

# Is it Worth to be Patient? Analysis and Optimization of Delayed Mobile Data Offloading

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**Abstract**—Operators have recently resorted to WiFi offloading to deal with increasing data demand and induced congestion. Researchers have further suggested the use of “delayed offloading”: if no WiFi connection is available, (some) traffic can be delayed up to a given deadline, or until WiFi becomes available. Nevertheless, there is no clear consensus as to the benefits of delayed offloading, with a couple of recent experimental studies largely diverging in their conclusions. Nor is it clear how these benefits depend on network characteristics (e.g. WiFi availability), user traffic load, etc. In this paper, we propose a queueing analytic model for delayed offloading, and derive the mean delay, offloading efficiency, and other metrics of interest, as a function of the user’s “patience”, and key network parameters. We validate the accuracy of our results using a range of realistic scenarios, and use these expressions to show how to optimally choose deadlines.

**Keywords**—Mobile data offloading, Deadlines, Queueing, 2D Markov chain, Optimization.

## I. INTRODUCTION

Lately, an enormous growth in the mobile data traffic has been reported. This increase is due to a significant penetration of smartphones and tablets in the market, as well as Web 2.0 and streaming applications which have high-bandwidth requirements. Cisco [1] reports that by 2017 the mobile data traffic will increase by 13 times, and will climb to 13.2 exabytes per month. Mobile video traffic will comprise 66% of the total traffic, compared to 51% in 2012 [1].

This increase in traffic demand is overloading cellular networks, forcing them to operate close to (and often beyond) their capacity limits. Upgrading to LTE or LTE-advanced, as well as the deployment of additional network infrastructure could help alleviate this capacity crunch [2], but reports already suggest that such solutions are bound to face the same problems [3]. Furthermore, these solutions may not be cost-effective from the operators’ perspective: they imply an increased cost (for power, location rents, deployment and maintenance), without a similar increase in revenues [4].

A more cost-effective way to cope with the problem of highly congested mobile networks is by offloading some of the traffic through Femtocells (SIPTO, LIPA [5]), and the use of WiFi. In 2012, 33% of the total mobile data traffic was offloaded [1]. Projections say that this will increase to 46% by 2017 [1]. Out of these, data offloading through WiFi has become a popular solution. Some of the advantages often cited compared to Femtocells are: lower cost, higher data rates, lower ownership cost [2], etc. Also, wireless operators have

already deployed or bought a large number of WiFi access points (AP) [2].

There exist two types of WiFi offloading. The usual way of offloading is *on-the-spot offloading*: when there is WiFi available, all traffic is sent over the WiFi network; otherwise, *all* traffic is sent over the cellular interface. More recently, “delayed” offloading has been proposed: if there is currently no WiFi availability, (some) traffic can be delayed instead of being sent/received immediately over the cellular interface. In the simplest case, traffic is delayed until WiFi connectivity becomes available. This is already the case with current smartphones, where the user can select to send synchronization or backup traffic (e.g. Dropbox, Google+) only over WiFi. A more interesting case is when the user (or the device on her behalf) can choose a deadline (e.g. per application, per file, etc.). If up to that point no AP is detected, the data are transmitted through the cellular network [6], [7].

We have already analyzed the case of on-the-spot offloading in [8]. Delayed offloading offers additional flexibility and promises potential performance gains to both the operator and user. First, more traffic could be offloaded, further decongesting the cellular network. Second, if a user defers the transmission of less delay-sensitive traffic, this could lead to energy savings [9]. Finally, with more operators moving away from flat rate plans towards usage-based plans [10], users have incentives to delay “bulky” traffic to conserve their plan quotas or to receive better prices [11].

Nevertheless, there is no real consensus yet as to the added value of delayed offloading, if any. