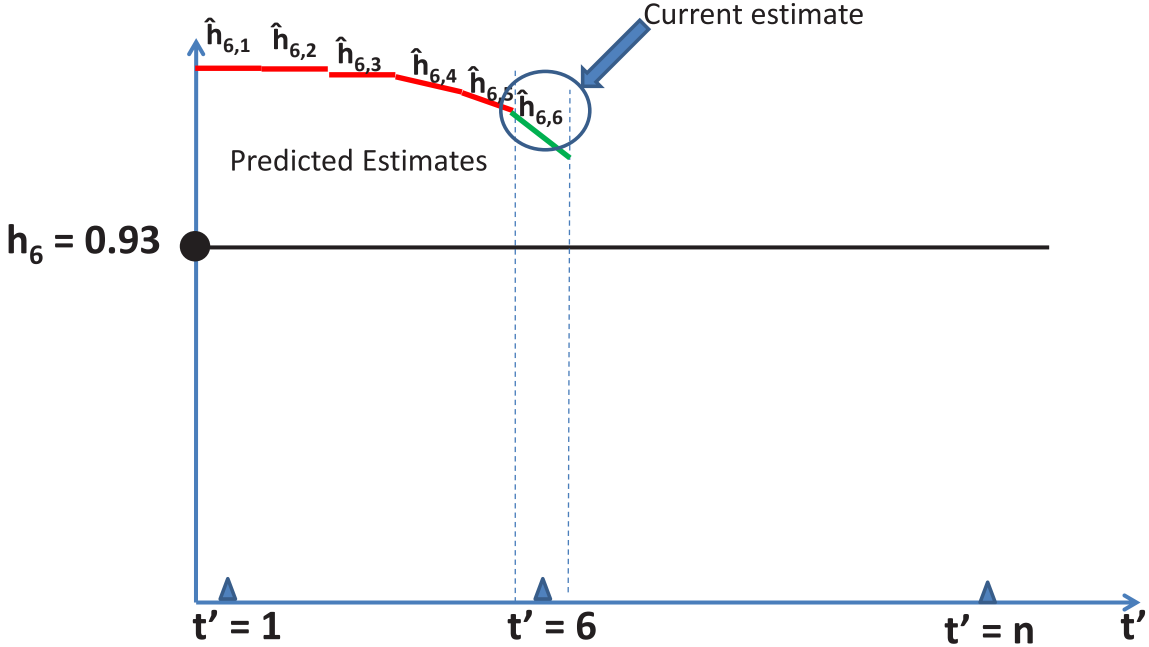


Knowledge at time t’ = 1,2,3.....

'Current estimate' of $\boldsymbol{h}_6$ at $t' = t = 6$

What do we know - at any point in time t' - about channel $\mathbf{h}_6$?

$\hat{\mathbf{h}}_{6,t'}$ **t' = 1,2,3.....**

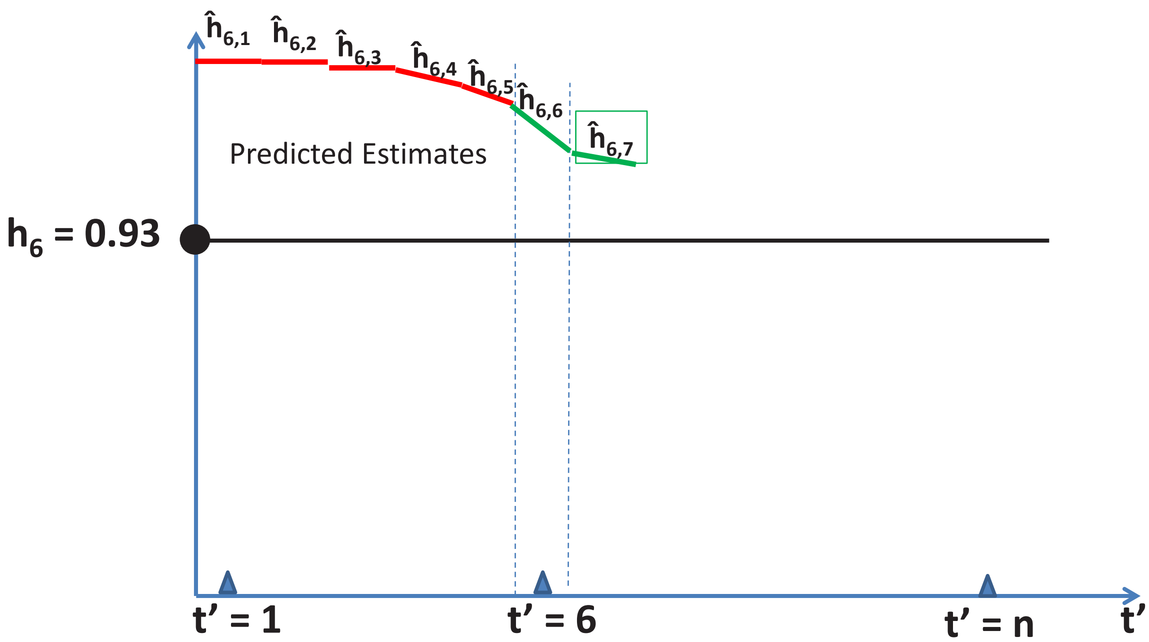


Knowledge at time t' = 1,2,3.....

‘Delayed estimates’ at $t' > t = 6, \quad t' \leq n$

What do we know - at any point in time t’ - about channel $\mathbf{h_6}$?

$\hat{\mathbf{h}}_{6,t'}$ **t’ = 1,2,3…..**

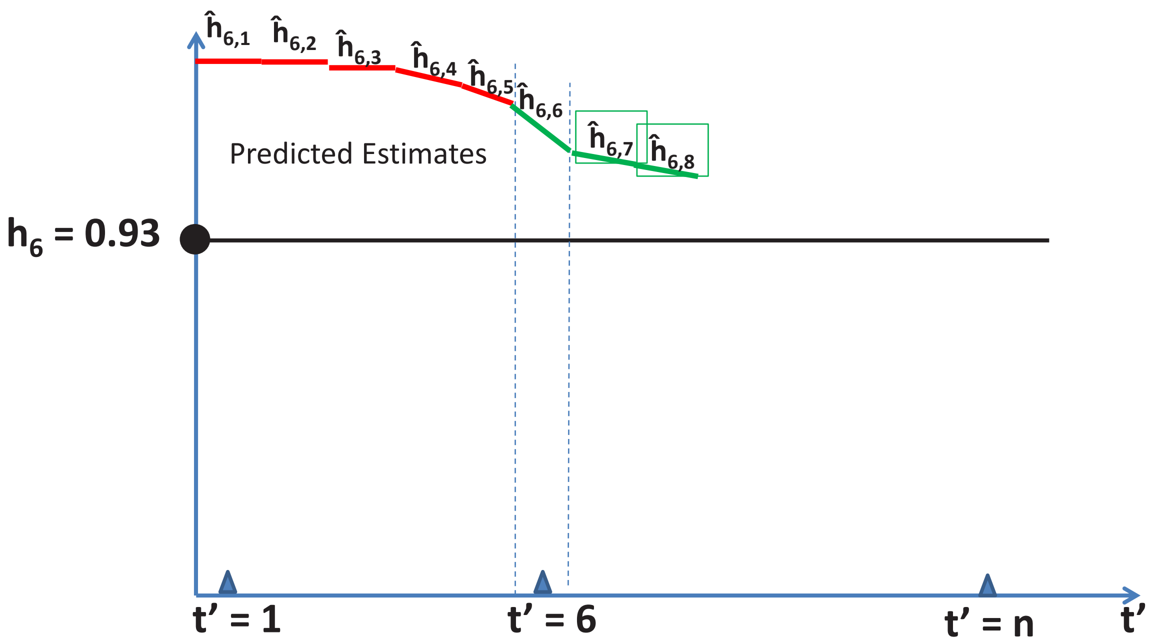


Knowledge at time t’ = 1,2,3…..

‘Delayed estimate’ at $t' > t = 6, \quad t' \leq n$

What do we know - at any point in time t’ - about channel $\mathbf{h_6}$?

$\hat{\mathbf{h}}_{6,t'}$ **t’ = 1,2,3…..**

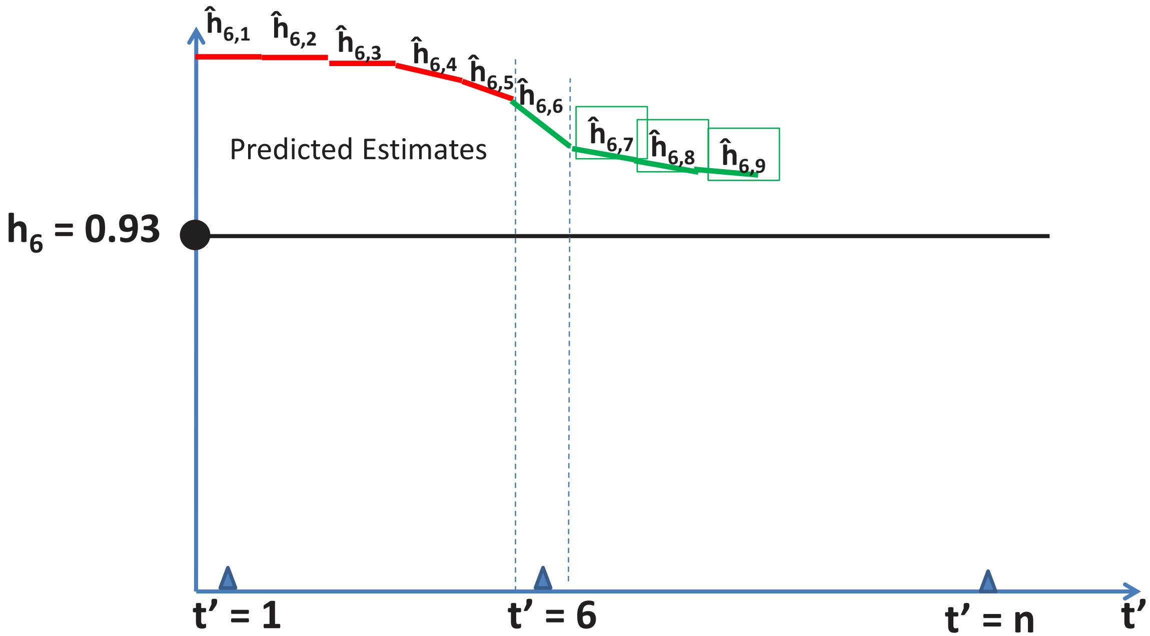


Knowledge at time t’ = 1,2,3…..

'Delayed estimate' at $t' > t = 6, \quad t' \leq n$

What do we know - at any point in time t' - about channel $\mathbf{h_6}$?

$\hat{\mathbf{h}}_{6,t'}$ **t' = 1,2,3.....**

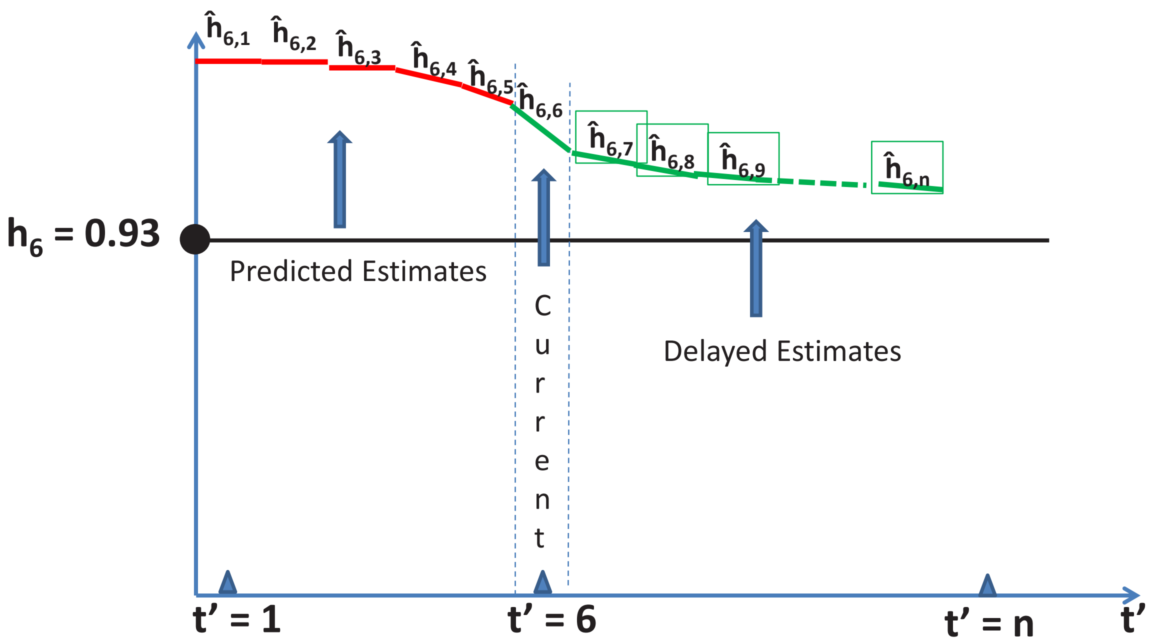


Knowledge at time t' = 1,2,3.....

‘Delayed estimate’ at $t' > t = 6, \quad t' \leq n$

What do we know - at any point in time t’ - about channel $\mathbf{h}_6$?

$\hat{\mathbf{h}}_{6,t'}$ **t’ = 1,2,3…..**

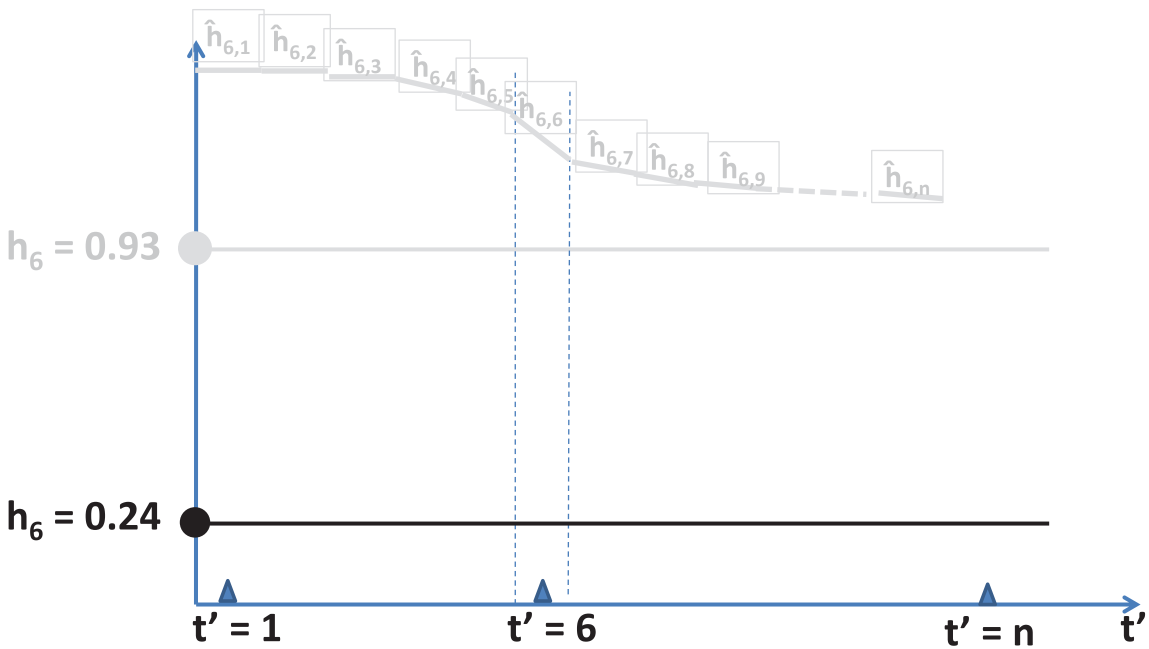


Knowledge at time t’ = 1,2,3…..

And similarly another channel instance for $\boldsymbol{h}_6$

What do we know - at any point in time t' - about channel $\mathbf{h}_6$?

$\hat{\mathbf{h}}_{6,t'}$ **t' = 1,2,3.....**

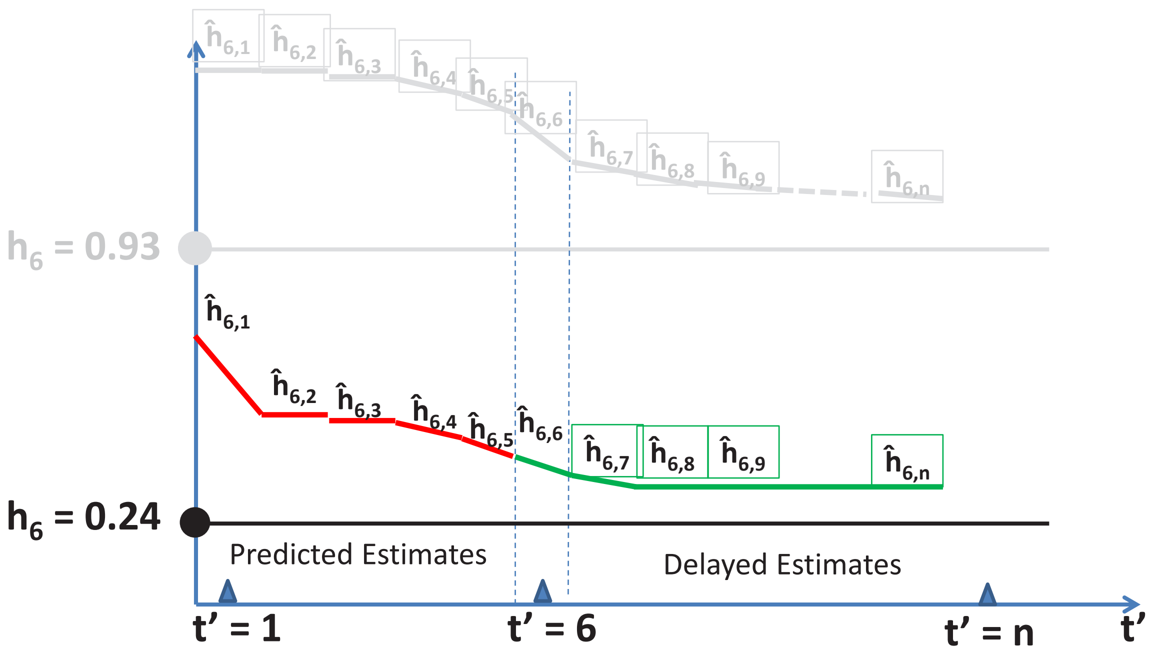


Knowledge at time t' = 1,2,3.....

And another CSIT estimate instance: $t' = 1 \to n$

What do we know - at any point in time t' - about channel $\mathbf{h}_6$?

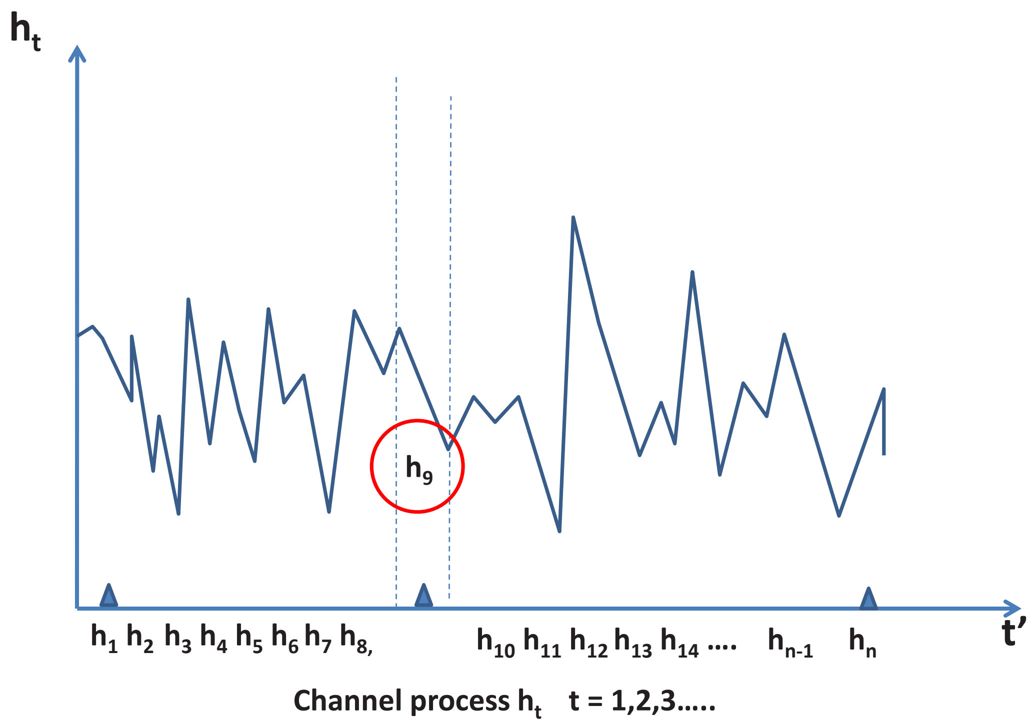
$\hat{\mathbf{h}}_{6,t'}$ **t' = 1,2,3.....**



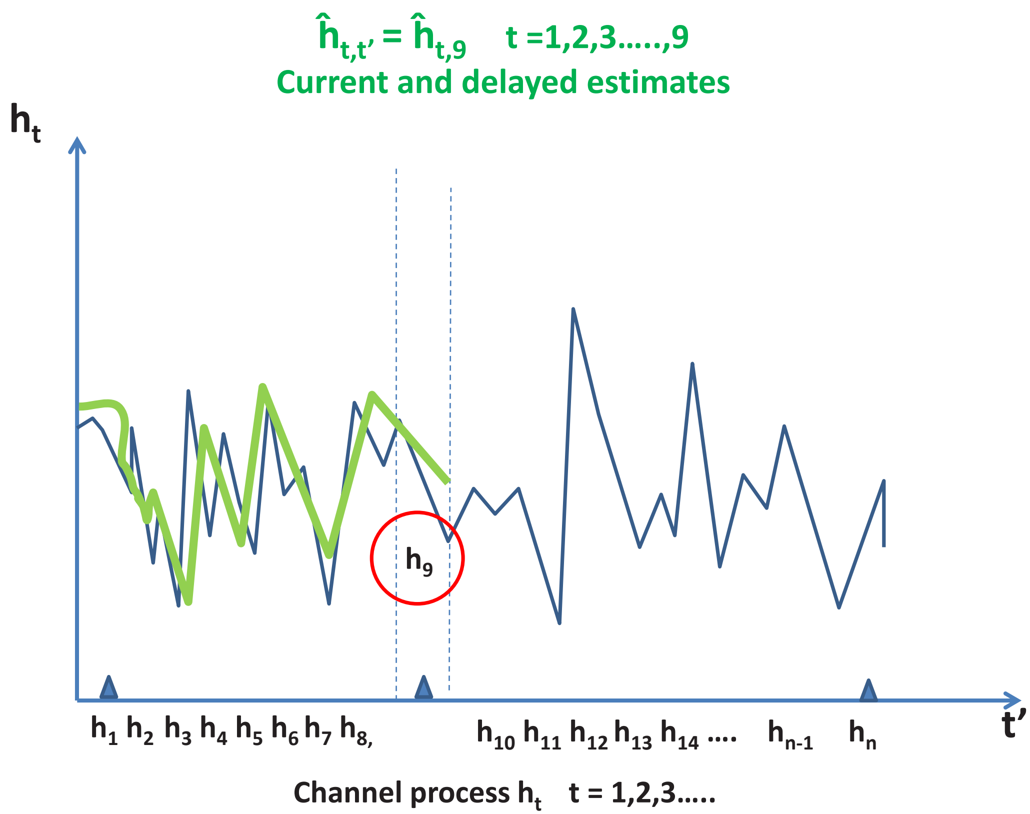
Knowledge at time t' = 1,2,3.....

Yet another point of view - knowledge of channel process

What do we know at time t’, about the channel process (say t’=9)

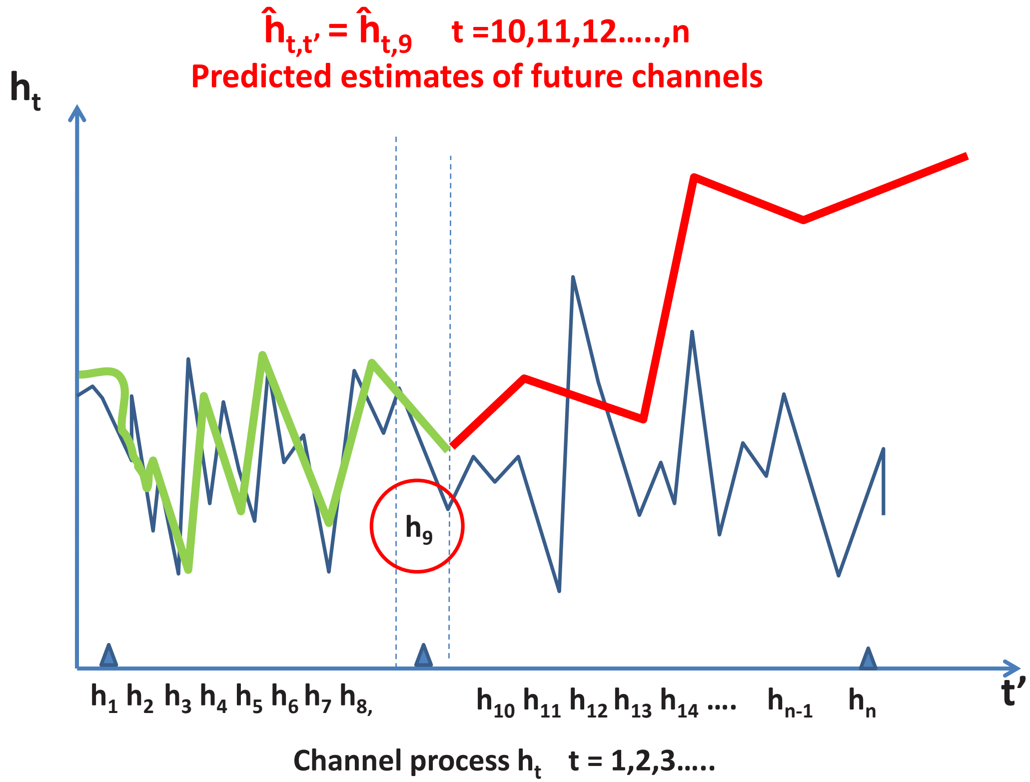


What do we know at time t', about the channel process (say t'=9)



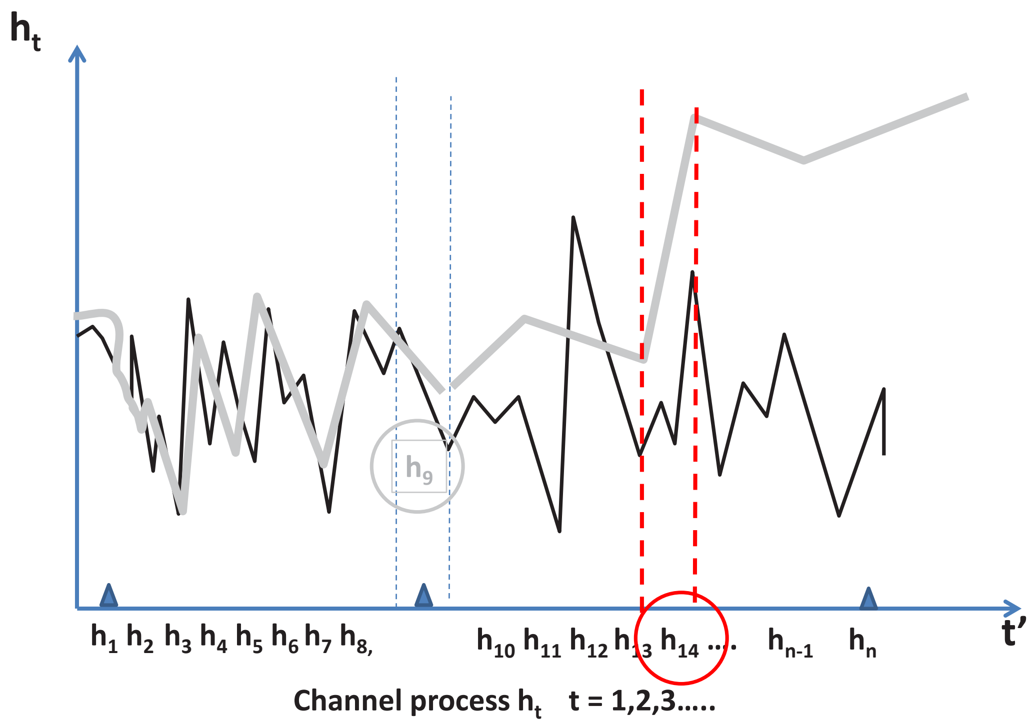
What we know at $t' = 9$, about future channels

What do we know at time t', about the channel process (say t'=9)



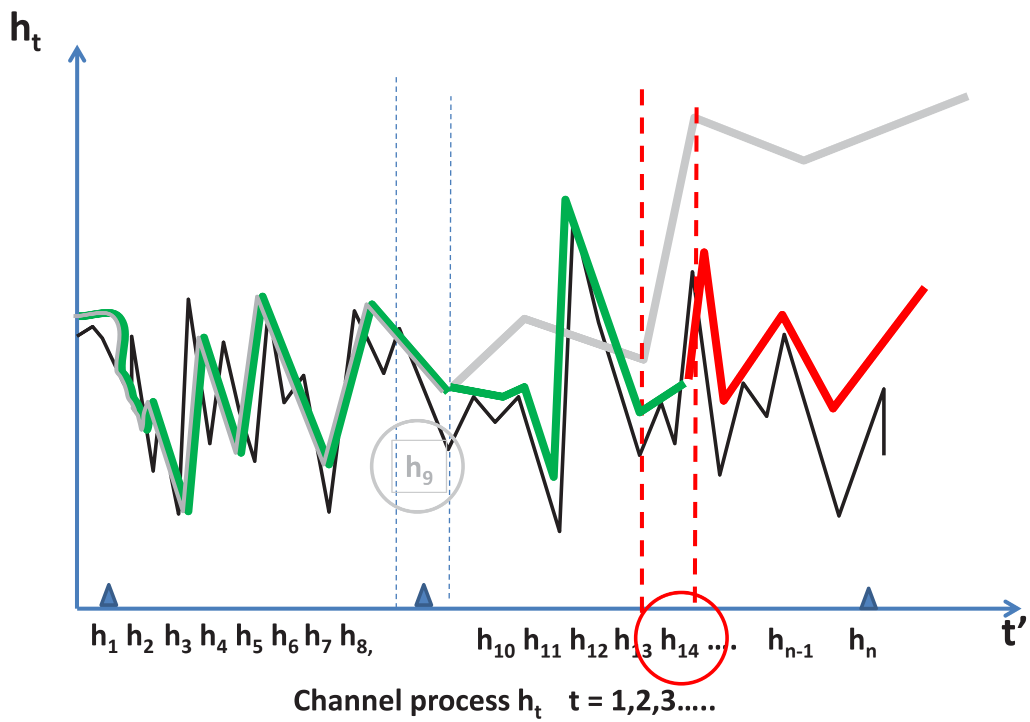
What is our knowledge at time $t' = 14$?

What do we know - at time t' = 14 – about the channel process?



Good for past, not so good for future

What do we know - at time t' = 14 – about the channel process?

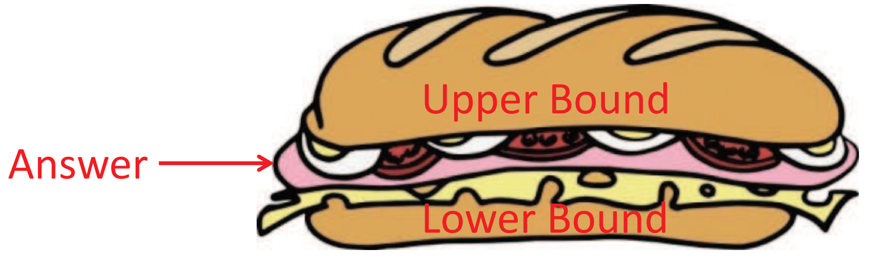


Learning tools of the trade

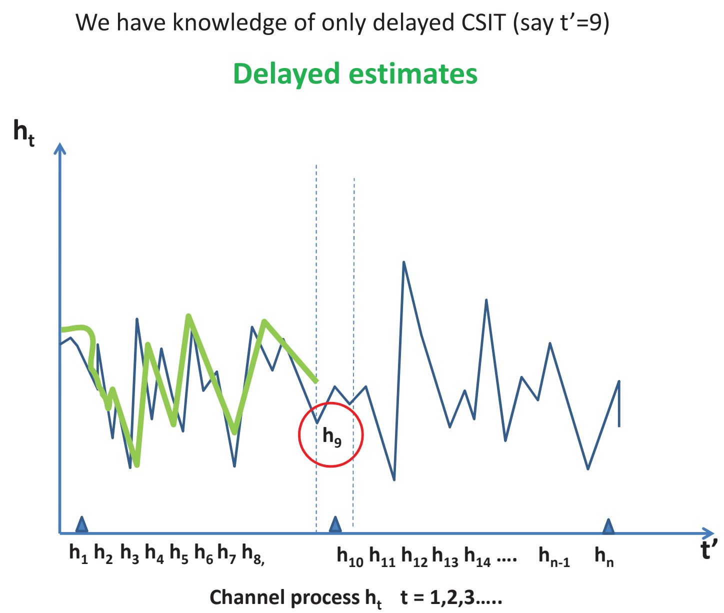
Let us learn how to utilize different tools of the trade

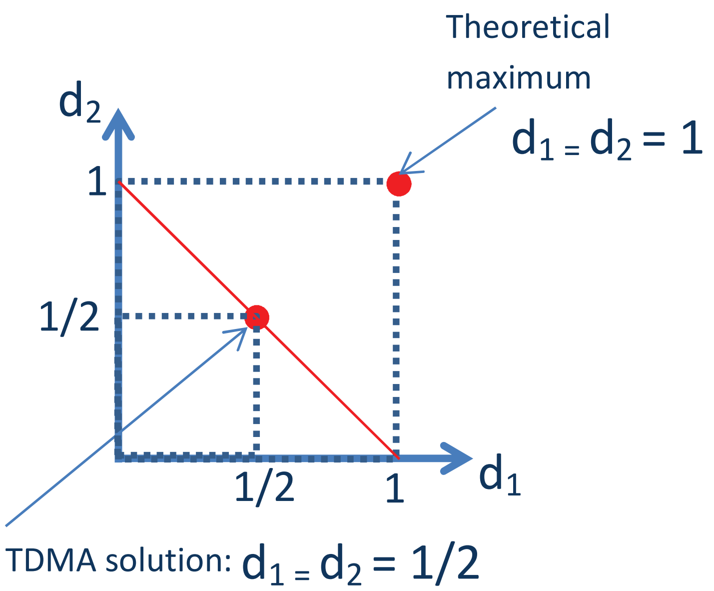
Answers in the form of:

- Novel precoders/decoders that cleverly use feedback
- Information theoretic outer bounds (try to prove optimality)

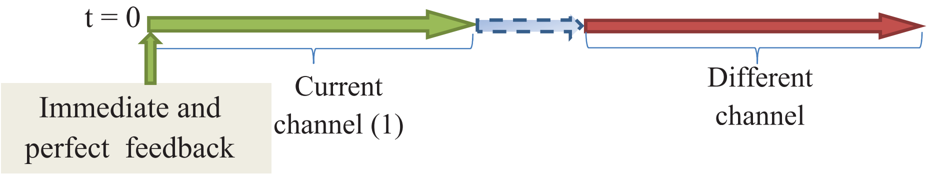


HOW TO UTILIZE DELAYED FEEDBACK?





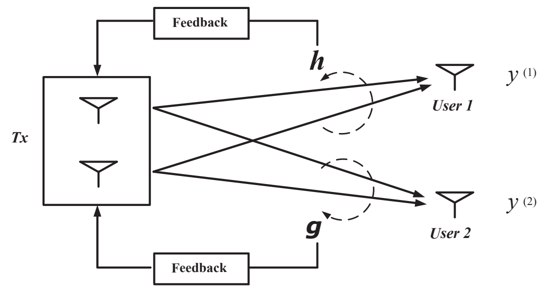
Delayed vs. current CSIT in block fading

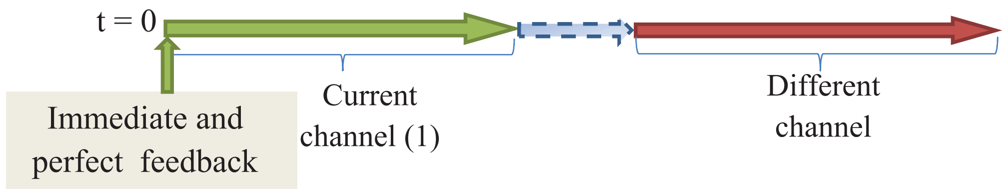


- Perfect current CSIT is that which arrives immediately
  - ⋆ At the very beginning of the coherence period of the channel
  - ⋆ At time t: $\boldsymbol{h}_t, \boldsymbol{g}_t$ unknown to transmitter

- Delayed CSIT:
  - ⋆ At time $t + \tau$, $\tau > T \triangleq T_c$: $\boldsymbol{h}_t, \boldsymbol{g}_t$ perfectly known to transmitter
  - ⋆ Feedback comes with substantial delay - after channel changes

Utilizing Delayed CSIT - MAT



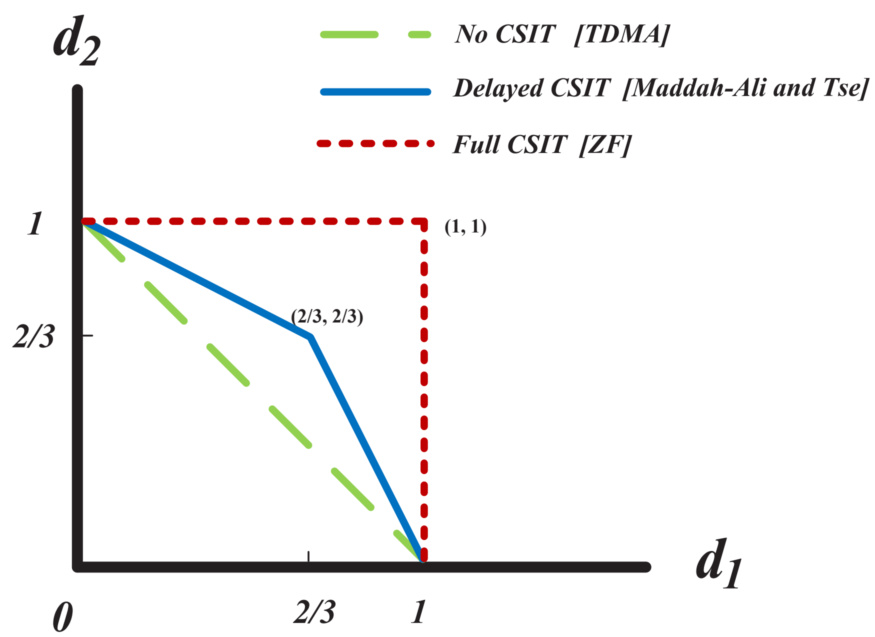


No current CSIT BUT perfect delayed CSIT

coherence block	1	2	3	4	$\cdots$
	$-$	$\boldsymbol{h}_1$	$\boldsymbol{h}_2$	$\boldsymbol{h}_3$	$\cdots$
	$-$	$\boldsymbol{g}_1$	$\boldsymbol{g}_2$	$\boldsymbol{g}_3$	$\cdots$

- *Theorem (Maddah-Ali and Tse): Optimal DoF*

$$d_1 = d_2 = 2/3$$



Maddah-Ali and Tse (MAT) intuition

- Intuition 1: interference alignment in space and time
- Intuition 2: current interference known at transmitter at later time
- Intuition 3: do the damage now, and fix it later

Maddah-Ali and Tse (MAT) scheme

- Tx sends symbols a_1, a_2 for user 1, and b_1, b_2 for user 2, in 3 channel uses
  - ⋆ WOLOG consider $T_{\text{coh}} = 1$ (unit coherence period)
  - ⋆ Duration $T = 3$: Tx sequentially sends vectors $\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3 \in \mathbb{C}^2$

- In the first two channel uses:

$$t = 1 : \ \boldsymbol{x}_1 = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \qquad \begin{aligned} y_1^{(1)} &= \boldsymbol{h}_1^\top \boldsymbol{x}_1 + \text{noise} \\ y_1^{(2)} &= \boldsymbol{g}_1^\top \boldsymbol{x}_1 + \text{noise} \end{aligned}$$

$$t = 2 : \ \boldsymbol{x}_2 = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \qquad \begin{aligned} y_2^{(1)} &= \boldsymbol{h}_2^\top \boldsymbol{x}_2 + \text{noise} \\ y_2^{(2)} &= \boldsymbol{g}_2^\top \boldsymbol{x}_2 + \text{noise} \end{aligned}$$

Maddah-Ali and Tse (MAT) scheme[1]

- After two coherence blocks: Tx reconstructs $\boldsymbol{g}_1^\top \boldsymbol{x}_1$ and $\boldsymbol{h}_2^\top \boldsymbol{x}_2$
  - ⋆ using knowledge of delayed CSIT

$$t = 3 : \ \boldsymbol{x}_3 = \begin{bmatrix} \boldsymbol{h}_2^\top \boldsymbol{x}_2 + \boldsymbol{g}_1^\top \boldsymbol{x}_1 \\ 0 \end{bmatrix}, \qquad \begin{matrix} y_3^{(1)}/h_{3,1} = \boldsymbol{h}_2^\top \boldsymbol{x}_2 + \boldsymbol{g}_1^\top \boldsymbol{x}_1 + \text{noise} \\ y_3^{(2)}/g_{3,1} = \boldsymbol{h}_2^\top \boldsymbol{x}_2 + \boldsymbol{g}_1^\top \boldsymbol{x}_1 + \text{noise} \end{matrix}$$

- $\boldsymbol{h}_t \triangleq [h_{t,1} \ h_{t,2}]^\top$, $\boldsymbol{g}_t \triangleq [g_{t,1} \ g_{t,2}]^\top$, then user 1 has

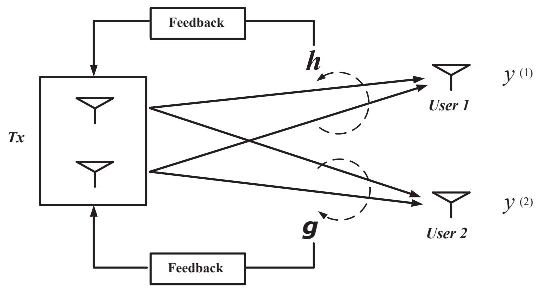
$$\tilde{\boldsymbol{y}}^{(1)} \triangleq \begin{bmatrix} y_1^{(1)} \\ y_3^{(1)}/h_{3,1} - y_2^{(1)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{h}_1^\top \\ \boldsymbol{g}_1^\top \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \text{noise}$$

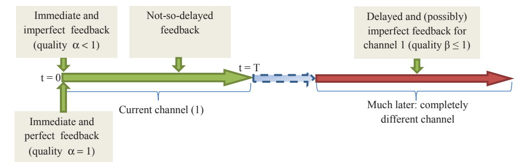
- Each user decodes two symbols in three timeslots: $d_1 = d_2 = 2/3$
- Intuition: Space-time interference alignment, retrospective interference cancelation using delayed CSIT

Feedback asymmetry: one user has more feedback

FEEDBACK ASYMMETRY: ONE USER HAS MORE FEEDBACK

One user has more feedback: Maleki, Jafar and Shamai

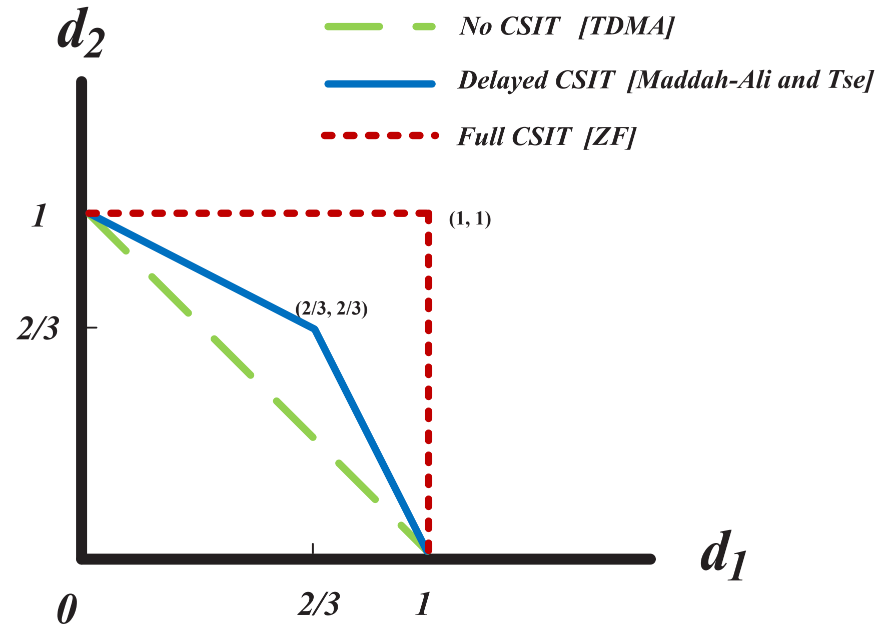




- Current CSIT for $\boldsymbol{h}_t$ (of 1st user): Perfectly and instantly known at Tx
- Delayed CSIT for $\boldsymbol{g}_t$ (of 2nd user): Perfectly known to Tx after coherence period passes

coherence block	1	2	3	4	$\cdots$
	$\boldsymbol{h}_1$	$\boldsymbol{h}_2$	$\boldsymbol{h}_3$	$\boldsymbol{h}_4$	$\cdots$
	$-$	$\boldsymbol{g}_1$	$\boldsymbol{g}_2$	$\boldsymbol{g}_3$	$\cdots$

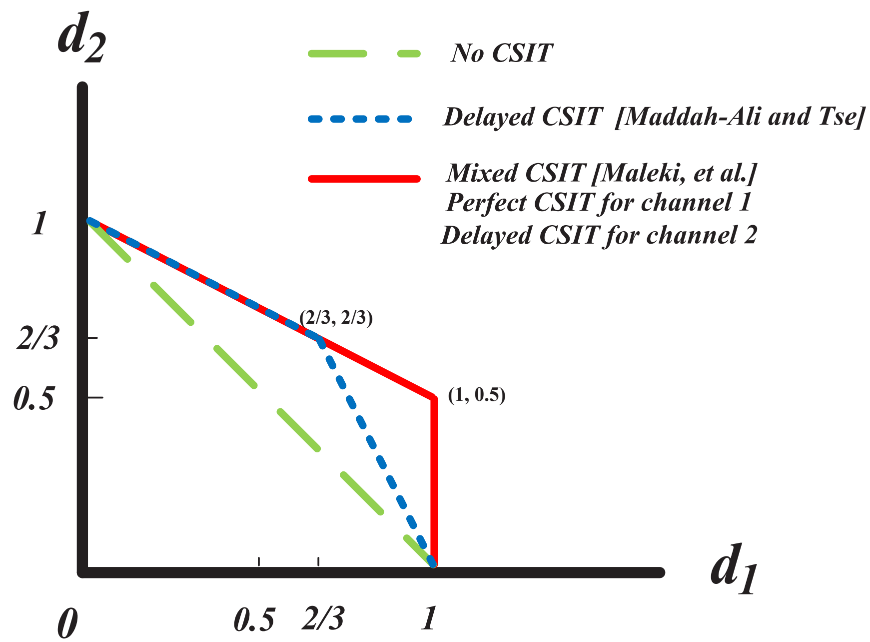
- Recall: if both users only had delayed feedback



One user has more feedback: Maleki, Jafar and Shamai$_2$

- Now One user has delayed, the other had perfect

- *Theorem: Derived optimal DoF is* $d_1 = 1,\ d_2 = 1/2,$ *(sum DoF* $3/2 \geq 4/3$*)*



$$d_1 = 1,\ d_2 = 1/2,\ (\text{sum DoF } 3/2)$$

Maleki et al.: asymmetric scheme

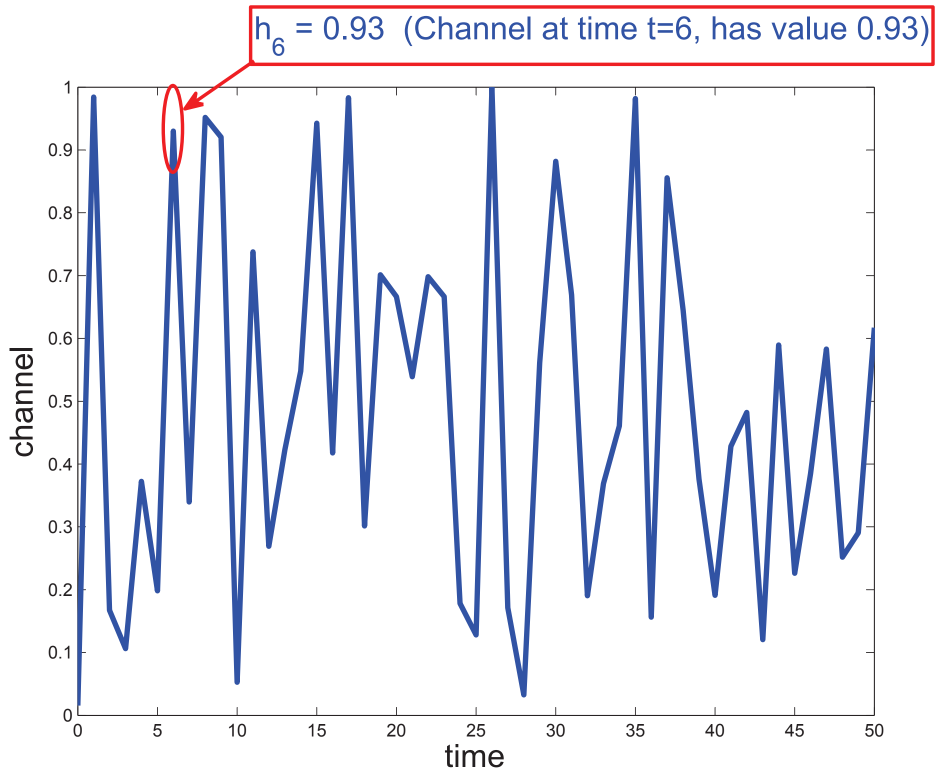
- Tx sends symbols a_1, a_2 for user 1, and b for user 2, in 2 channel uses
  - ⋆ WOLOG: one channel use $=$ one coherence block
  - ⋆ Tx will sequentially send signal vectors $\boldsymbol{x}_1, \boldsymbol{x}_2 \in \mathbb{C}^2$
  - ⋆ note use of symbol $\perp \rightarrow$ (orthogonal)

$$\boldsymbol{x}_1 = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \boldsymbol{h}_1^{\perp} b, \quad \boldsymbol{x}_2 = \begin{bmatrix} \boldsymbol{g}_1^{\top} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \\ 0 \end{bmatrix} + \boldsymbol{h}_2^{\perp} b$$

- Intuitions:
  - ⋆ Current CSIT can be used for instantaneous interference mitigation
  - ⋆ Delayed CSIT can be used for retrospective interference cancelation

INTRODUCING FEEDBACK <u>QUALITY</u> CONSIDERATIONS

Introducing feedback QUALITY considerations$_1$

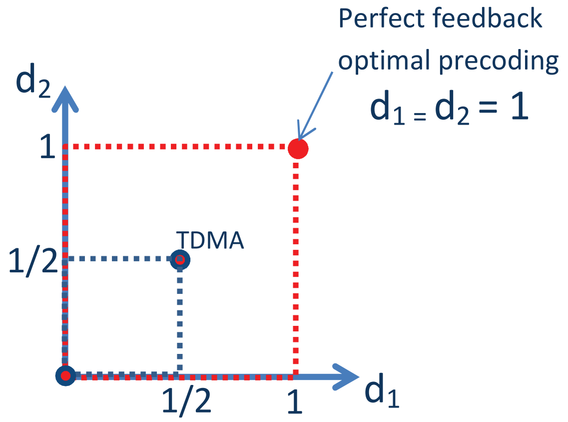


Introducing feedback QUALITY considerations$_2$

- Jindal et al., Caire et al: *"Optimal DoF does not need infinite number of feedback bits"*
  - ⋆ Let $\hat{\boldsymbol{h}}_t$ be the INSTANTANEOUS estimate of channel $\boldsymbol{h}_t$
  - ⋆ Let $\hat{\boldsymbol{g}}_t$ be the INSTANTANEOUS estimate of channel $\boldsymbol{g}_t$
  - ⋆ Then if

$$\mathbb{E}[\|\hat{\boldsymbol{h}}_t - \boldsymbol{h}_t\|^2] \approx P^{-1}, \qquad \mathbb{E}[\|\hat{\boldsymbol{g}}_t - \boldsymbol{g}_t\|^2] \approx P^{-1}$$

  - ⋆ you can achieve the optimal DoF



Refining quality considerations

- Motivation: Note $\mathbb{E}[\|\hat{\boldsymbol{h}}_t - \boldsymbol{h}_t\|^2] \approx P^{-1}$ corresponds to sending about $\log P$ bits of feedback per scalar (rate distortion theory - not optimal)

- What if you cannot send so many bits?

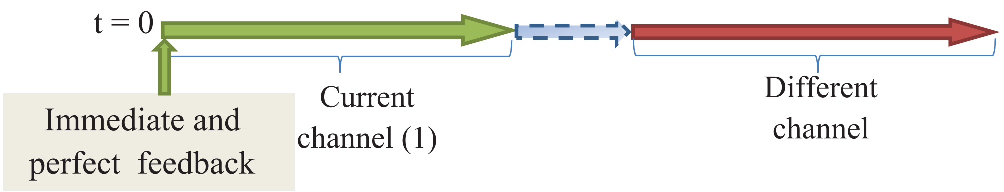
KOBAYASHI-YANG-YI-GESBERT:
CURRENT CSIT ESTIMATION ERRORS WITH POWER $P^{-\alpha}$

- Current CSIT quality exponent

$$\alpha = -\lim \frac{\log \mathbb{E}[\|\hat{\boldsymbol{h}}_t - \boldsymbol{h}_t\|^2]}{\log P} = -\lim \frac{\log \mathbb{E}[\|\hat{\boldsymbol{g}}_t - \boldsymbol{g}_t\|^2]}{\log P}, \qquad \alpha : 0 \to 1$$

Combining current and delayed CSIT (Yang-Gesbert et al.)

- Perfect delayed CSIT + imperfect current CSIT



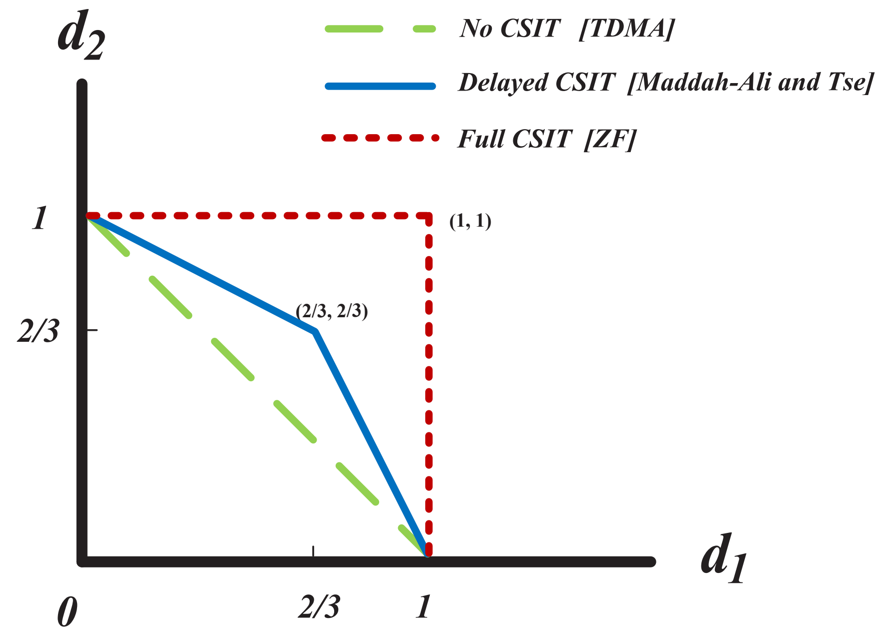
Coherence block	1	2	3	4	$\cdots$
Current estimates (quality α)	$\widehat{\boldsymbol{h}}_1, \widehat{\boldsymbol{g}}_1$	$\widehat{\boldsymbol{h}}_2, \widehat{\boldsymbol{g}}_2$	$\widehat{\boldsymbol{h}}_3, \widehat{\boldsymbol{g}}_3$	$\widehat{\boldsymbol{h}}_4, \widehat{\boldsymbol{g}}_4$	$\cdots$
Delayed estimates (exact) $\rightarrow$		$\boldsymbol{h}_1, \boldsymbol{g}_1$	$\boldsymbol{h}_2, \boldsymbol{g}_2$	$\boldsymbol{h}_3, \boldsymbol{g}_3$	$\boldsymbol{h}_4, \boldsymbol{g}_3$

- Current CSIT: PARTIAL instantaneous interference mitigation
- Delayed CSIT: retrospective interference management, at later time

Combining current and delayed CSIT (Yang-Gesbert et al.)[1]

Recall: if both users only had delayed feedback

$$(\Rightarrow \alpha = 0)$$

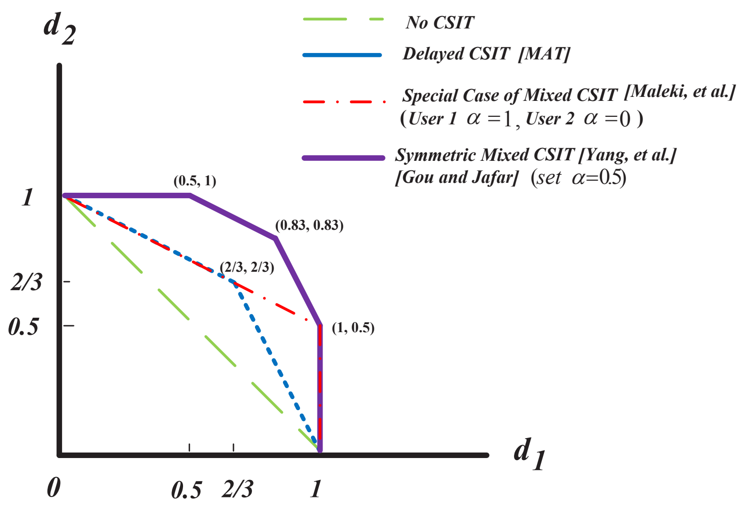


Perfect delayed, and imperfect current CSIT

NOW EACH HAS DELAYED + IMPERFECT CURRENT ESTIMATES
($\Rightarrow \alpha > 0$)

- *Theorem*[1]*: Perfect delayed CSIT and α-quality current CSIT, gives:*

$$d_1 = d_2 = \frac{2+\alpha}{3}$$



[1] Yang-Kobayashi-Yi-Gesbert, Gou-Jafar 2012

Scheme: Yang-Kobayashi-Yi-Gesbert

- Tx to communicate in three channel uses, sending $\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3 \in \mathbb{C}^2$
- First information symbols $a_1, a_1^{'}$ for user 1
- First information symbols $b_1, b_1^{'}$ for user 2
- Two phases: phase 1 ($t = 1$), phase 2 ($t = 2, 3$)
- Unit coherence period (WOLOG)

- During phase 1 ($t = 1$), the transmitter sends ($\boldsymbol{u}_1 = \hat{\boldsymbol{g}}_1^{\perp},\ \boldsymbol{v}_1 = \hat{\boldsymbol{h}}_1^{\perp}$)

$$\boldsymbol{x}_1 = \underbrace{\overbrace{\boldsymbol{u}_1}^{\hat{\boldsymbol{g}}_1^{\perp}} a_1}_{\text{power } P,\ \text{rate prelog } r=1} + \underbrace{\overbrace{\boldsymbol{v}_1}^{\hat{\boldsymbol{h}}_1^{\perp}} b_1}_{P,\ r=1} + \underbrace{\overbrace{\boldsymbol{u}_1^{'}}^{\text{random}} a_1^{'}}_{P^{1-\alpha},\ r=1-\alpha} + \underbrace{\overbrace{\boldsymbol{v}_1^{'}}^{\text{random}} b_1^{'}}_{P^{1-\alpha},\ r=1-\alpha}$$

- Users receive

$$y_1^{(1)} = \boldsymbol{h}_1^{\mathsf{T}}\boldsymbol{u}_1 a_1 + \boldsymbol{h}_1^{\mathsf{T}}\boldsymbol{u}_1^{'} a_1^{'} + \overbrace{\underbrace{\tilde{\boldsymbol{h}}_1^{\mathsf{T}}\boldsymbol{v}_1 b_1 + \boldsymbol{h}_1^{\mathsf{T}}\boldsymbol{v}_1^{'} b_1^{'}}_{\text{power } P^{1-\alpha}}}^{\text{interference } \iota_1^{(1)}} + \text{noise},$$

$$y_1^{(2)} = \overbrace{\underbrace{\tilde{\boldsymbol{g}}_1^{\mathsf{T}}\boldsymbol{u}_1 a_1 + \boldsymbol{g}_1^{\mathsf{T}}\boldsymbol{u}_1^{'} a_1^{'}}_{\text{power } P^{1-\alpha}}}^{\text{interference } \iota_1^{(2)}} + \boldsymbol{g}_1^{\mathsf{T}}\boldsymbol{v}_1 b_1 + \boldsymbol{g}_1^{\mathsf{T}}\boldsymbol{v}_1^{'} b_1^{'} + \text{noise}.$$

Scheme: Yang-Kobayashi-Yi-Gesbert$_2$

- At the end of phase 1. Reconstruct interference using delayed CSIT

$$\iota_1^{(1)} = \tilde{\boldsymbol{h}}_1^{\mathsf{T}} \boldsymbol{v}_1 b_1 + \boldsymbol{h}_1^{\mathsf{T}} \boldsymbol{v}_1^{'} b_1^{'}, \quad \iota_1^{(2)} = \tilde{\boldsymbol{g}}_1^{\mathsf{T}} \boldsymbol{u}_1 a_1 + \boldsymbol{g}_1^{\mathsf{T}} \boldsymbol{u}_1^{'} a_1^{'}$$

- Quantize interference into $\bar{\iota}_1^{(i)}$
  - ⋆ quantization rate: $(1-\alpha)\log P$ bits $\rightarrow$ bounded quant. error
- Map all quantization bits of $\{\bar{\iota}_1^{(i)}\}_{t=1}^{2} \rightarrow$ into $\{c_t\}_{t=2}^{3}$
- Send these symbols during next phase
  - ⋆ a) to cancel interference
  - ⋆ b) to get extra observation for MIMO decoding

- Phase 2, $t = 2, 3$, Tx sends c_t and extra a_t, b_t

$$\boldsymbol{x}_t = \underbrace{\boldsymbol{w}_t c_t}_{P,\ \ r=1-\alpha} + \underbrace{\hat{\boldsymbol{g}}_t^{\perp} a_t}_{P^{\alpha},\ \ r=\alpha} + \underbrace{\hat{\boldsymbol{h}}_t^{\perp} b_t}_{P^{\alpha},\ \ r=\alpha}$$

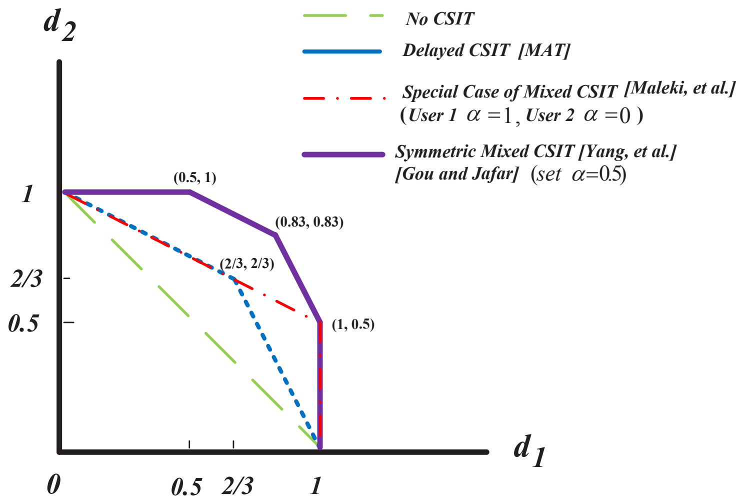
  - ⋆ Successive decoding: $c_t \to a_t$ at user 1, $c_t \to b_t$ at user 2
  - ⋆ Reconstructing approximate interference:$\{c_t\}_{t=2}^{3} \to \{\bar{\check{\iota}}_1^{(i)}\}_{t=1}^{2}$
  - ⋆ Go back to phase 1, and decode a_1, a_2 at user 1, and b_1, b_2 at user 2

$$\begin{bmatrix} y_1^{(1)} - \bar{\check{\iota}}_1^{(1)} \\ \bar{\check{\iota}}_1^{(2)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{h}_1^{\mathsf{T}} \\ \boldsymbol{g}_1^{\mathsf{T}} \end{bmatrix} \begin{bmatrix} \boldsymbol{u}_1 & \boldsymbol{u}_1^{'} \end{bmatrix} \begin{bmatrix} a_1 \\ a_1^{'} \end{bmatrix} + \text{noise}$$

$$d_1 = d_2 = \frac{2+\alpha}{3}$$

Intuition of benefits

- To achieve $d_1 = d_2 = \frac{2+\alpha}{3}$, send a total of $\log P$ feedback bits
  - ⋆ $\alpha \log P$ bits sent immediately
  - ⋆ $(1-\alpha)\log P$ bits sent at any point after coherence period



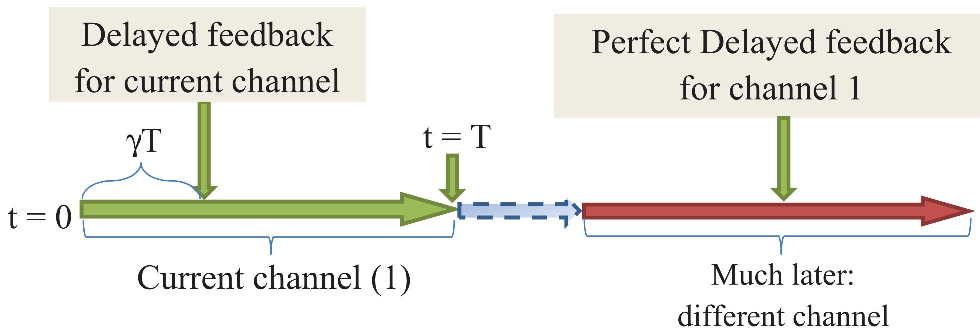
Can imperfect feedback give optimal performance?

Can imperfect feedback give optimal performance?

- Answer: yes, if we have more receiving nodes
  - ⋆ Example: MISO BC, with 3 users

Even delayed CSIT can achieve max DoF

- Setting (Lee and Heath 2012)
  - ⋆ MISO BC, two transmitter antennas, <u>three</u> users
  - ⋆ Send perfect feedback at γ fraction of coherence period



- *Theorem (Lee and Heath 2012): The optimal sum-DoF* $d_1 + d_2 + d_3 = 2$ *is achieved for any delay* $0 \leq \gamma \leq \frac{1}{3}$

Lee and Heath scheme for not-too-Delayed CSIT

- Phase 1
  - ⋆ Phase duration: one 'time slot'. No current CSIT available.
  - ⋆ Tx sends a total of six different data symbols; two per user

$$\boldsymbol{x}_1 = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} + \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

  - ⋆ User $i = 1, 2, 3$, receives a signal consisting of three linear combinations $L_1^{(i)}(\boldsymbol{a}) = \boldsymbol{h}_1^{(i)\top}\boldsymbol{a}$, $L_1^{(i)}(\boldsymbol{b}) = \boldsymbol{h}_1^{(i)\top}\boldsymbol{b}$, $L_1^{(i)}(\boldsymbol{c}) = \boldsymbol{h}_1^{(i)\top}\boldsymbol{c}$

$$\begin{aligned} y_1^{(1)} &= L_1^{(1)}(\boldsymbol{a}) + L_1^{(1)}(\boldsymbol{b}) + L_1^{(1)}(\boldsymbol{c}), \\ y_1^{(2)} &= L_1^{(2)}(\boldsymbol{a}) + L_1^{(2)}(\boldsymbol{b}) + L_1^{(2)}(\boldsymbol{c}), \\ y_1^{(3)} &= L_1^{(3)}(\boldsymbol{a}) + L_1^{(3)}(\boldsymbol{b}) + L_1^{(3)}(\boldsymbol{c}), \end{aligned}$$

- Phase 2
  - ⋆ Two time slots = two independent channel blocks ($t = 2, 3$)
  - ⋆ current CSIT for channel at $t = 2, 3$
  - ⋆ with delayed CSIT of the channel corresponding to $t = 1$
  - ⋆ Construct same interference experienced at time $t = 1$

$$\boldsymbol{x}_t = \boldsymbol{V}_t^{(a)}\boldsymbol{a} + \boldsymbol{V}_t^{(b)}\boldsymbol{b} + \boldsymbol{V}_t^{(c)}\boldsymbol{c}, \quad t = 2, 3$$

  - ⋆ With good precoders $\boldsymbol{V}_t^{(a)}, \boldsymbol{V}_t^{(b)}, \boldsymbol{V}_t^{(c)}$, users receive signals of the form

$$\begin{aligned} y_t^{(1)} &= L_t^{(1)}(\boldsymbol{a}) + L_1^{(1)}(\boldsymbol{b}) + L_1^{(1)}(\boldsymbol{c}), \\ y_t^{(2)} &= L_1^{(2)}(\boldsymbol{a}) + L_t^{(2)}(\boldsymbol{b}) + L_1^{(2)}(\boldsymbol{c}), \\ y_t^{(3)} &= L_1^{(3)}(\boldsymbol{a}) + L_1^{(3)}(\boldsymbol{b}) + L_t^{(3)}(\boldsymbol{c}), \end{aligned}$$

Lee and Heath Scheme$_2$

- So, the precoders are chosen as

$$
\begin{aligned}
\boldsymbol{V}_t^{(a)} &= \begin{bmatrix} \boldsymbol{h}_t^{(2)\top} \\ \boldsymbol{h}_t^{(3)\top} \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{h}_1^{(2)\top} \\ \boldsymbol{h}_1^{(3)\top} \end{bmatrix} \\
\boldsymbol{V}_t^{(b)} &= \begin{bmatrix} \boldsymbol{h}_t^{(1)\top} \\ \boldsymbol{h}_t^{(3)\top} \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{h}_1^{(1)\top} \\ \boldsymbol{h}_1^{(3)\top} \end{bmatrix} \\
\boldsymbol{V}_t^{(c)} &= \begin{bmatrix} \boldsymbol{h}_t^{(1)\top} \\ \boldsymbol{h}_t^{(2)\top} \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{h}_1^{(1)\top} \\ \boldsymbol{h}_1^{(2)\top} \end{bmatrix}
\end{aligned}
$$

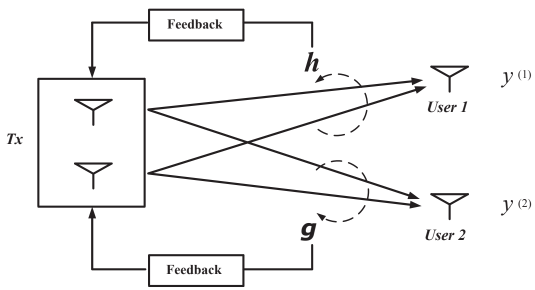
- User 1 decoding

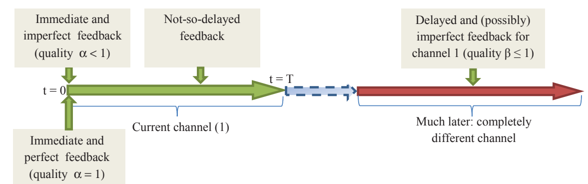
$$
\begin{bmatrix} y_1^{(1)} - y_2^{(1)} \\ y_1^{(1)} - y_3^{(1)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{h}_1^{(1)T} - \boldsymbol{h}_2^{(1)T}\boldsymbol{V}_2^{(a)} \\ \boldsymbol{h}_1^{(1)T} - \boldsymbol{h}_3^{(1)T}\boldsymbol{V}_3^{(a)} \end{bmatrix} \boldsymbol{a} + \text{noise}
$$

- Each user decodes 2 symbols in 3 channel uses- optimal sum DoF (2)
- Intuitions: Not-too-delayed CSIT is used for interference reconstruction and interference cancellation

ALTERNATING CSIT[2]

Feedback alternates from user to user





Time t	*1*	*2*	*3*	*4*	*5*	*6*	*7*	...
CSIT of channel h	*P*	*D*	*N*	*P*	*P*	*N*	*N*	...
CSIT of channel g	*D*	*P*	*N*	*N*	*N*	*P*	*P*	...

[2] Tandon-Jafar-Shamai-Poor 2012

Alternating CSIT$_1$

- CSIT for each user's channel, at a specific time, can be either perfect (P), delayed (D) or not available (N)
  - ⋆ $I_1, I_2 \in \{P, D, N\}$
  - ⋆ For example, in a specific time: $I_1 = P,\ I_2 = D$

- $\lambda_{I_1 I_2}$ is the fraction of time associated with CSIT states I_1, I_2
  - ⋆ Symmetric assumption $\lambda_{I_1 I_2} = \lambda_{I_2 I_1}$
- $\lambda_P = \lambda_{PP} + \lambda_{PD} + \lambda_{PN}$
- $\lambda_D = \lambda_{DP} + \lambda_{DD} + \lambda_{DN}$

- *Theorem: Derived DoF*

$$d = \min\{\frac{2 + \lambda_P}{3}, \frac{1 + \lambda_P + \lambda_D}{2}\}$$

Alternating CSIT$_2$

(Recall: $d_{\text{opt}} = \min\{\frac{2+\lambda_P}{3}, \frac{1+\lambda_P+\lambda_D}{2}\}$)

Example:

- First half $I_1 = P,\ I_2 = D$, second half $I_1 = D,\ I_2 = P$ $\quad \begin{bmatrix} P & D \\ D & P \end{bmatrix}$
- $\lambda_{PD} = \lambda_{DP} = 0.5$ ($\lambda_P = \lambda_D = 1/2 \quad \Rightarrow \min\{\frac{2+1/2}{3}, \frac{1+1/2+1/2}{2}\} = \frac{5}{6}$)
- Then optimal DoF $d_1 = d_2 = 5/6$

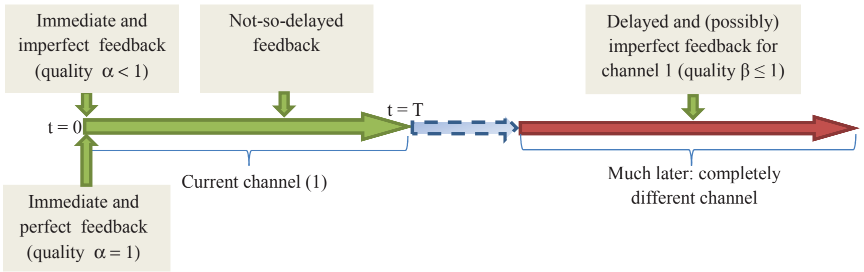
Symmetry 'beats' alternating

- Asymmetry: $\lambda_{PD} = 1 \quad \Rightarrow \quad d_1 + d_2 = 3/2$ (Maleki et al.)
  - ⋆ Instantaneous perfect CSIT for channel of user 1 $I_1 = P$
  - ⋆ Delayed CSIT for channel of user 2 $I_2 = D$
- Symmetry: $\lambda_{PD} = 0.5, \lambda_{DP} = 0.5$

$$\Rightarrow \quad d_1 + d_2 = 5/3 \geq 3/2$$

  - ⋆ Half of time $I_1 = P, \; I_2 = D$, other half $I_1 = D, \; I_2 = P$
- Same feedback cost, but symmetric provides gain $5/3 - 3/2$

Summary: Part-1



- No CSIT $d_1 = d_2 = 1/2$
- Perfect CSIT $d_1 = d_2 = 1$
- Delayed CSIT-MAT $d_1 = d_2 = 2/3$
- Perfect CSIT for channel 1, delayed CSIT for channel 2 - Maleki et al. $d_1 = 1,\ d_2 = 1/2$
- Imperfect current CSIT α, perfect delayed CSIT - Sheng et al. and Gou and Jafar $d_1 = d_2 = (2+\alpha)/3$
- Not-too-delayed CSIT can be optimal - Lee and Heath ($\gamma \leq \frac{1}{3}$, 2×3 setting)

Common themes of what we have seen

- Motivated by timeliness-and-quality considerations
- Timeliness and quality might be hard to get over limited feedback links
- Timeliness and quality affect performance
  - ⋆ Feedback delays and imperfections generally reduce performance
- A corresponding clear delay-and-quality question....

The fundamental question

How much feedback is necessary, and when, in order to achieve a certain performance?

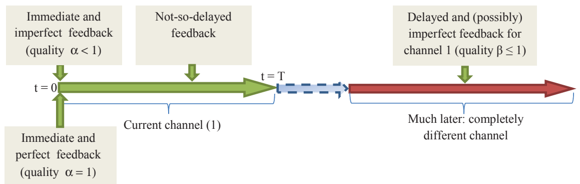
Answering a broad range of practical questions

"Answering a broad range of practical performance-vs-feedback questions, up to a sublogarithmic *factor of P"*

WHAT WOULD ENGINEERS ASK?

- What is the role of MIMO in reducing feedback quality?
- When is delayed feedback necessary?
- When is predicted feedback necessary?
- What is better: less feedback early, or more feedback later?
- How to exploit feedback of imperfect quality?
- How to exploit feedback with predictions?
- How to exploit feedback with delayed information?
- How much feedback, where, and when, for a certain performance?

Addressing interesting and practical scenarios



- Can a specific accumulation-rate of feedback bits, guarantee a certain target DoF performance?

  - ⋆ If we send $\frac{1}{10}\log P$ feedback bits without delay (at $t = 0$),
  - ⋆ then send $\frac{1}{8}\log P$ bits at $t = T_{coh}/3$
  - ⋆ then send $\frac{1}{9}\log P$ bits at $t = 2T_{coh}/3$
  - ⋆ and $\frac{1}{6}\log P$ bits at any time $t > T_{coh}$
  - ⋆ then what performance can be guaranteed?

A unified performance-vs-feedback framework

A unified performance-vs-feedback framework

Fundamental formulation of performance-vs-feedback problem

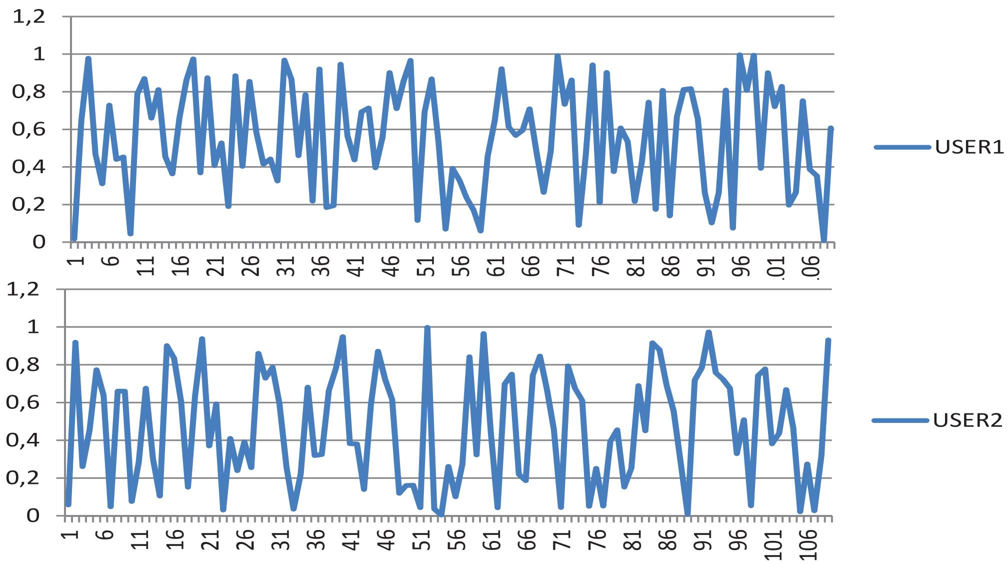
Fundamental formulation:step 1

STEP 1: COMMUNICATION OF DURATION n (n IS LARGE)

Fundamental formulation:step 2

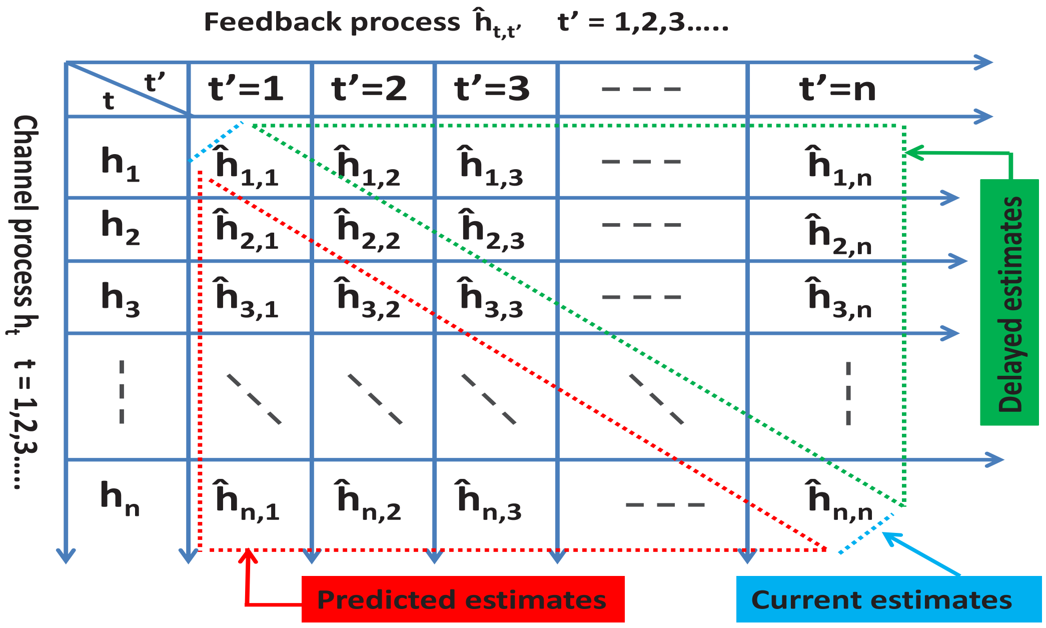
STEP 2: COMMUNICATION ENCOUNTERS AN ARBITRARY CHANNEL PROCESS

$$\begin{array}{llllll} \text{user } 1: & \boldsymbol{h}_1 & \boldsymbol{h}_2 & \boldsymbol{h}_3 & \cdots & \boldsymbol{h}_n \\ \text{user } 2: & \boldsymbol{g}_1 & \boldsymbol{g}_2 & \boldsymbol{g}_3 & \cdots & \boldsymbol{g}_n \end{array}$$



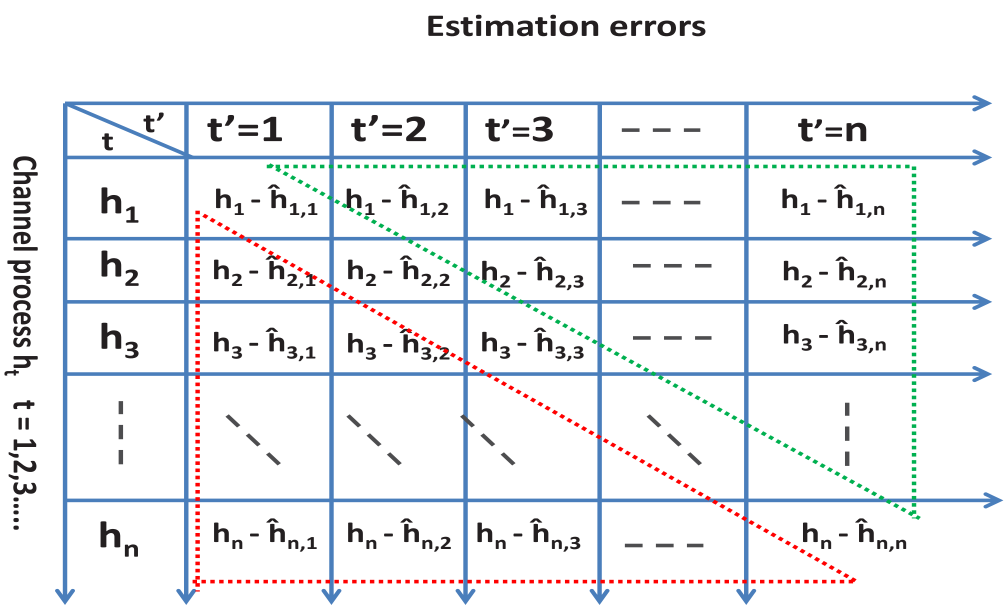
Step 3: An arbitrary feedback process

What do we know - at any time t'– about any channel $\mathbf{h}_t$?

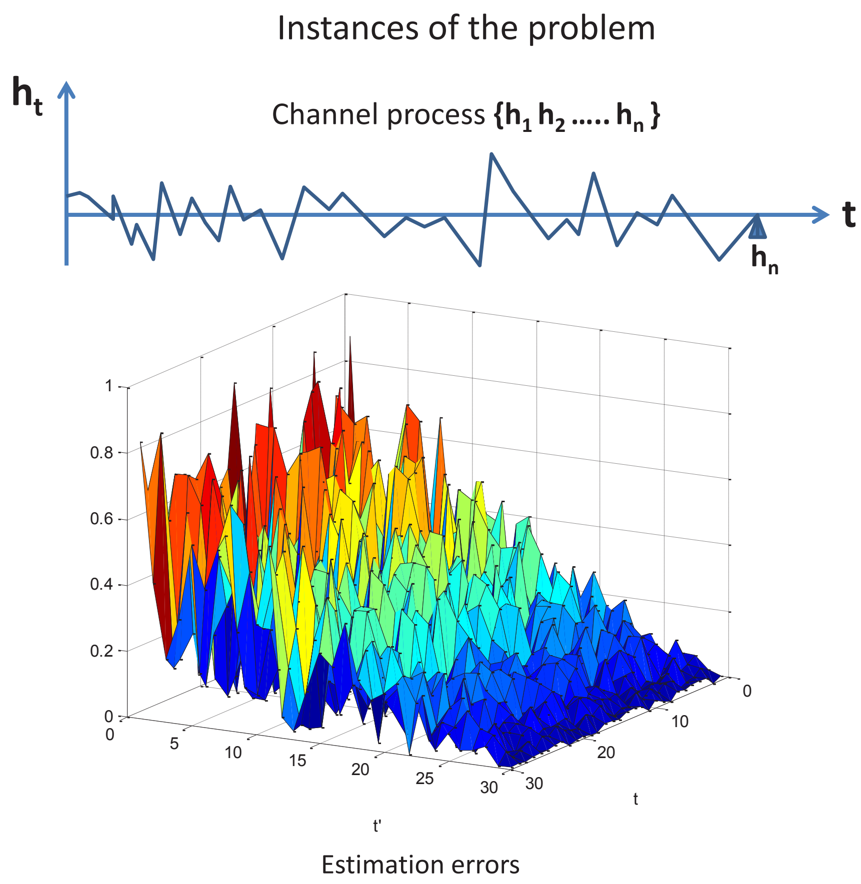


Fundamental formulation:step 4

STEP 4: A 'PRIMITIVE' MEASURE OF FEEDBACK 'GOODNESS'

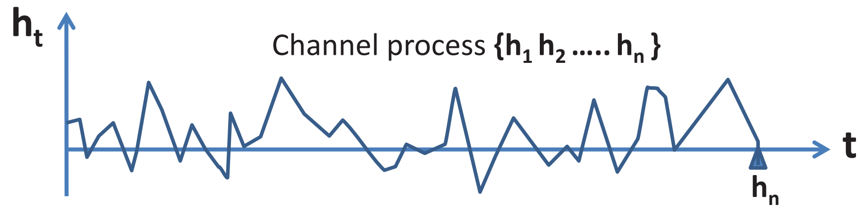


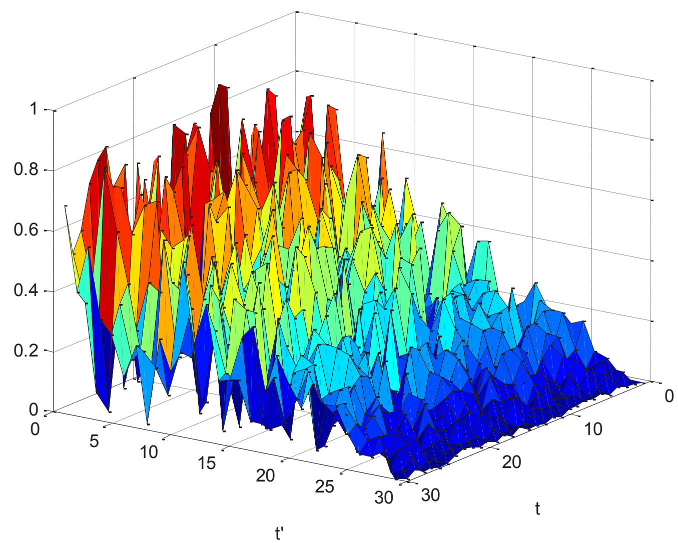
Remember the problem is random



Remember the problem is random$_1$

Instances of the problem

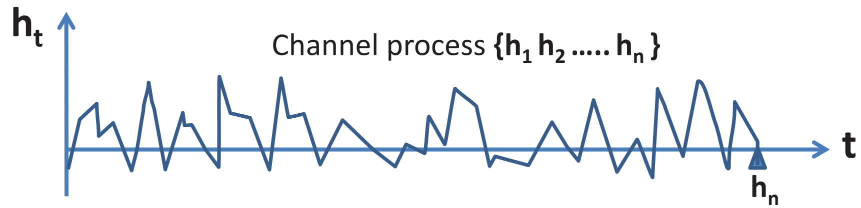


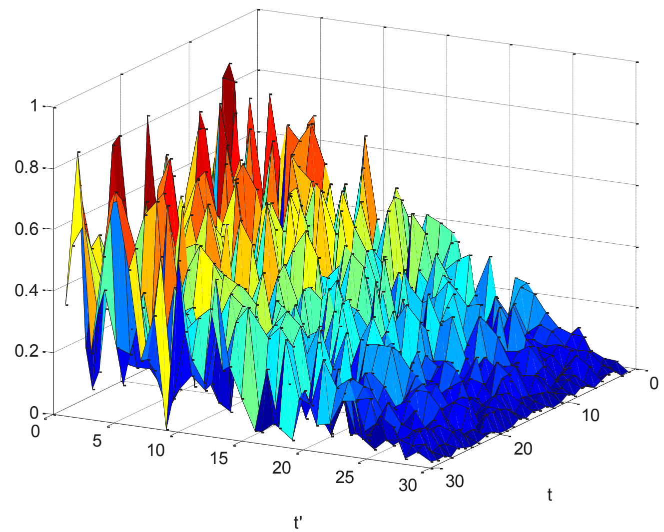


Estimation errors

Remember the problem is random$_2$

Instances of the problem

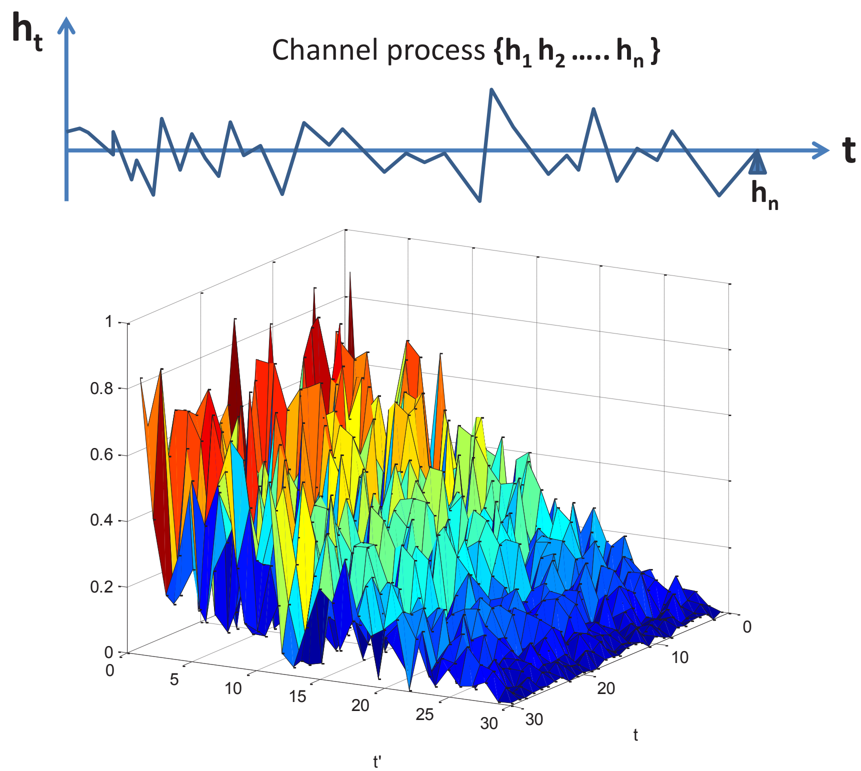




Estimation errors

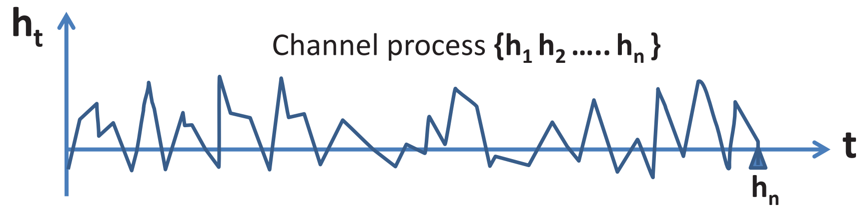
Remember the problem is random$_3$

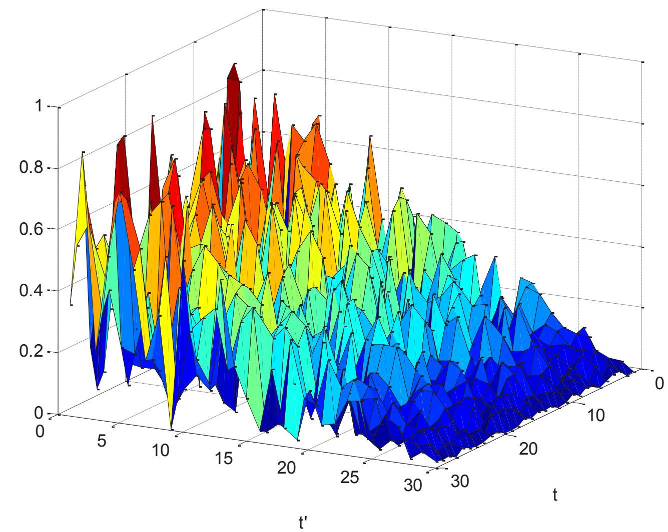
Instances of the problem



Estimation errors

Instances of the problem

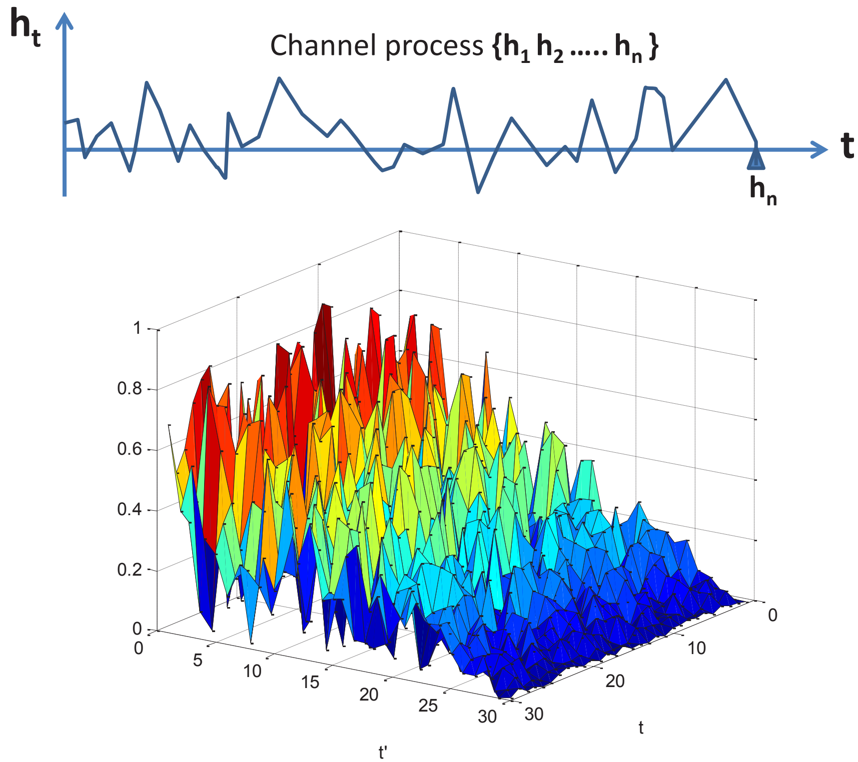




Estimation errors

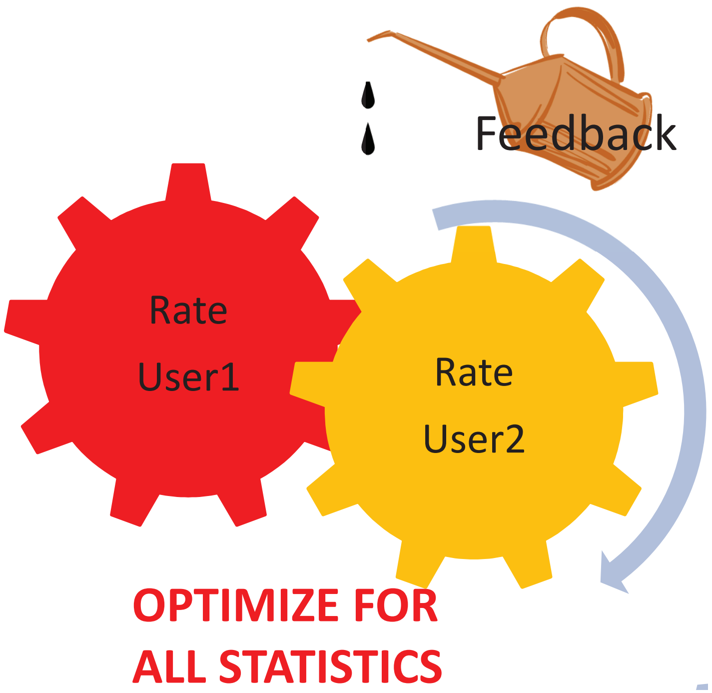
Remember the problem is random$_5$

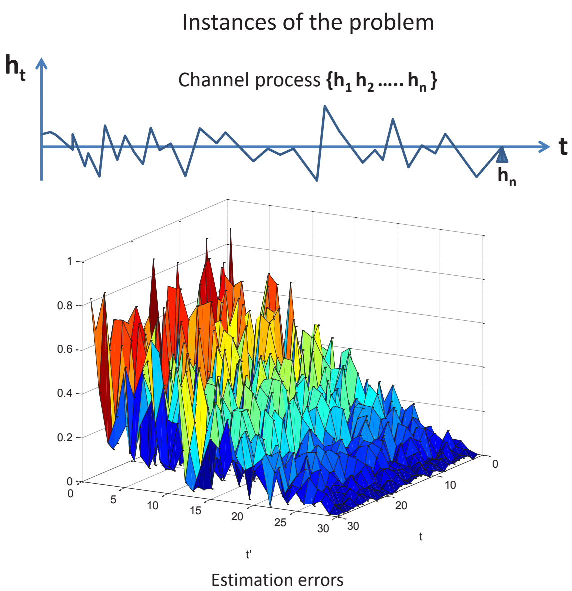
Instances of the problem



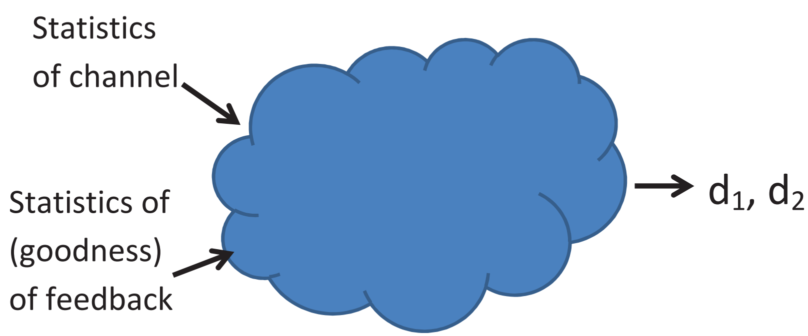
Estimation errors

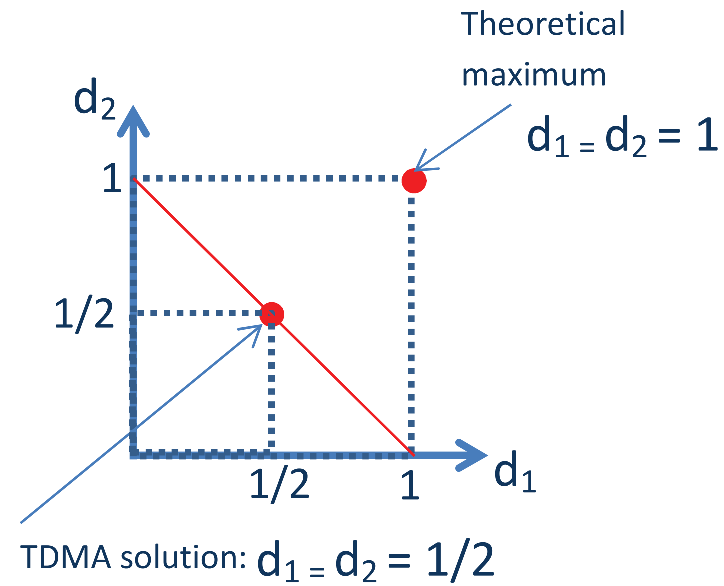
Challenge - optimize user's rates for given feedback





Recall: performance in degrees-of-freedom (DoF)





$$d_i = \lim_{P \to \infty} \frac{R_i}{\log P}, \;\; i = 1, 2$$

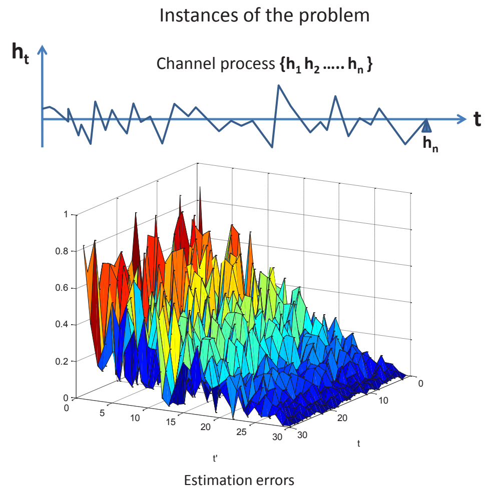
- (R_1, R_2): achievable rate pair $\quad R_i \approx d_i \log P$

Brief notations, conventions and assumptions

Notation

Quality of *current* CSIT for channel at time t

$$\alpha_t^{(1)} \triangleq -\lim_{P\to\infty} \frac{\log \mathbb{E}[||\boldsymbol{h}_t - \hat{\boldsymbol{h}}_{t,t}||^2]}{\log P} \qquad \alpha_t^{(2)} \triangleq -\lim_{P\to\infty} \frac{\log \mathbb{E}[||\boldsymbol{g}_t - \hat{\boldsymbol{g}}_{t,t}||^2]}{\log P}$$



Notation$_1$

Quality of *delayed* CSIT for channel at time t

$$\beta_t^{(1)} \triangleq -\lim_{P\to\infty} \frac{\log \mathbb{E}[||\boldsymbol{h}_t - \hat{\boldsymbol{h}}_{t,t+\eta}||^2]}{\log P} \qquad \beta_t^{(2)} \triangleq -\lim_{P\to\infty} \frac{\log \mathbb{E}[||\boldsymbol{g}_t - \hat{\boldsymbol{g}}_{t,t+\eta}||^2]}{\log P}$$

for some large $\eta < \infty$.

Quality range (WOLOG): $\quad 0 \leq \alpha_t^{(i)} \leq \beta_t^{(i)} \leq 1$

Notation and common conventions

- Average of exponent sequences

$$\bar{\alpha}^{(1)} \triangleq \frac{1}{n}\sum_{t=1}^{n} \alpha_t^{(1)} \qquad \bar{\alpha}^{(2)} \triangleq \frac{1}{n}\sum_{t=1}^{n} \alpha_t^{(2)}$$

$$\bar{\beta}^{(1)} \triangleq \frac{1}{n}\sum_{t=1}^{n} \beta_t^{(1)} \qquad \bar{\beta}^{(2)} \triangleq \frac{1}{n}\sum_{t=1}^{n} \beta_t^{(2)}$$

- Common conventions:
  - ⋆ Gaussian estimation errors
  - ⋆ Current estimate error is statistically independent of current and past estimates
  - ⋆ Wait for delayed-CSIT only for a finite amount of time
  - ⋆ Perfect and global knowledge of channel state information at receivers

Performance vs. CSIT timeliness and quality

THE FOLLOWING RESULTS HOLD FOR GENERAL SETTING

- Communication over (large) n time slots
- Channel $\left\{ \boldsymbol{h}_t, \boldsymbol{g}_t \right\}_t^n$, Feedback $\left\{ \hat{\boldsymbol{h}}_{t,t'}, \hat{\boldsymbol{g}}_{t,t'} \right\}_{t,t'=1}^n$
- 'Goodness' measure: statistics of error sets

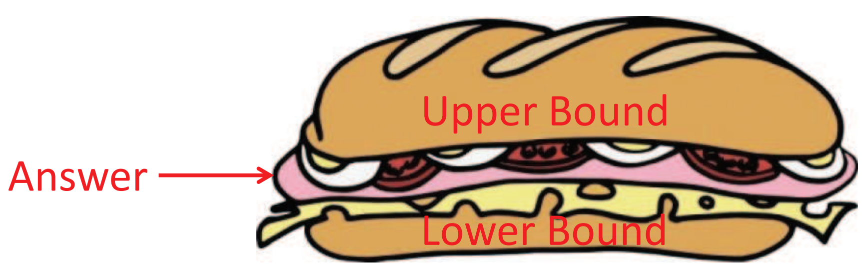
$$\left\{ (\boldsymbol{h}_t - \hat{\boldsymbol{h}}_{t,t'}), (\boldsymbol{g}_t - \hat{\boldsymbol{g}}_{t,t'}) \right\}_{t,t'=1}^n$$

- Challenge: derive DoF region

Answers in the form of bounds

Recall: Answers in the form of

- Novel precoders/decoders that cleverly use feedback
- Information theoretic outer bounds (try to prove optimality)



Magical reduction in difficulty of problem

Theorem: *(Chen-Elia 2013) The DoF region*

$$d_1 \leq 1, \quad d_2 \leq 1$$

$$2d_1 + d_2 \leq 2 + \bar{\alpha}^{(1)}$$

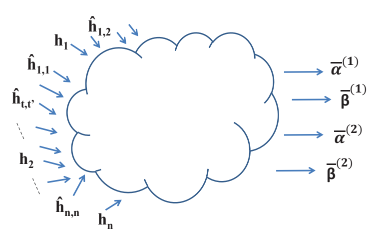
$$2d_2 + d_1 \leq 2 + \bar{\alpha}^{(2)}$$

$$d_1 + d_2 \leq \frac{1}{2}(2 + \bar{\beta}^{(1)} + \bar{\beta}^{(2)})$$

is achievable for a large range of parameters.

MAGICALLY, RESULT A FUNCTION OF JUST 4 STATISTICAL PARAMETERS!!!!

Complexity of the problem is captured by only 4 parameters



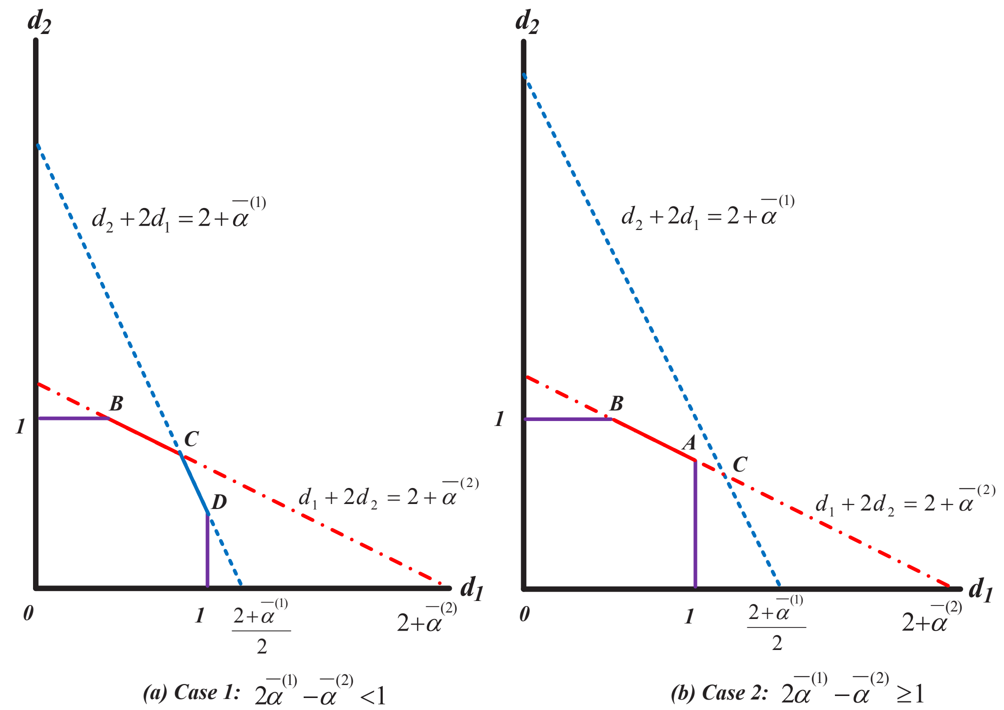
Specifically: Optimal DoF for sufficiently good delayed CSIT

Theorem: *(Chen-Elia) The optimal DoF of the two-user MISO BC with a CSIT process* $\left\{\hat{\boldsymbol{h}}_{t,t'}, \hat{\boldsymbol{g}}_{t,t'}\right\}_{t=1,t'=1}^{n}$ *of quality* $\left\{(\boldsymbol{h}_t - \hat{\boldsymbol{h}}_{t,t'}), (\boldsymbol{g}_t - \hat{\boldsymbol{g}}_{t,t'})\right\}_{t=1,t'=1}^{n}$ *is given by*

$$
\begin{aligned}
d_1 \le 1, \quad d_2 \le 1 \\
2d_1 + d_2 \le 2 + \bar{\alpha}^{(1)} \\
2d_2 + d_1 \le 2 + \bar{\alpha}^{(2)}
\end{aligned}
$$

for any sufficiently good delayed-CSIT process such that

$$\min\{\bar{\beta}^{(1)}, \bar{\beta}^{(2)}\} \ge \min\{\frac{1 + \bar{\alpha}^{(1)} + \bar{\alpha}^{(2)}}{3}, \frac{1 + \min\{\bar{\alpha}^{(1)}, \bar{\alpha}^{(2)}\}}{2}\}$$



- *Optimal DoF regions for the two-user MISO BC with sufficiently good delayed CSIT.*

DoF lower bound for case of 'weak' delayed CSIT

Proposition: *For a CSIT process* $\left\{\hat{\boldsymbol{h}}_{t,t'}, \hat{\boldsymbol{g}}_{t,t'}\right\}_{t=1,t'=1}^{n}$ *for which* $\min\{\bar{\beta}^{(1)}, \bar{\beta}^{(2)}\} < \min\{\frac{1+\bar{\alpha}^{(1)}+\bar{\alpha}^{(2)}}{3}, \frac{1+\bar{\alpha}^{(2)}}{2}\}$*, the DoF region is inner bounded by the polygon described by*

$$\begin{aligned} d_1 \leq 1, \quad d_2 &\leq 1 \\ 2d_1 + d_2 &\leq 2 + \bar{\alpha}^{(1)} \\ 2d_2 + d_1 &\leq 2 + \bar{\alpha}^{(2)} \\ d_1 + d_2 &\leq 1 + \min\{\bar{\beta}^{(1)}, \bar{\beta}^{(2)}\}. \end{aligned}$$

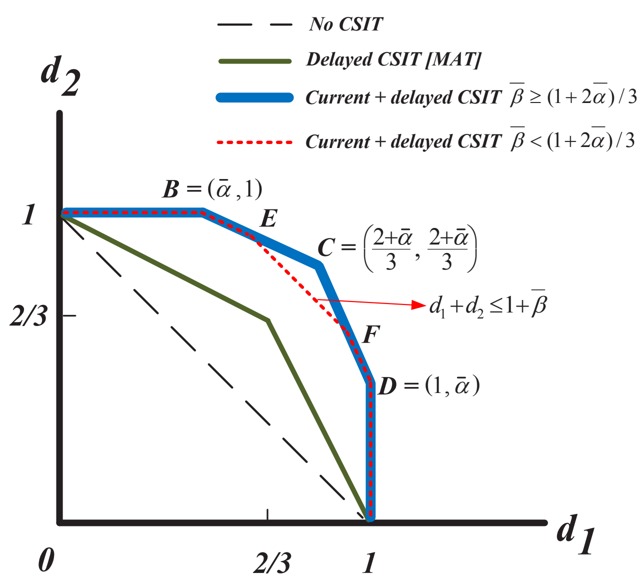
3

3

- We suspect loose outer bound
- Generalization of Lapidoth-Shamai-Wigger 2005 conjecture:
  - ⋆ *for* $\beta^{(1)} = \beta^{(2)} = \alpha^{(1)} = \alpha^{(2)} = 0$) that $d_1 = d_2 \in [\frac{1}{2}, \frac{2}{3}]$

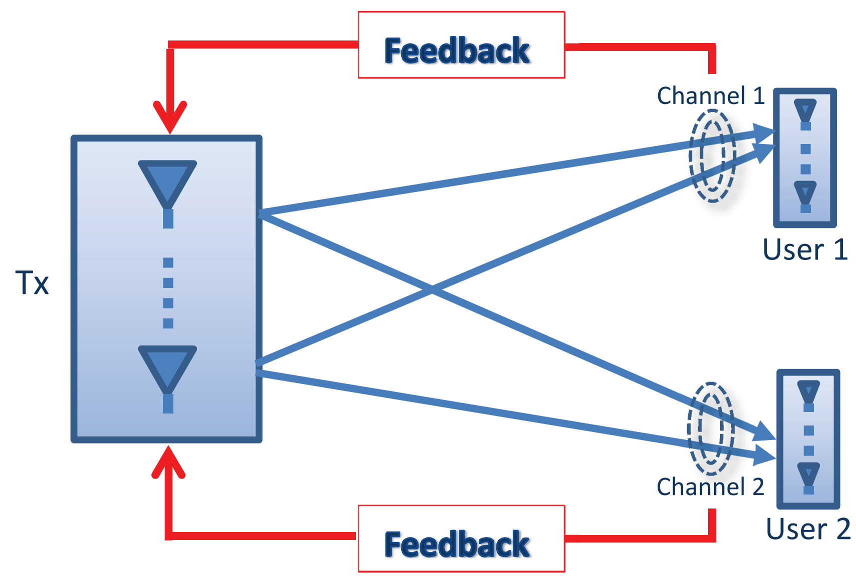
Users have similar long-term feedback capabilities

$$\bar{\alpha}^{(1)} = \bar{\alpha}^{(2)} = \bar{\alpha}$$
$$\bar{\beta}^{(1)} = \bar{\beta}^{(2)} = \bar{\beta}$$



MIMO BC

What if I have many transmit and receive antennas?



Theorem: *The optimal DoF region of the Two-user Symmetric $M\times(N,N)$ MIMO BC with sufficiently good delayed CSIT* [4]

$$d_1 + d_2 \le \langle 2N\rangle'$$

$$d_1 \le \langle N\rangle'; \quad \frac{d_1}{\langle N\rangle'} + \frac{d_2}{\langle 2N\rangle'} \le 1 + \frac{\langle 2N\rangle' - \langle N\rangle'}{\langle 2N\rangle'}\ \bar{\alpha}^{(1)}$$

$$d_2 \le \langle N\rangle'; \quad \frac{d_1}{\langle 2N\rangle'} + \frac{d_2}{\langle N\rangle'} \le 1 + \frac{\langle 2N\rangle' - \langle N\rangle'}{\langle 2N\rangle'}\ \bar{\alpha}^{(2)}$$

[4] $\langle\bullet\rangle' = \min\{\bullet, M\}$. 'Sufficiently good delayed CSIT': $\min\{\bar{\beta}^{(1)},\bar{\beta}^{(2)}\} \ge \min\{1, M - N', \frac{N(1+\bar{\alpha}^{(1)}+\bar{\alpha}^{(2)})}{\langle 2N\rangle'+N}, \frac{N(1+\min\{\bar{\alpha}^{(1)}+\bar{\alpha}^{(2)}\})}{\langle 2N\rangle'}\}$.

Theorem: *(Chen-Elia) The optimal DoF region of the Two-user Symmetric* $(M, M) \times (N, N)$ *IC with sufficiently good delayed CSIT, is*

$$d_1 + d_2 \leq \min\{2M, 2N, \max\{M, N\}\}$$

$$d_1 \leq \langle N \rangle'; \quad \frac{d_1}{\langle N \rangle'} + \frac{d_2}{\langle 2N \rangle'} \leq 1 + \frac{\langle 2N \rangle' - \langle N \rangle'}{\langle 2N \rangle'} \bar{\alpha}^{(1)}$$

$$d_2 \leq \langle N \rangle'; \quad \frac{d_1}{\langle 2N \rangle'} + \frac{d_2}{\langle N \rangle'} \leq 1 + \frac{\langle 2N \rangle' - \langle N \rangle'}{\langle 2N \rangle'} \bar{\alpha}^{(2)}$$

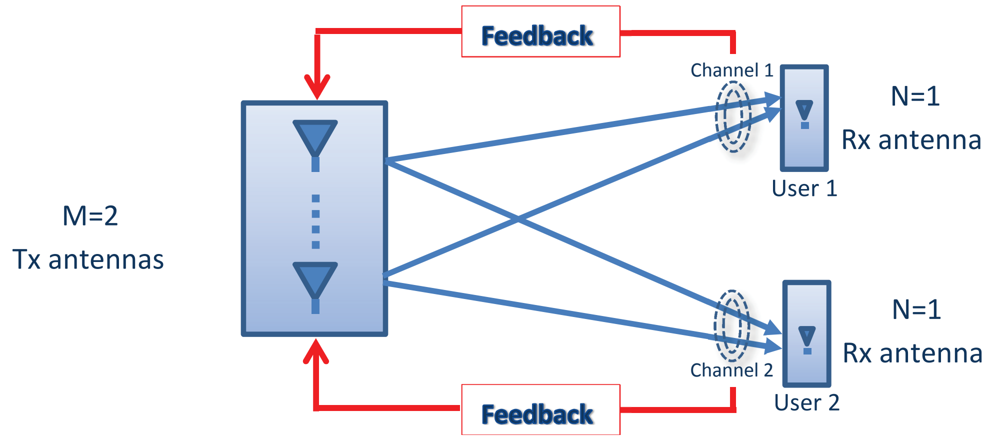
INSIGHT

Insight: more antennas for less CSIT quality

What is the role of MIMO in reducing necessary feedback quality?

Can, having more receive antennas, allow for reduced feedback quality?

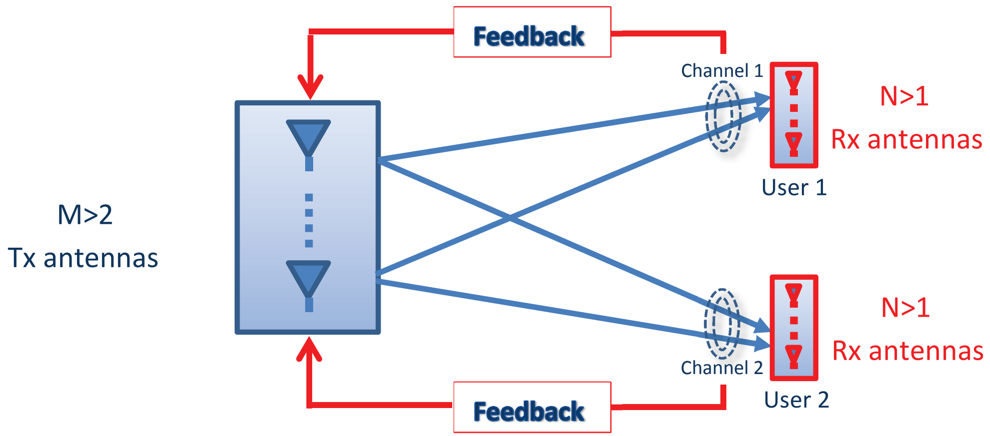
Insight: more antennas for less CSIT quality[1]



- Previous results show that, to achieve $d_1 = d_2 = 1$, we need constantly 'perfect' feedback.

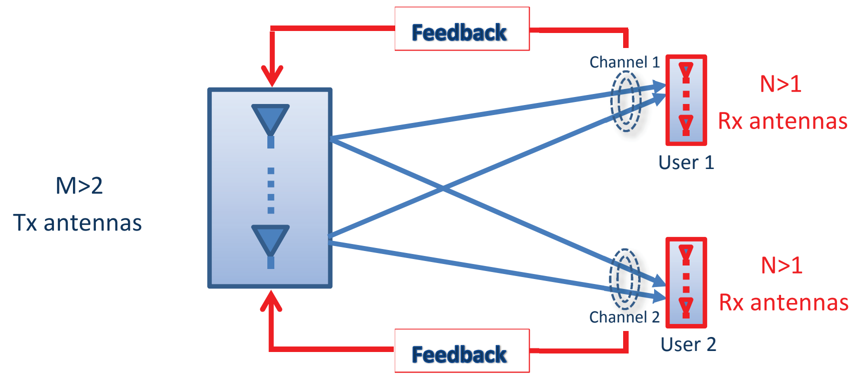
$$\alpha_t^{(1)} = \alpha_t^{(2)} = 1, \ \forall t \quad \Rightarrow \bar{\alpha}^{(1)} = \bar{\alpha}^{(2)} = 1$$

But what if we have more antennas?



- Do we still need constantly 'perfect' feedback, to achieve the (respective) optimal DoF?

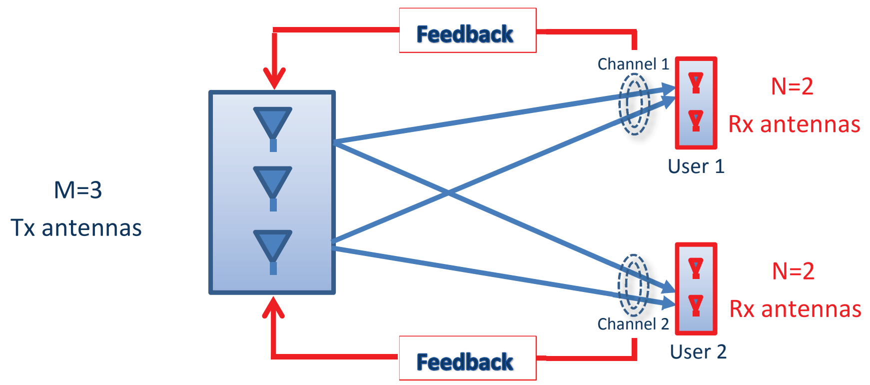
Insight: more antennas for less CSIT quality$_3$



Corollary: *(Chen-Elia) A CSIT process with* $\bar{\alpha}^{(1)}+\bar{\alpha}^{(2)} \geq \min\{M, 2N\}/N$, *achieves the optimal sum-DoF associated to perfect feedback*[5].

[5] Interested in $M > N$ (recall that if $M \leq N$, then no CSIT is needed)

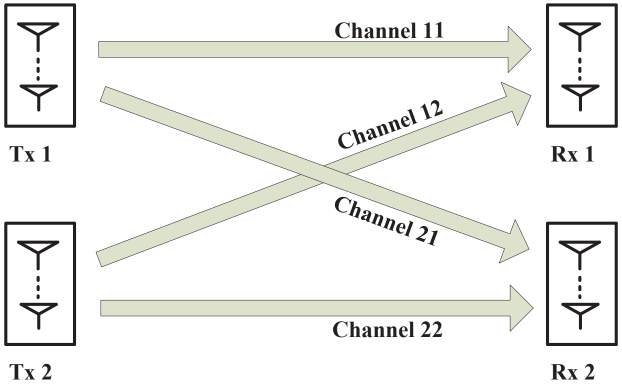
Insight: more antennas for less CSIT quality$_4$



EXAMPLE: $M = 3, N = 2$

- Note: perfect CSIT ($\bar{\alpha}^{(1)} = \bar{\alpha}^{(2)} = 1$) gives optimal sum-DoF of 3
- BUT: same sum DoF with $\bar{\alpha}^{(1)} + \bar{\alpha}^{(2)} = 3/2$
  - ⋆ e.g. $\bar{\alpha}^{(1)} = \bar{\alpha}^{(2)} = 3/4$

Insight: MIMO-IC

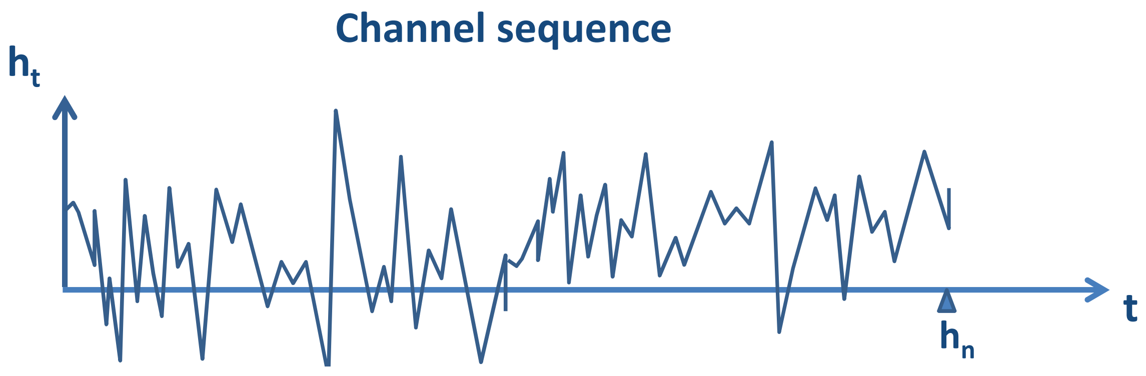


Corollary: *In the IC, no CSIT is needed for the direct links.*

Insight: Minimizing the total number of feedback bits

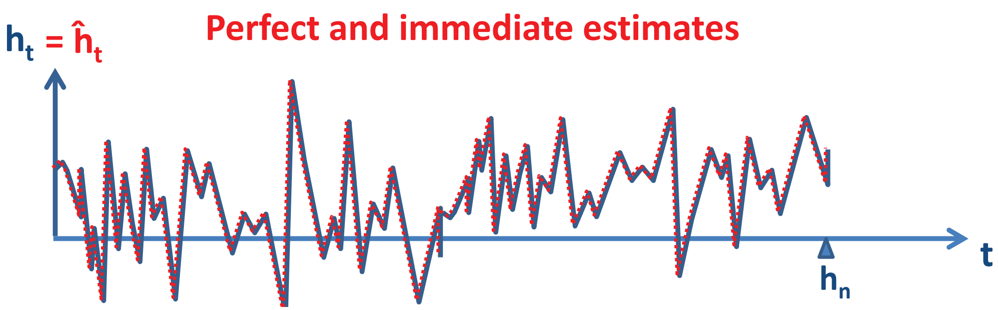
Minimizing the total number of feedback bits

- Assume you want to feedback a certain i.i.d. channel process of duration n



Assume perfect and immediate feedback

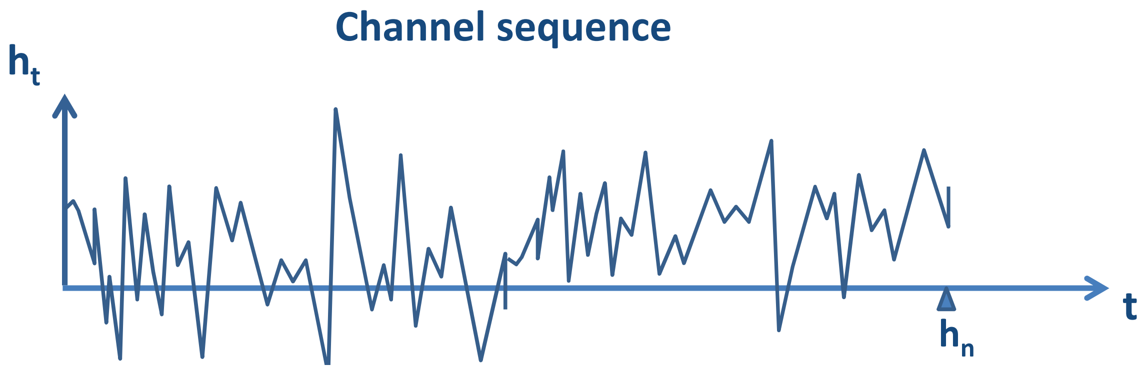
- Assume perfect and immediate feedback



- Need to send $n \times X$ bits
- X is number of bits required to perfectly describe a channel scalar

Now assume perfect but delayed feedback

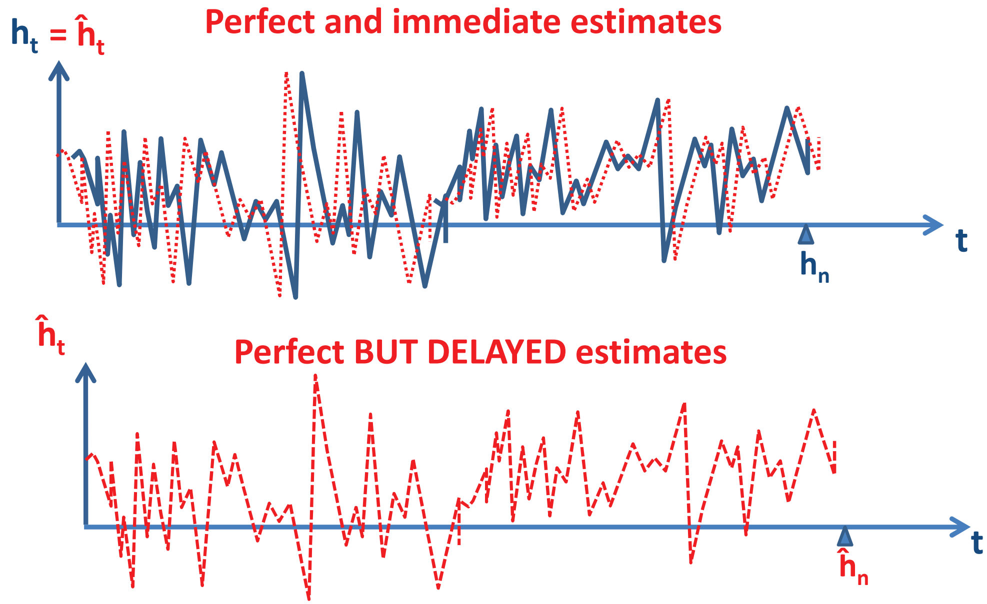
- Assume same channel process



Now assume perfect but delayed feedback

- Assume same channel process
- With perfect-quality BUT DELAYED feedback

Now assume perfect but delayed feedback₁



- Need to send $\approx n \times X$ bits
- Just shifted the time-scale of the problem
- Did not drastically reduce feedback amount

- Need to reduce quality of delayed feedback also
- We will see more of this, later on, but for now...

Insight: Reducing total number of feedback bits

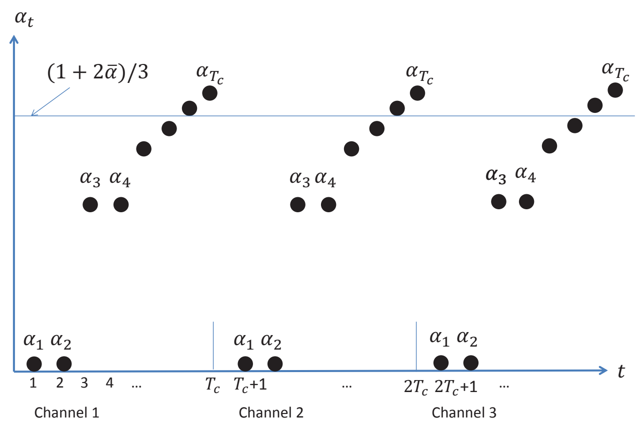
Corollary: *Having*

$$\min\{\bar{\beta}^{(1)},\bar{\beta}^{(2)}\} \geq \min\{1, M-\min\{M,N\}, \frac{N(1+\bar{\alpha}^{(1)}+\bar{\alpha}^{(2)})}{\min\{M,2N\}+N}, \frac{N(1+\bar{\alpha}^{(2)})}{\min\{M,2N\}}\}$$

is like having perfect delayed CSIT (i.e., like having $\bar{\beta}^{(1)} = \bar{\beta}^{(2)} = 1$*).*

Insight: how much delayed feedback is necessary?

- Along the same lines

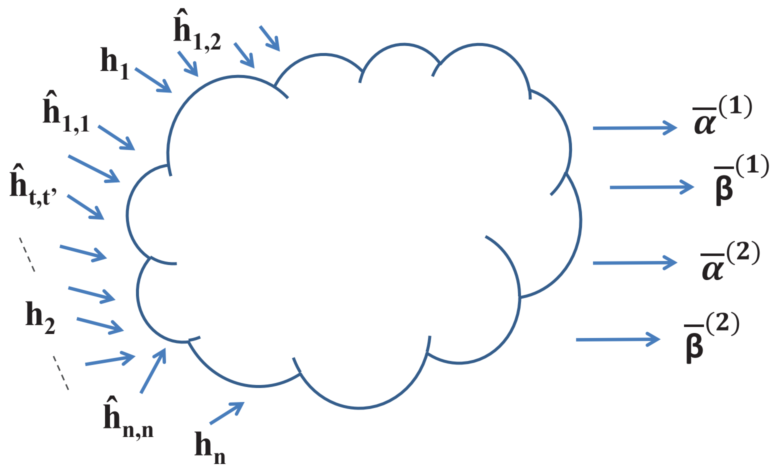


- *Corollary: Having delayed-CSIT quality* $\beta \geq \frac{1+2\bar{\alpha}}{3}$ *is equivalent to having perfect delayed CSIT.*

- *Corollary: If* $\alpha_{T_c} \geq \frac{1+2\bar{\alpha}}{3}$, *there is no need for any delayed CSIT* (More later)
  - ⋆ i.e., no need for feedback after coherence block.

Insight: reduced 'problem complexity'

Complexity of the problem is captured by only 4 parameters



Insight: reduced 'problem complexity'$_1$

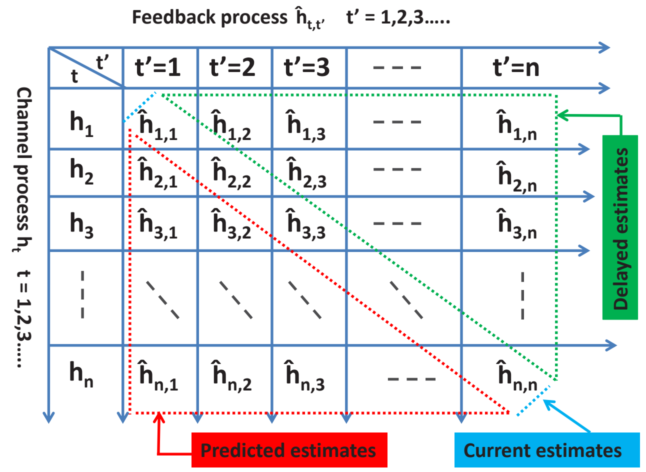
- Gaussianity $\Rightarrow$ Statistics of $\left\{(\boldsymbol{h}_t - \hat{\boldsymbol{h}}_{t,t'}), (\boldsymbol{g}_t - \hat{\boldsymbol{g}}_{t,t'})\right\}_{t,t'=1}^{n}$ captured by covariance matrix

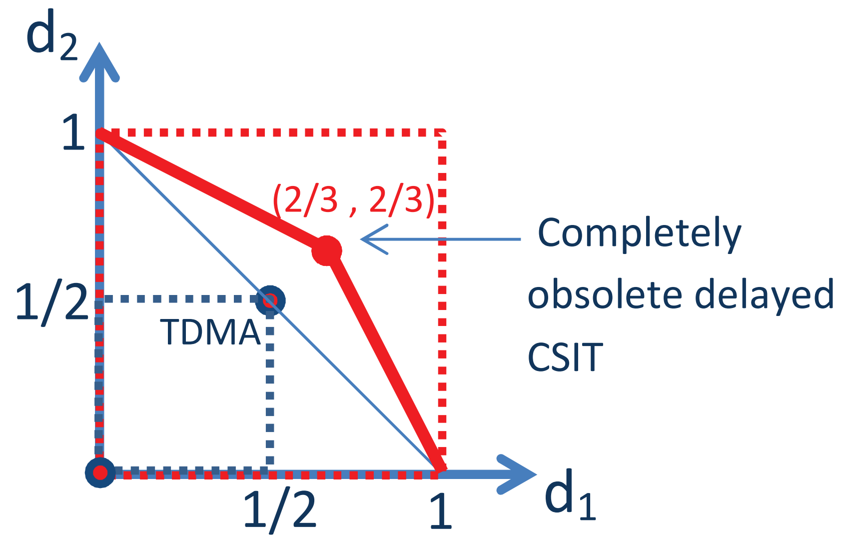
$$Cov\left(vect(\left\{(\boldsymbol{h}_t - \hat{\boldsymbol{h}}_{t,t'}), (\boldsymbol{g}_t - \hat{\boldsymbol{g}}_{t,t'})\right\}_{t,t'=1}^{n})\right) \in \mathbb{C}^{2n^2 \times 2n^2}$$

- Diagonal entries of $Cov(\bullet)$ are $\left\{\frac{1}{M}\mathbb{E}[||\boldsymbol{h}_t - \hat{\boldsymbol{h}}_{t,t'}||^2], \frac{1}{M}\mathbb{E}[||\boldsymbol{g}_t - \hat{\boldsymbol{g}}_{t,t'}||^2]\right\}_{t,t'=1}^{n}$. Some of them are represented by the exponents
- But, the rest, plus the off-diagonal entries not used by scheme
- But, scheme meets outer bound that holds irrespective of these other entries
- $\Rightarrow$ exponents faithfully represent problem
- In the end only the four averages show up

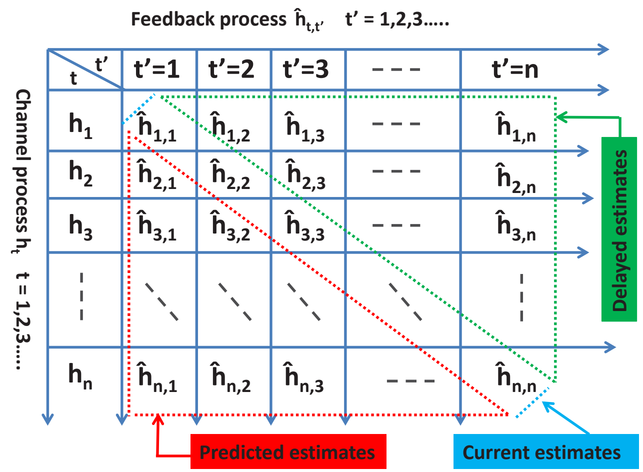
Insight: delayed CSIT?

Theorem: *(Maddah-Ali and Tse) (Have seen). Completely obsolete feedback helps.*



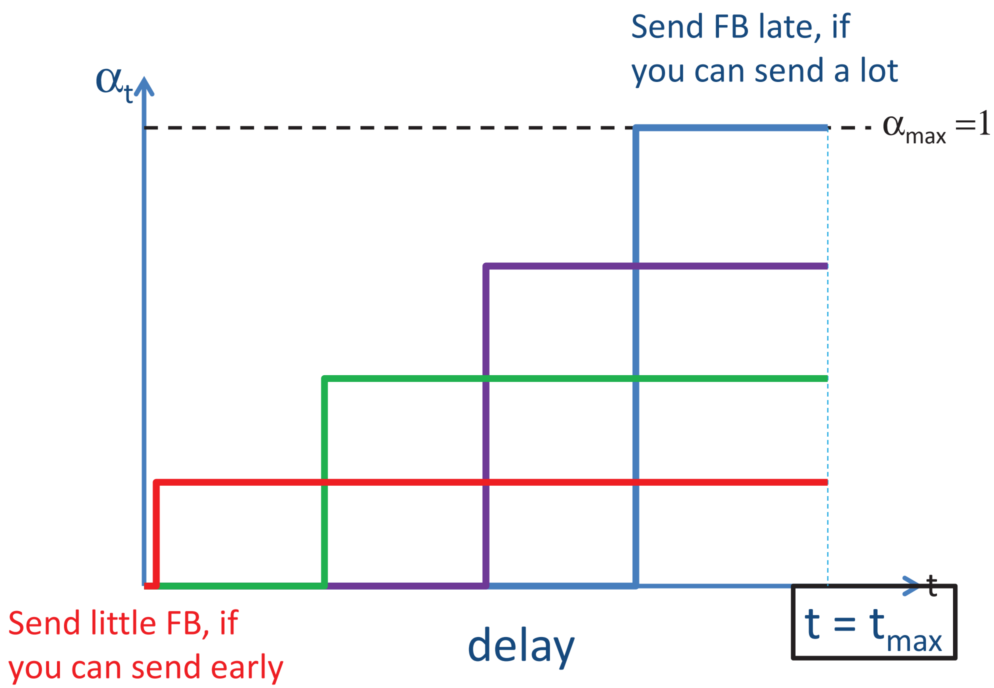


Corollary: *(Chen-Elia) There is no DoF gain in using predicted CSIT*[6]*.*



[6] For sufficiently good delayed CSIT. Same conclusion also holds based on inner bounds.

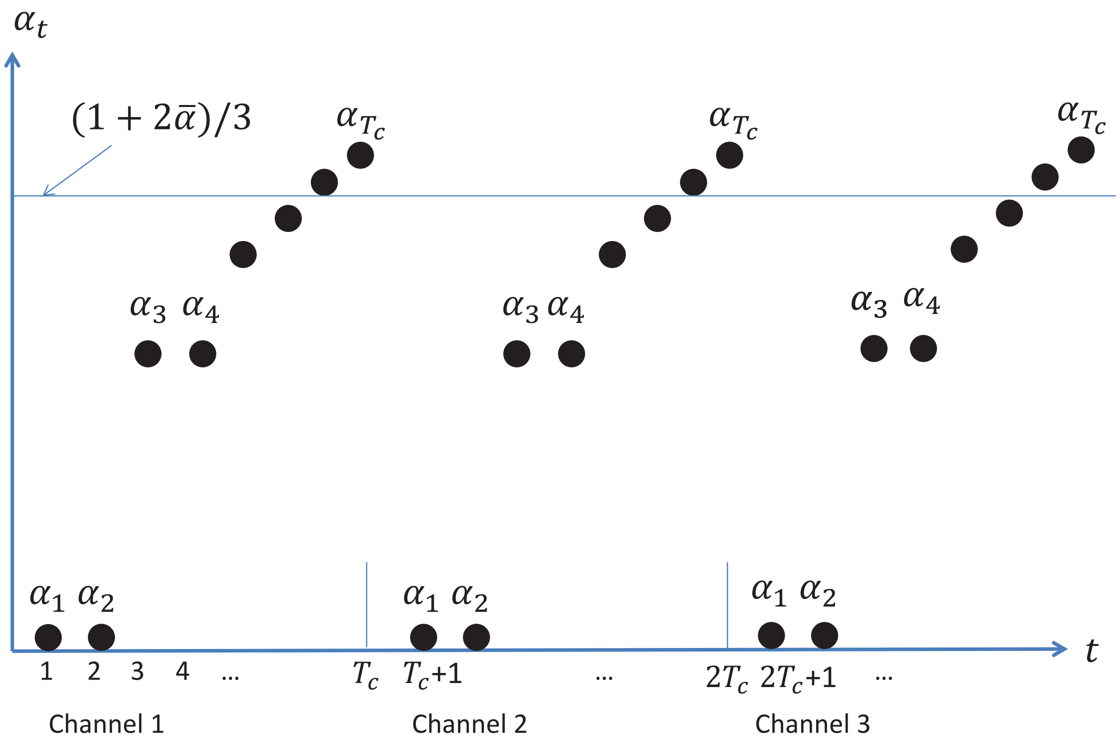
Insight: Less feedback early, or more feedback later?



Evolving CSIT with gradual feedback

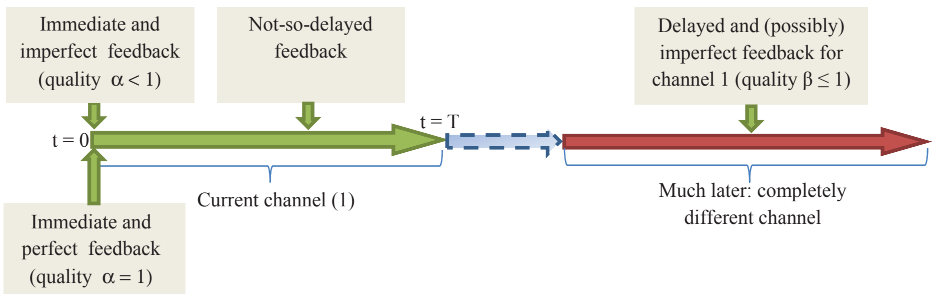
- A useful tool
- Answering many fundamental questions

Insight: Evolving feedback and block fading



Evolving CSIT with gradual feedback

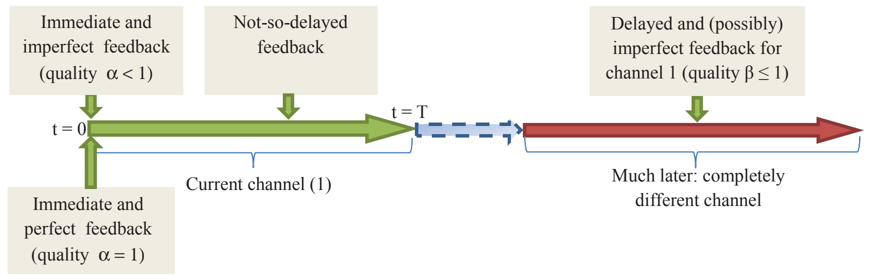
Evolving CSIT with gradual feedback[7]



- Feedback comes in steps
- A gradual accumulation of feedback bits can result in a progressively increasing CSIT quality
  - ⋆ As time progresses across the coherence period (T channel uses - current CSIT), or at any time after

[7]Chen-Elia 2012

General setting of evolving CSIT



- Block fading: coherence block of duration T
- Current estimates at time t

$$\hat{\boldsymbol{h}}_t, \ \hat{\boldsymbol{g}}_t$$

- Quality of current estimates: α_t

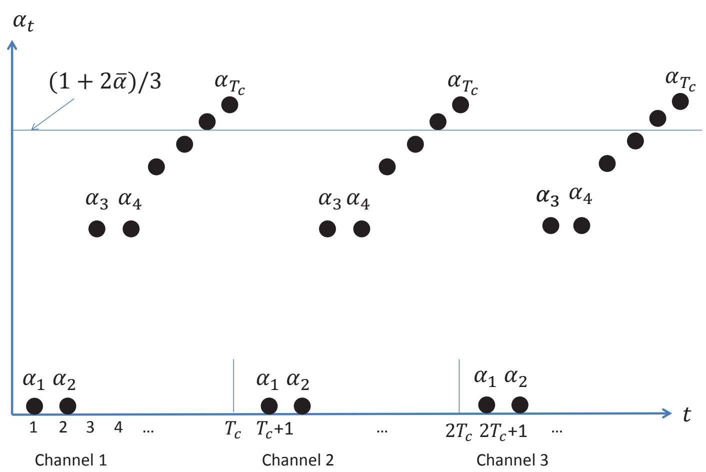
$$\mathbb{E}||\boldsymbol{h} - \hat{\boldsymbol{h}}_t||^2 = \mathbb{E}||\boldsymbol{g} - \hat{\boldsymbol{g}}_t||^2 \approx P^{-\alpha_t}$$

- Delayed CSIT of quality β
- Evolving CSIT: $0 \leq \alpha_1 \leq \alpha_2 \leq \cdots \leq \alpha_T \leq \beta \leq 1$

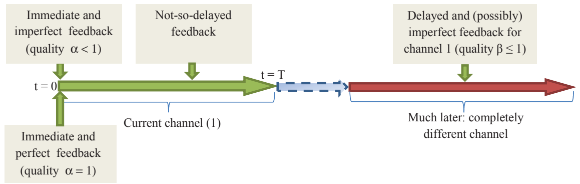
General setting of evolving CSIT$_1$

EVOLVING EXPONENTS

time	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$\cdots$	$t = T$	$t > T$
quality exponent	$0 \leq \alpha_1$	α_2	α_3	α_4	$\cdots$	α_T	$\beta \leq 1$

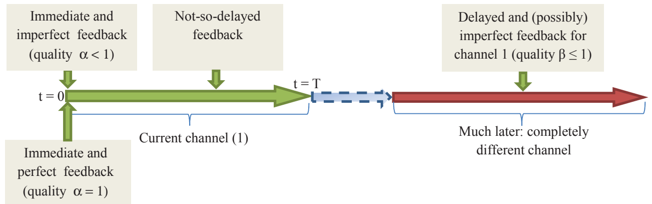


General setting of evolving CSIT$_2$



- Example (recall: each coherence block has T channel uses)
  - ⋆ If we send no feedback bits at $t = 0$
  - ⋆ then send $1/3 \log P$ bits at $t = T/3$
  - ⋆ then send $1/3 \log P$ bits at $t = 2T/3$
  - ⋆ and $1/3 \log P$ bits at any time $t > T$
- then the corresponding evolving CSIT quality exponents are
  - ⋆ $\alpha_t = 0, \forall t \in [0, T/3)$
  - ⋆ $\alpha_t = 1/3, \forall t \in [T/3 + 1, 2T/3)$
  - ⋆ $\alpha_t = 2/3, \forall t \in [2T/3 + 1, T]$
  - ⋆ $\beta = 1$

General setting of evolving CSIT$_3$



Approach unifies previous works

- No CSIT ($\beta = \alpha_t = 0$)
- Full CSIT ($\alpha_1 = 1$)
- Maddah-Ali and Tse

$$\beta = 1, \alpha_t = 0, \ \forall t \leq T$$

- Imperfect current CSIT setting of Yang et al. and of Gou and Jafar

$$\beta = 1, \alpha_1 = \cdots = \alpha_T > 0$$

- Asymmetric setting of Maleki et al.

- Not-so-delayed CSIT setting of Lee and Heath

$$\beta = 1, \alpha_1 = \cdots = \alpha_\tau = 0, \quad \text{some } \tau < T$$

Evolving CSIT: results

Directly from previous results: Let

$$\bar{\alpha} \triangleq \frac{1}{T}\sum_{t=1}^{T}\alpha_t.$$

Then

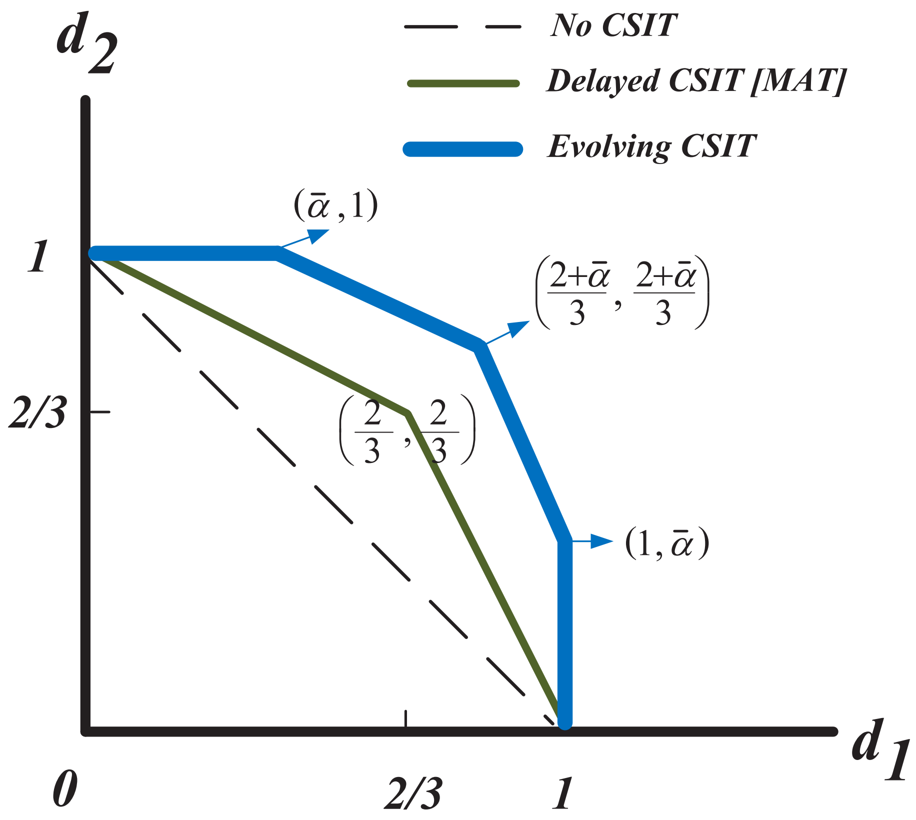
- *Theorem*[8]*: The optimal DoF region for symmetrically evolving current CSIT and perfect delayed CSIT is*

$$\begin{aligned} d_1 \leq 1, \quad d_2 &\leq 1 \\ 2d_1 + d_2 &\leq 2 + \bar{\alpha} \\ 2d_2 + d_1 &\leq 2 + \bar{\alpha} \end{aligned}$$

  and corresponds to the polygon with corner points

$$\{(0,0),(0,1),(\bar{\alpha},1),(\frac{2+\bar{\alpha}}{3},\frac{2+\bar{\alpha}}{3}),(1,\bar{\alpha}),(1,0)\}.$$

[8]Chen-Elia 2013



d_2
No CSIT
Delayed CSIT [MAT]
Evolving CSIT
$(\bar{\alpha}, 1)$
1
$\left(\frac{2+\bar{\alpha}}{3}, \frac{2+\bar{\alpha}}{3}\right)$
2/3
$\left(\frac{2}{3}, \frac{2}{3}\right)$
$(1, \bar{\alpha})$
d_1
0
2/3
1

Evolving CSIT: examples

EXAMPLE: How to achieve target DoF $d_1 = d_2 = d' = 7/9$?

- Recall sequence

$$\underbrace{\alpha_1 \leq \alpha_2 \leq \cdots \leq \alpha_T}_{\substack{\text{Progressive feedback} \\ \text{during coherence period}}} \quad \leq \quad \underbrace{\beta}_{\substack{\text{Delayed feedback} \\ \text{after coherence period}}}$$

- Optimal (symmetric) DoF was given:

$$d = \frac{2 + \bar{\alpha}}{3}$$

  ⋆ where $\bar{\alpha} = \text{average}(\alpha_1, \alpha_2, \cdots, \alpha_T)$

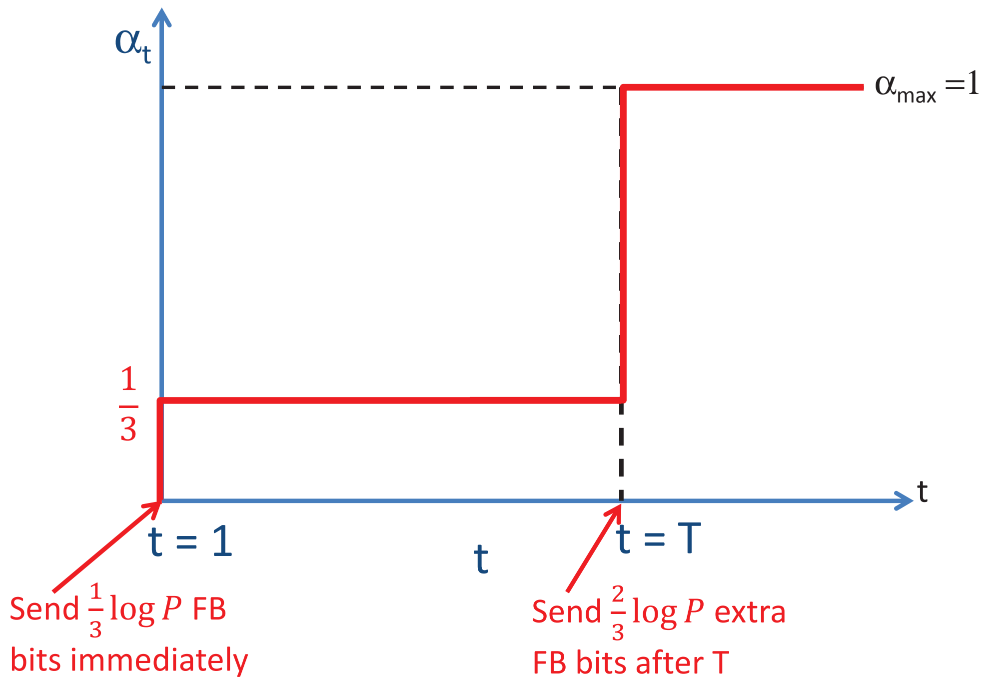
- Thus solve: We need

$$\bar{\alpha} \geq 3d' - 2 = 3 \cdot \frac{7}{9} - 2 = 1/3$$

- What are the feedback options?

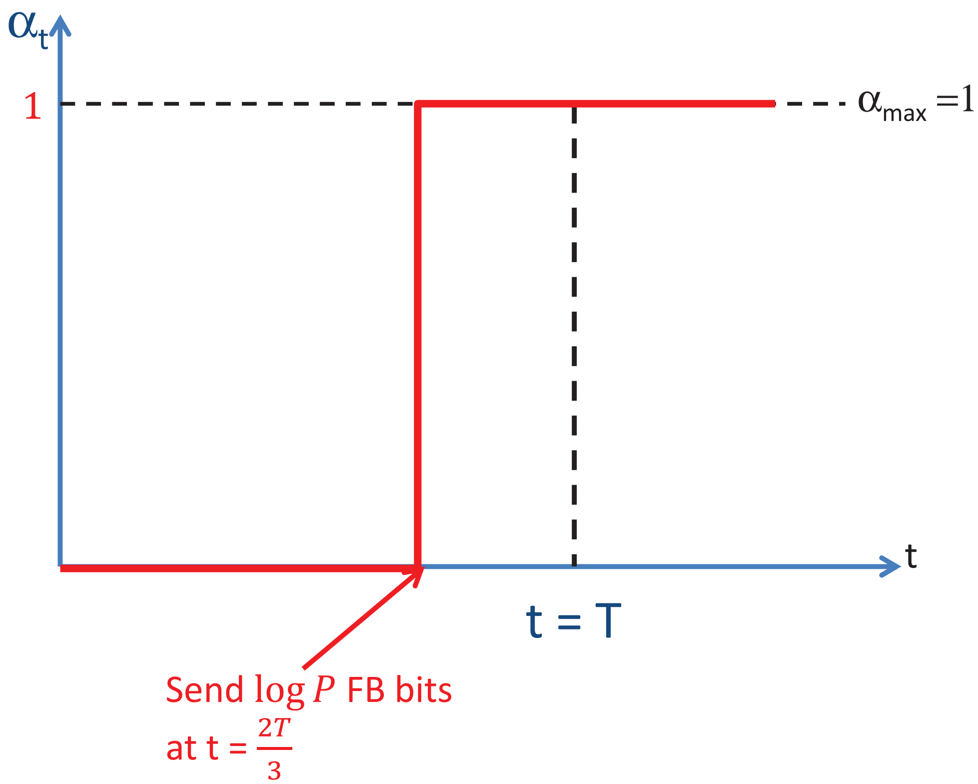
Evolving CSIT: examples[1]

$\bar{\alpha} = 1/3$: Option 1

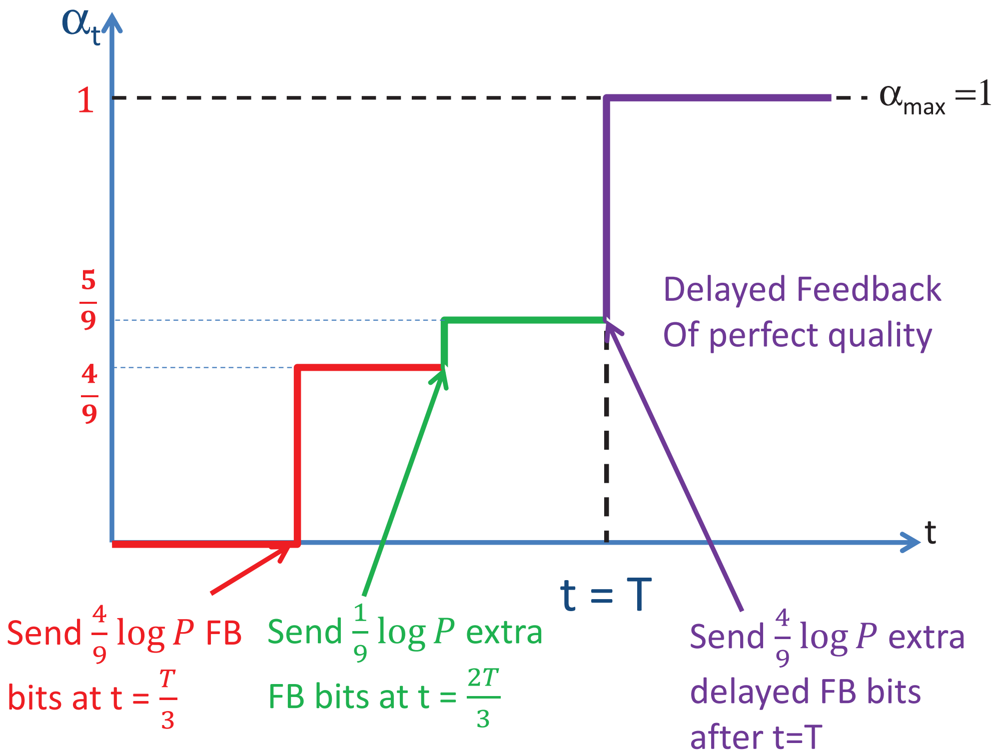


Evolving CSIT: examples₂

$\bar{\alpha} = 1/3$: OPTION 2



$\bar{\alpha} = 1/3$: OPTION 3



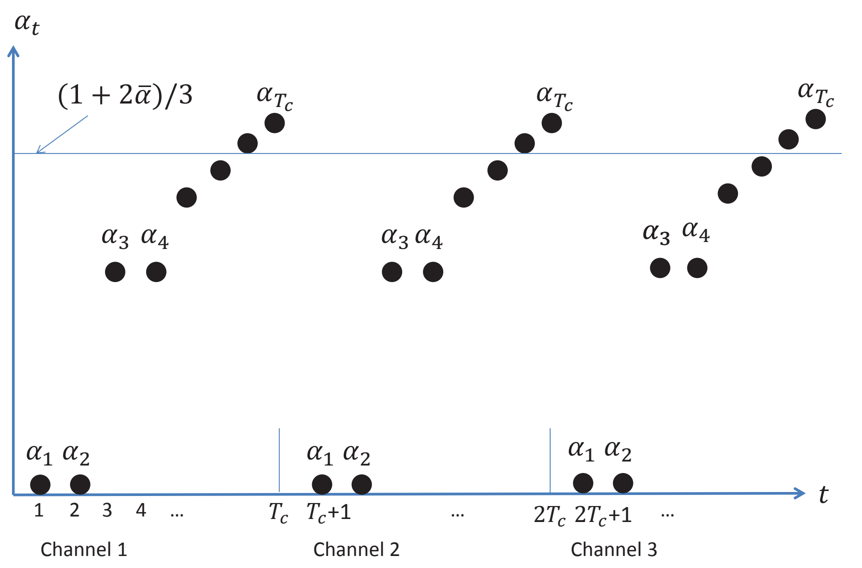
Evolving - Insight: Reducing total feedback

How to reduce total amount of feedback?

- Must reduce delayed feedback quality (reduce β)

Evolving - Insight: Reducing total feedback[1]

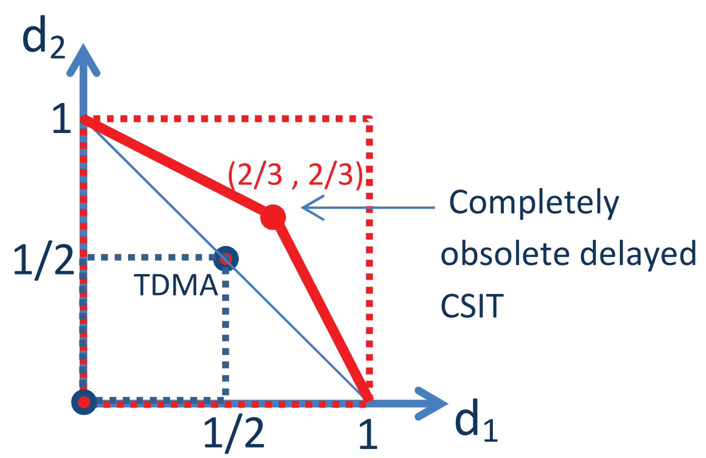
- How many (delayed) feedback bits must be gathered after channel changes?
- When is delayed feedback even necessary?
- Can imperfect delayed CSIT be as useful as perfect delayed CSIT?
- Can we achieve same performance as before with lesser total feedback?

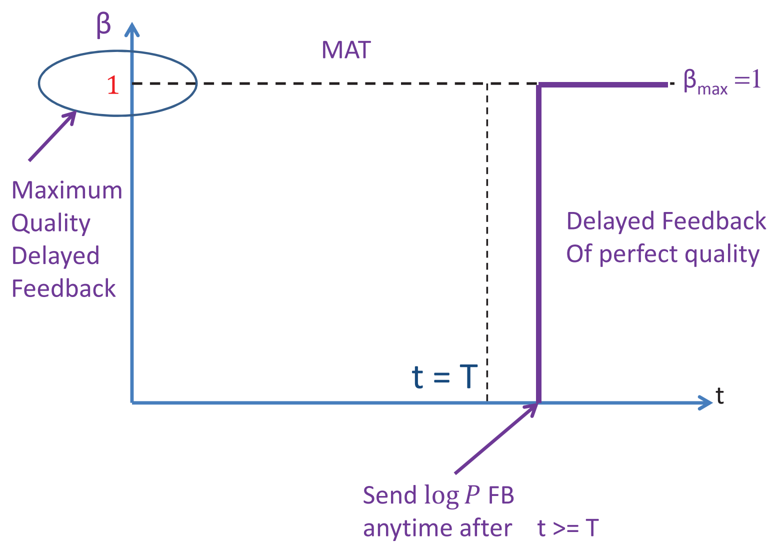


Example: imperfect delayed feedback

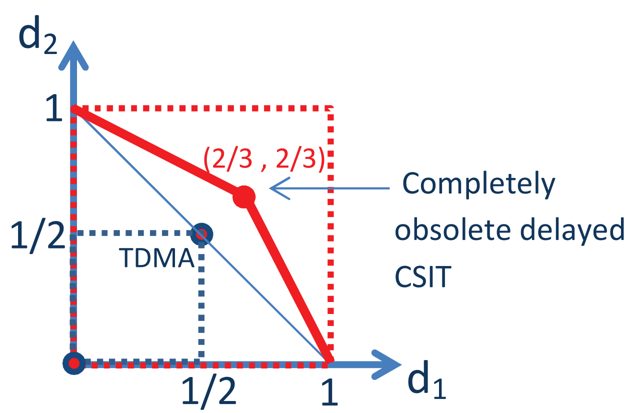
EXAMPLE:

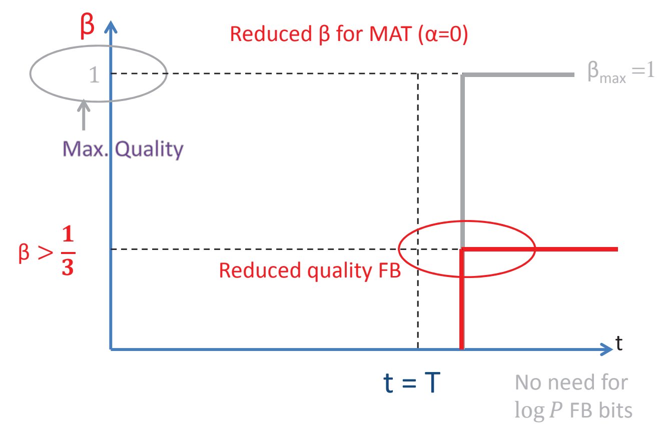
- Can we achieve the MAT $d = 2/3$, with less than a total of $\log P$ (current + delayed) feedback bits?
  - ⋆ I.e., with imperfect delayed feedback

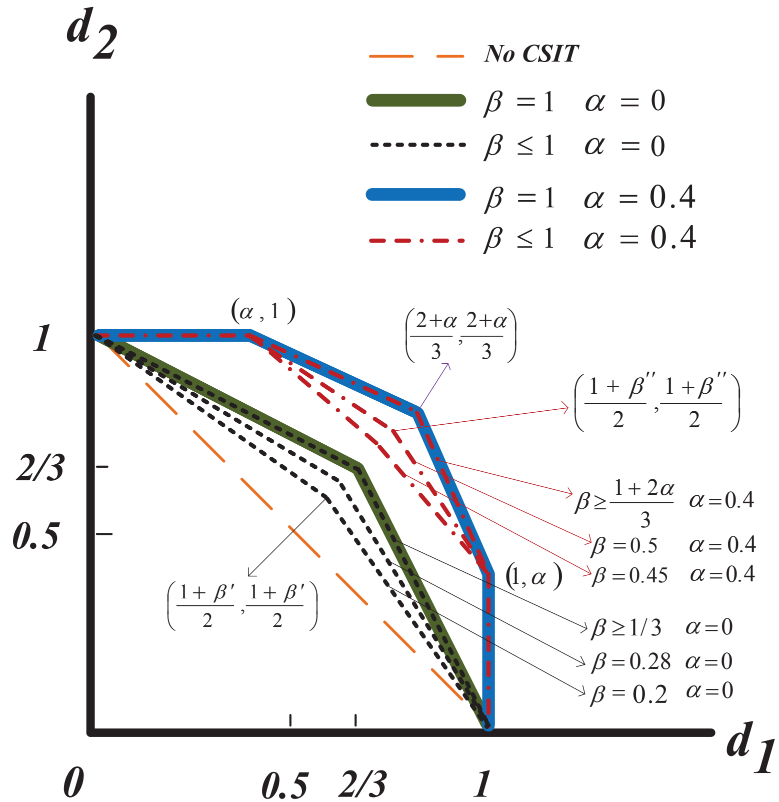




- *Corollary (Chen-Elia): MAT case (originally $\beta = 1, \alpha = 0$):* $\beta = 1/3$ suffices to achieve the optimal region ($d_1 = d_2 = 2/3$)

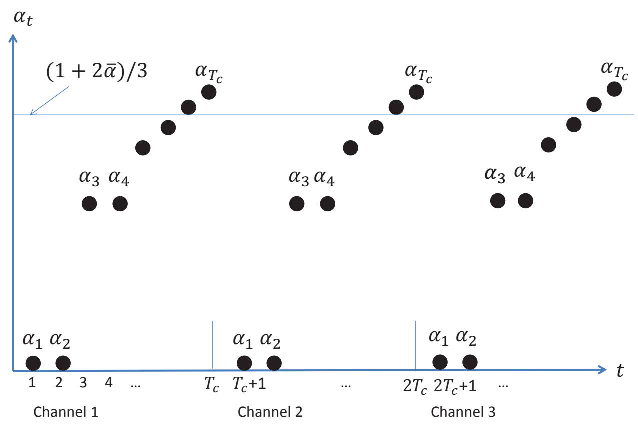


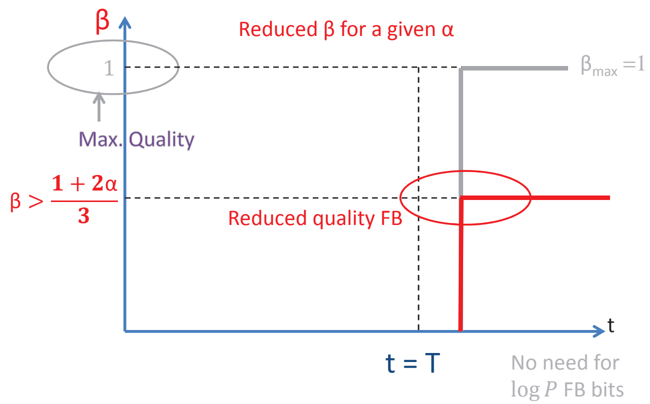




MISO BC with imperfect current and imperfect delayed CSIT.
$\beta' = \min\{\beta, \frac{1}{3}\}$ *and* $\beta'' = \min\{\beta, \frac{1+2\alpha}{3}\}$.

When is delayed feedback unnecessary?





- *Corollary: Having delayed-CSIT quality $\beta \geq \frac{1+2\bar{\alpha}}{3}$ is equivalent to having perfect delayed CSIT.*

- *Corollary: When $\alpha_T \geq \frac{1+2\bar{\alpha}}{3}$, there is no need for any delayed CSIT, i.e., do not send feedback after the end of the coherence block.*

Universal encoding-decoding scheme

Universal encoding-decoding scheme

Half the key of success

Schemes must lim-optimally utilize each and every bit feedback no matter how erroneous, delayed or premature

Universal encoding-decoding scheme

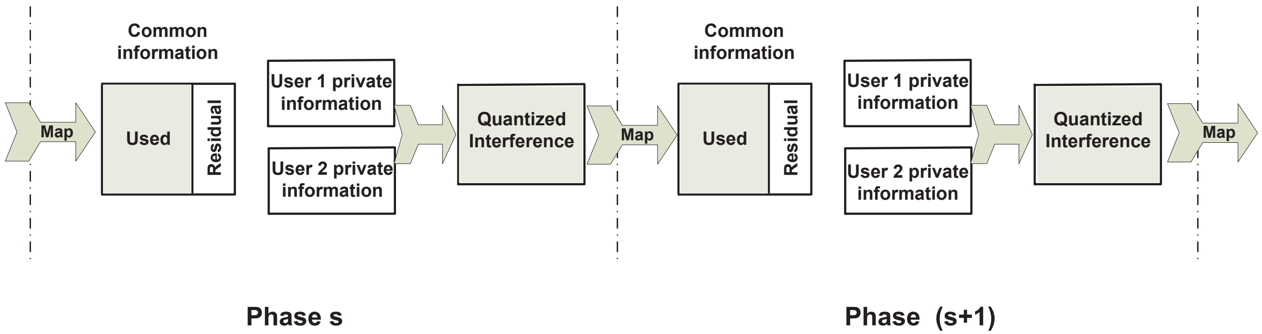
- Challenge: design scheme of duration n, that utilizes a CSIT process

$$\left\{ \hat{\boldsymbol{h}}_{t,t'}, \hat{\boldsymbol{g}}_{t,t'} \right\}_{t=1,t'=1}^{n}$$

- Get the help of quality exponents

- Novel schemes with a *phase-Markov* structure
  - ⋆ Schemes often meet outer bounds
  - ⋆ Apply in various settings (e.g. frequency selective: Hao-Clerckx 13)

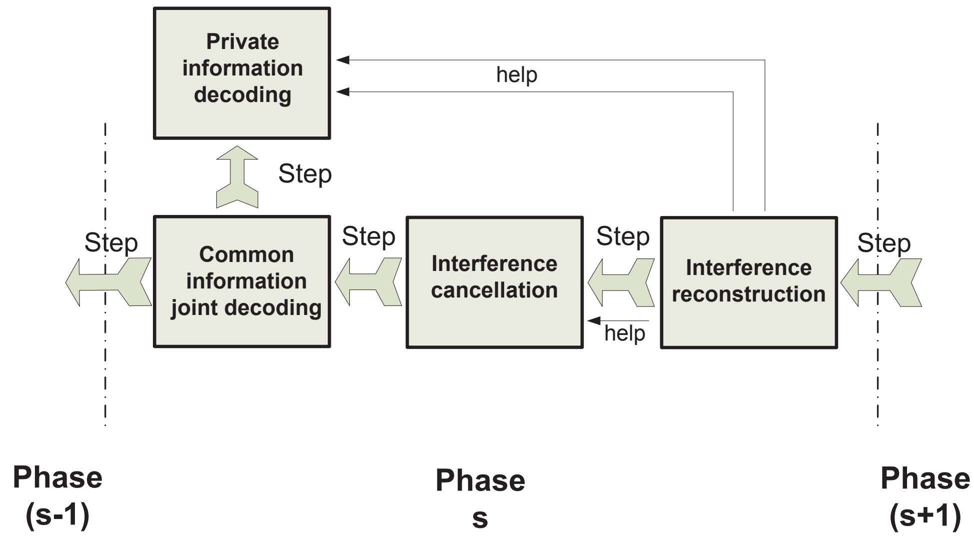
Universal encoding-decoding scheme[1]



Encoding and decoding phase-Markov scheme:

- Accumulated quantized interference bits of phase s, can be broadcasted to both users inside the common information symbols of the next phase
- while also a certain amount of common information can be transmitted to both users during phase s, which will then help resolve the accumulated interference of phase $(s-1)$.
- All parameters (power and rate allocation, etc) are functions of the (declared) quality exponents

Universal scheme: decoding



An information theoretic look to Block-Markov encoding

Shayevitz&Wigger Scheme for Generalized Feedback $\tilde{Y}$ (ISIT'10, IT-Trans March 2013)

(ISIT'10, IT-Trans March 2013)

- Block-Markov strategy
- In each block use Marton's nofeedback scheme to send fresh data $M_{1,b}$, $M_{2,b}$ & update infos $J_{0,b-1}$, $J_{1,b-1}$, $J_{2,b-1}$
- Update infos $J_{0,b-1}$, $J_{i,b-1}$ for Receiver i: compression indices for Marton-codewords and feedback-outputs $(U_{0,b-1}, U_{1,b-1}, U_{2,b-1}, \tilde{Y}_{b-1})$ given receiver-SI $Y_{i,b-1} \rightarrow V_{i,b-1}$

- Backward decoding:
  1. Use $J_{0,b}$, $J_{i,b}$, $Y_{i,b}$ to reconstruct compression $V_{i,b}$
  2. Decode $M_{i,b}$, $J_{0,b-1}$, $J_{i,b-1}$ based on improved outputs $(Y_{i,b}, V_{i,b})$

An information theoretic look to Block-Markov encoding[1]

Theorem:*[Region for Generalized Feedback $\tilde{Y}$](Shayevitz-Wigger'10)*

Achievable Region: (R_1, R_2) *achievable, if for some* $P_Q P_{U_0 U_1 U_2|Q}, P_{X|U_0 U_1 U_2 Q}, P_{V_0 V_1 V_2|U_0 U_1 U_2 \tilde{Y} Q}$:

$$
\begin{aligned}
R_1 &\le I(U_0, U_1; Y_1, V_1, Q) - I(U_0, U_1, U_2, \tilde{Y}; V_0, V_1 | Y_1, Q) \\
R_2 &\le I(U_0, U_2; Y_2, V_2, Q) - I(U_0, U_1, U_2, \tilde{Y}; V_0, V_2 | Y_2, Q) \\
R_1 + R_2 &\le I(U_1; Y_1, V_1 | U_0, Q) + I(U_2; Y_2, V_2 | U_0, Q) + \min_{i \in \{1,2\}} I(U_0; Y_i, V_i | Q) \\
&\quad - I(U_0, U_1, U_2, \tilde{Y}; V_1 | V_0, Y_1) - I(U_0, U_1, U_2, \tilde{Y}; V_2 | V_0, Y_2) \\
&\quad - I(U_1; U_2 | U_0, Q) - \max_{i \in \{1,2\}} I(U_0, U_1, U_2, \tilde{Y}; V_0 | Y_i, Q) \\
R_1 + R_2 &\le I(U_1, U_0; Y_1, V_1, Q) + I(U_2, U_0; Y_2, V_2, Q) - I(U_1; U_2 | U_0, Q) \\
&\quad - I(U_0, U_1, U_2, \tilde{Y}; V_0, V_1 | Y_1, Q) - I(U_0, U_1, U_2, \tilde{Y}; V_0, V_2 | Y_2, Q)
\end{aligned}
$$

Comments on the Shayevitz-Wigger'10 Region

- Update info should have common part $J_{0,b}$ useful to both rxs
- Tradeoff: update-info sent at expense of fresh data! $\rightarrow$ Identifying good update info/compression is hard in general
- Scheme applies to stale state information: $\tilde{Y} = S \rightarrow$ Maddah-Ali&Tse'10:

$$Q = \begin{cases} 0 & \text{w.p. } 1/3 \\ 1 & \text{w.p. } 1/3 \\ 2 & \text{w.p. } 1/3 \end{cases}, \quad V_0 = V_i = \begin{cases} \emptyset & \text{if } Q = 0 \\ Y_1 & \text{if } Q = 1 \\ Y_2 & \text{if } Q = 2 \end{cases}, \quad X = \begin{cases} U_0 & \text{if } Q = 0 \\ U_1 & \text{if } Q = 1 \\ U_2 & \text{if } Q = 2 \end{cases}$$

$\rightarrow$ Yang/Kobayashi/Gesbert/Yi'11: $Q \sim \text{Bern}(2/3)$,

$$V_0 = V_i = \begin{cases} \emptyset & \text{if } Q = 0 \\ (\hat{\eta}_1, \hat{\eta}_2) & \text{if } Q = 1 \end{cases}, \quad X = \begin{cases} U_1 + U_2 & \text{if } Q = 0 \\ U_0 + U_1 + U_2 & \text{if } Q = 1 \end{cases}$$

$\rightarrow$ Chen&Elia'13:$V_0 = V_1 = V_2 = (\bar{\tilde{l}}^{(1)}, \bar{\tilde{l}}^{(2)})$, $\quad X = U_0 + U_1 + U_2$

WU-WIGGER SCHEME '13 ITW 2013, ARXIV JAN. 2014

- Feedback rate-limited to R_{fb}, (receivers can code over feedback links)
- Scheme based on superposition coding and following ideas:
  - ⋆ feedback allows to occupy unused resource in superposition scheme
  - ⋆ new way to construct common info. useful for both receivers

Theorem:

(R_1, R_2) *achievable, if for some* $P_U P_{X|U} P_{\hat{Y}_1|UY_1}$:

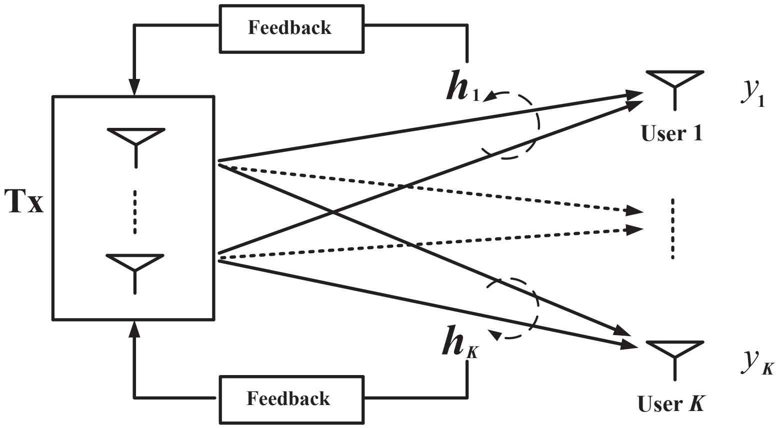
$$R_1 \leq I(U;Y_1) \tag{1}$$

$$R_2 \leq I(X;\hat{Y}_1 Y_2|U) = I(X;Y_2|U) + \underbrace{I(X;\hat{Y}_1|U,Y_2)}_{\text{purely beneficial}} \tag{2}$$

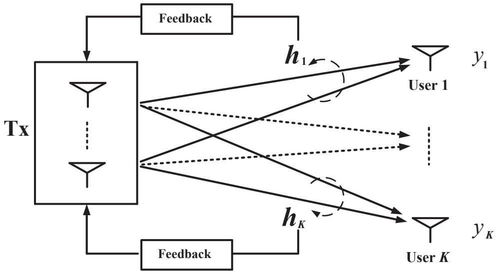
and $\quad I(\hat{Y}_1;Y_1|U,Y_2) \leq \min\{R_{fb}, I(U;Y_2) - I(U;Y_1)\}.$

- Any $R_{\text{fb}} > 0$ increases capacity of strictly less-noisy DMBCs
  - ⋆ Ex.: BSC-BC or BEC-BC with unequal cross-over/erasure prob.

K-USER MISO BC
A WIDE RANGE OF OPEN PROBLEMS



Similar channel model: K-user MISO BC



$$
\begin{aligned}
y_{1,t} &= \boldsymbol{h}_{1,t}^{\mathsf{T}} \boldsymbol{x}_t + z_{1,t} \\
y_{2,t} &= \boldsymbol{h}_{2,t}^{\mathsf{T}} \boldsymbol{x}_t + z_{2,t} \\
&\vdots \\
y_{K,t} &= \boldsymbol{h}_{K,t}^{\mathsf{T}} \boldsymbol{x}_t + z_{K,t}
\end{aligned}
$$

- M-transmit antenna, K single-antenna users
- $\boldsymbol{x}_t$ transmitted vector at time t
- Power constraint $\mathbb{E}[||\boldsymbol{x}_t||^2] \leq P$ (SNR)
- $z_{k,t}$ AWGN noise

Same DoF measure of performance

$$d_i = \lim_{P \to \infty} \frac{R_i}{\log P}, \;\; i = 1, 2, \cdots, K$$

- $(R_1, R_2, \cdots, R_K)$: achievable rate tuple
- Corresponding DoF region: The set of all achievable DoF tuples

Some prior work: Imperfect and delayed feedback

Emphasis on the K-user case $(K \geq 2)$

- ...
- Delayed CSIT [Maddah-Ali and Tse 10]
- Not-so-delayed CSIT [Lee and Heath 12]
- Alternating CSIT [Tandon et al. 12]
- ...

K-user MISO BC with only delayed feedback

Theorem: *(Maddah-Ali and Tse) The optimal sum-DoF*

$$d_\Sigma \triangleq \sum_{k=1}^{K} d_k$$

of the K-user MISO BC with delayed feedback, takes the form

$$d_{MAT} \triangleq \frac{K}{1 + \frac{1}{\min\{2,M\}} + \frac{1}{\min\{3,M\}} + \cdots + \frac{1}{\min\{K,M\}}}$$

Glass half-full or half-empty

Corollary 1 *(Maddah-Ali and Tse) When* $M \geq K \to \infty$ *then*

$$d_{MAT} \approx \frac{K}{\ln K}$$

- Recall that no feedback gives $d_\Sigma = 1$
- Recall that perfect feedback gives $d_\Sigma = K$
- Good news:

$$d_{\text{MAT}} \approx \frac{K}{\ln K} >> 1 \ \ (\text{scales with } K)$$

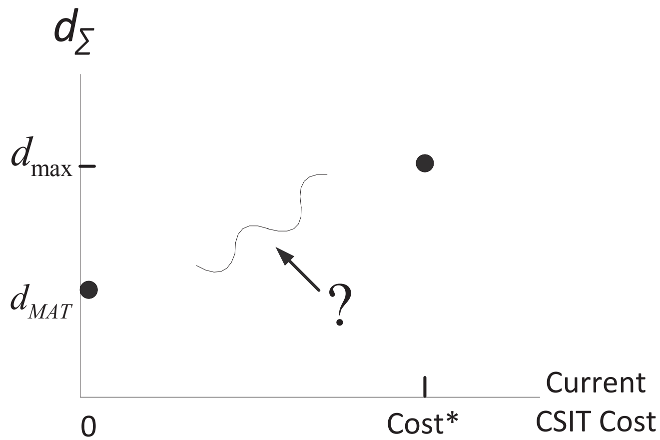
- Bad news:

$$\frac{d_{\text{MAT}}}{K} \approx \frac{1}{\ln K} \to 0 \ \ (\text{unbounded gap from optimal performance})$$

K-user BC with only delayed feedback$_1$

K-USER PROBLEM LARGELY OPEN

- Strong need for understanding role of current feedback



  ⋆ [Tandon et al. 12] [Lee and Heath 12]

- Strong need for outer bounds [Tandon et al. 12][Chen-Yang-Elia 13]

Same problem formulation

- Communication of duration n (n is large)
- An arbitrary channel fading process (random)

$$\left\{\boldsymbol{h}_{k,t}\right\}_{k=1,\ t=1}^{K\quad n}$$

- An arbitrary feedback process (CSIT)

$$\left\{\hat{\boldsymbol{h}}_{k,t,t'}\right\}_{k=1,\ t=1,\ t'=1}^{K\quad n\quad n}$$

  - ⋆ $\hat{\boldsymbol{h}}_{k,t,t'}$: CSIT estimate at any time t', of channel $\boldsymbol{h}_{k,t}$ (at time t)

- A 'primitive' measure of feedback 'goodness'

$$\left\{(\boldsymbol{h}_{k,t}-\hat{\boldsymbol{h}}_{k,t,t'})\right\}_{k=1,\ t=1,\ t'=1}^{K\quad n\quad n}$$

Same notation

Quality of *current* CSIT for channel $\boldsymbol{h}_{k,t}$ at time t

$$\alpha_t^{(k)} \triangleq -\lim_{P\to\infty} \frac{\mathbb{E}[||\boldsymbol{h}_{k,t} - \hat{\boldsymbol{h}}_{k,t,t}||^2]}{\log P} \qquad (user\ k)$$

- $\hat{\boldsymbol{h}}_{k,t,t'}$: CSIT estimate at any time t', of channel $\boldsymbol{h}_{k,t}$ (at time t)

Quality of *delayed* CSIT for channel $\boldsymbol{h}_{k,t}$ at time t

$$\beta^{(k)} \triangleq -\lim_{P\to\infty} \frac{\mathbb{E}[||\boldsymbol{h}_{k,t} - \hat{\boldsymbol{h}}_{k,t,t+\eta}||^2]}{\log P} \qquad (user\ k)$$

For any sufficiently large finite integer $\eta > 0$.

Same notation$_1$

Quality range (WOLOG)

$$0 \leq \alpha_t^{(k)} \leq \beta^{(k)} \leq 1$$

- $\beta_t^{(k)} = 1 \ \rightarrow$ perfect delayed CSIT (about channel $\boldsymbol{h}_{k,t}$ at time t)
- $\alpha_t^{(k)} = 1 \ \rightarrow$ perfect current (full) CSIT (about channel $\boldsymbol{h}_{k,t}$ at time t).

Average of exponent sequences $\{\alpha_t^{(k)}\}_{t=1}^n$

- Averages of the quality exponents (*current CSIT cost*)

$$\bar{\alpha}^{(k)} \triangleq \frac{1}{n}\sum_{t=1}^{n} \alpha_t^{(k)}, \quad k = 1, 2, \cdots, K$$

- π denotes a permutation of the ordered set $\{1, 2, \cdots, K\}$, $\pi(k)$ denotes the k th element of set π.

New outer bound (DoF Region)

For general setting: general channel process (large duration n), general feedback process

Theorem: *[DoF region outer bound] (Chen-Elia): The DoF region of the K-user $M \times 1$ MISO BC with a general CSIT feedback process, is outer bounded as*

$$\sum_{k=1}^{K} \frac{d_{\pi(k)}}{\min\{k, M\}} \leq 1 + \sum_{k=1}^{K-1} \left(\frac{1}{\min\{k, M\}} - \frac{1}{\min\{K, M\}} \right) \bar{\alpha}^{(\pi(k))}$$

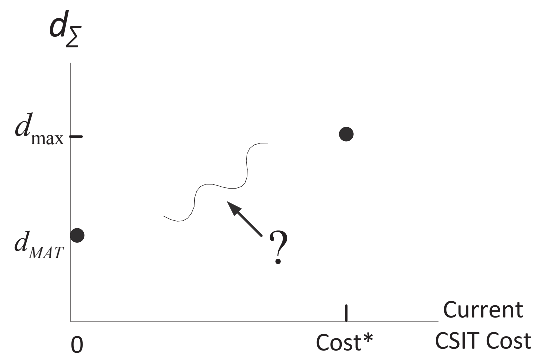
$$d_k \leq 1, \quad k = 1, 2, \cdots, K$$

New outer bound (Sum DoF)

Corollary: *[Sum DoF outer bound] For the K-user $M \times 1$ MISO BC, the sum DoF is outer bounded as*

$$d_\Sigma \leq d_{MAT} + \left(1 - \frac{d_{MAT}}{\min\{K, M\}}\right) \sum_{k=1}^{K} \bar{\alpha}^{(k)}$$

Current CSIT cost vs sum DoF



What is the current CSIT cost for a certain $d_\Sigma \in [d_{\text{MAT}}, d_{\max}]$?

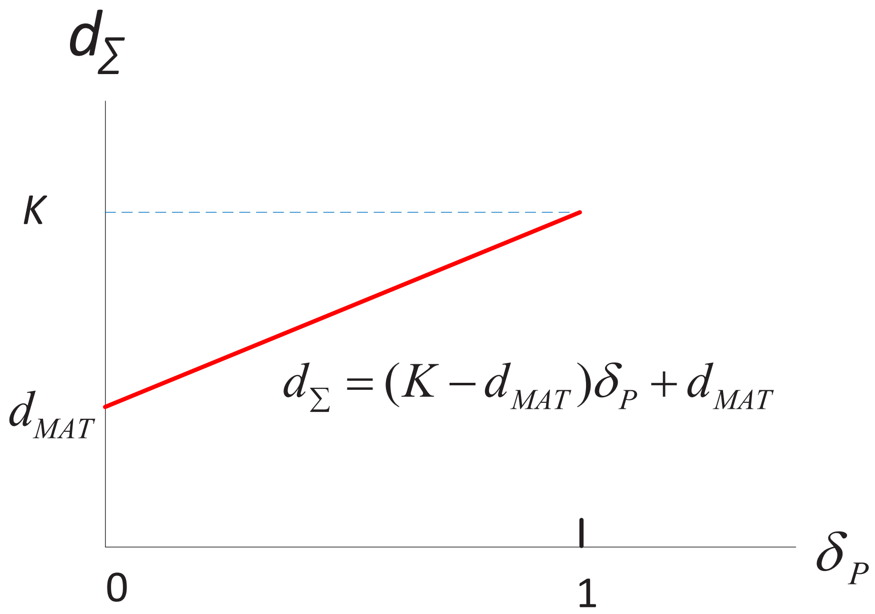
- E.g, for the case with $M = 2, K = 3$ ($d_{\text{MAT}} = \frac{3}{2}, d_{\max} = 2$)
  - ⋆ What is the current CSIT cost for $d_\Sigma = \frac{7}{4}$?
  - ⋆ What is the current CSIT cost for $d_\Sigma = \frac{5}{3}$?

Current CSIT cost vs sum DoF

Theorem: *[Optimal cases, d_Σ vs $\bar{\alpha}$] For the K-user MISO BC with $M \geq K$ or with $M = 2, K = 3$, and given a current CSIT cost $\bar{\alpha}$, the optimal sum DoF is characterized as*

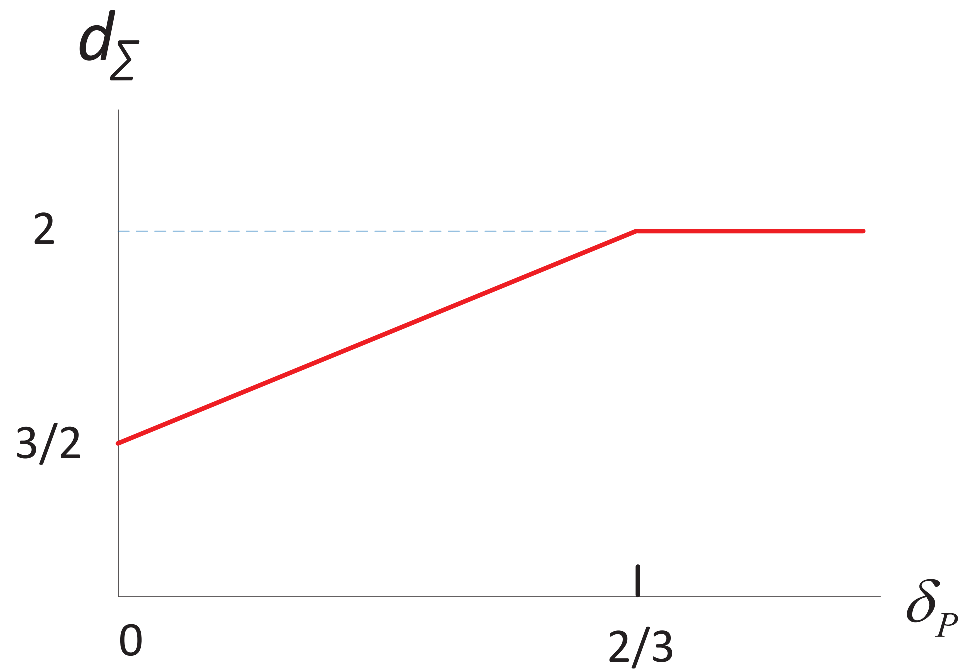
$$d_\Sigma = d_{MAT} + \left(K - \frac{K d_{MAT}}{\min\{K, M\}}\right) \min\left\{\bar{\alpha}, \frac{\min\{K, M\}}{K}\right\}$$

Current CSIT cost vs sum DoF$_1$



Optimal sum DoF d_Σ vs. $\bar{\alpha} =: \delta_p$ for the MISO BC with $M \geq K$

Current CSIT cost vs sum DoF$_2$



Optimal sum DoF (d_Σ) vs. $\bar{\alpha} =: \delta_p$ for the MISO BC with $M = 2, K = 3$

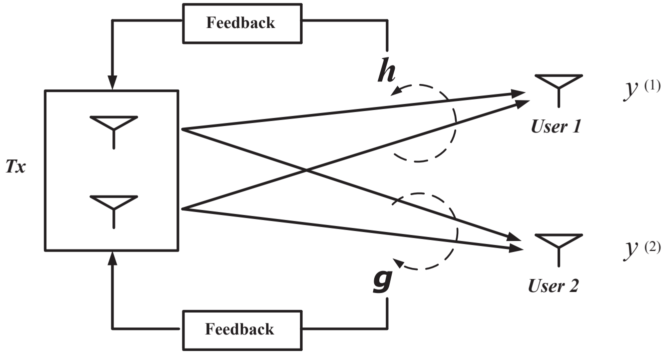
($\bar{\alpha} = 1/3$ for $d_\Sigma = \frac{7}{4}$) and ($\bar{\alpha} = 2/9$ for $d_\Sigma = \frac{5}{3}$)

Novel outer bounds

- Challenge: Outer bound for general K-user MISO BC (with general feedback process)
  - ⋆ Best known bound 1: for two-user [Yang et al., Gou and Jafar, Tandon et al., Chen and Elia, 12]
  - ⋆ Best known bound 2: for K-user, only for the maximum sum DoF point, i.i.d channel [Tandon et al. 12]
- Techniques
  - ⋆ Degraded BC construction
  - ⋆ Gaussian input maximizes the weighted difference of two (degraded) differential entropies [Weingarten et al. 09]
  - ⋆ MIMO techniques
  - ⋆ Statistical techniques

Global CSIR

Global channel state information at receivers (global CSIR)



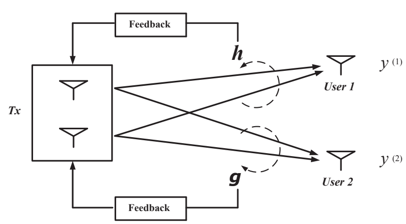
- Global CSIR: A user must know the channels of the other users

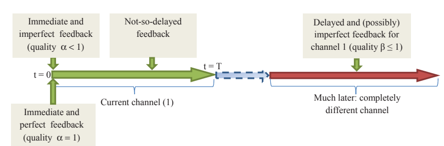
The challenge of global CSIR

Great challenge in distributing perfect global CSIR (see Kobayashi-Caire ISIT 2012)

- Training and limited-capacity/limited-reliability feedback links
- Challenge extreme as number of users increases
- Problem: Achilles' heel of delayed-CSIT approaches

Consider imperfect and delayed global CSIR





Imperfect and delayed Global CSIR[9]

- CSIT: No current, imperfect delayed ($\alpha = 0,\ 0 \leq \beta \leq 1$)
- Global CSIR: No current, imperfect delayed (β)
- No receiver access to CSIT estimates at transmitter

Theorem: DoF inner bounds

$$\{(0,0),(0,1),(\frac{1+\beta}{2},\frac{1+\beta}{2}),(1,0)\}, \quad \beta < \frac{1}{3}$$
$$\{(0,0),(0,1),(\frac{2}{3},\frac{2}{3}),(1,0)\}^{*}, \quad \beta \geq \frac{1}{3}$$

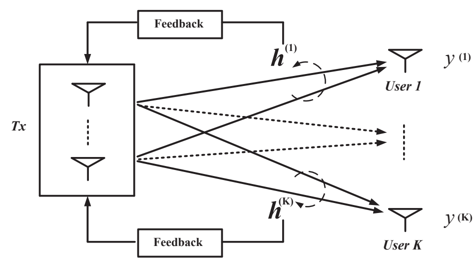
$*$ Optimal and previously associated to perfect delayed CSIT and perfect global CSIR

[9] Chen-Elia 2012

Open problems regarding Global CSIR

- How to use imperfect and delayed global CSIR when there are many users?
- How to use imperfect and delayed global CSIR when $\alpha > 0$?
- Tightening of existing bounds
- How to use imperfect and delayed global CSIR in interference settings?

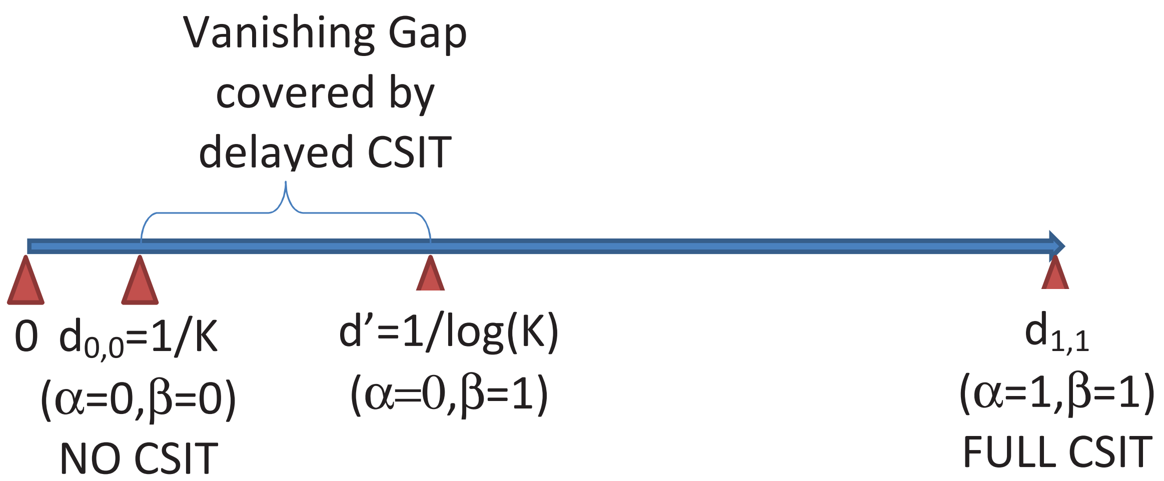
Open problems in the K-user setting



- TDMA (No CSIT) $d = 1/K$
- Perfect CSIT $d = 1$
- Only delayed CSIT [Maddah-Ali and Tse]

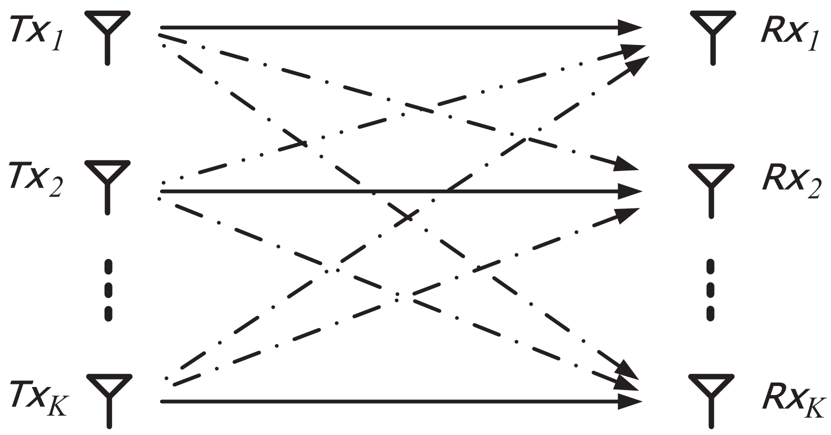
$$d \approx 1/\ln K, \quad \text{for large } K$$

Open problems in the K-user setting[1]



- Glass half full or half empty?
- How best to complement delayed feedback?
- Novel schemes (index coding)
- Novel information theoretic bounds:
  - ⋆ GRAND CHALLENGE: outer bounds and constructions

Open problem in K-user interference setting



K-pair interference channel

- Before: TDMA $\quad d = 1/K$
- Some extensions of seen work by Yang et al. (from BC to two-user IC)
    - ⋆ Most approaches are limited to two-user IC

Interference alignment with delayed CSIT

A PROMISING DIRECTION:
INTERFERENCE ALIGNMENT WITH DELAYED FEEDBACK

- Interference alignment (IA) [Cadambe and Jafar 08] $\quad d = 1/2$
  - ⋆ "Each user gets half of the cake"
  - ⋆ IA concept first introduced for the X channel by Maddah-Ali, Motahari and Khandani
  - ⋆ Powerful tool but!
  - ⋆ Global and perfect CSIT is required for IA

Interference alignment with delayed CSIT

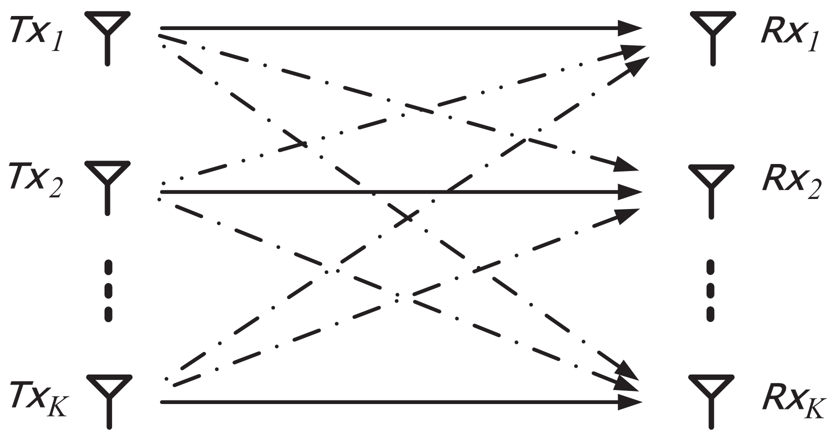
SOME EXISTING APPROACHES

- With delayed CSIT 3×3 SISO IC can achieve $\frac{9}{8}$ sum DoF
  - ⋆ [Maleki, Jafar and Shamai 11]
- With delayed CSIT 3×3 SISO IC can achieve $\frac{36}{31}$ sum DoF
  - ⋆ [Abdoli, Ghasemi and Khandani 11]

THE MAIN OPEN PROBLEM:

WHAT IS OPTIMAL DOF FOR $K \times K$ SISO IC WITH DELAYED CSIT?

Extension to X channel



X channel: Each transmitter has a message to be communicated with each receiver

- With perfect global CSIT, $M \times N$ SISO X channel has sum DoF $\frac{MN}{M+N-1}$
  - ⋆ [Cadambe and Jafar 2009]
  - ⋆ Example: 2×2 : sum Dof $= \frac{4}{3}$

Extension to X channel[1]

- With delayed CSIT 2×2 SISO X channel can achieve $\frac{8}{7}$ sum DoF
  - ⋆ [Maleki et al. 2011]
- With delayed CSIT 2×2 SISO X channel can achieve $\frac{6}{5}$ sum DoF
  - ⋆ [Ghasemi, Motahari and Khandani 11]
- With delayed CSIT 3×3 SISO X channel can achieve $\frac{5}{4}$ sum DoF
  - ⋆ [Ghasemi, Motahari and Khandani 11]

THE MAIN OPEN PROBLEM:

WHAT IS OPTIMAL DOF FOR $K \times K$ SISO X CHANNEL WITH DELAYED AND IMPERFECT CSIT?

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Thank you

Thank you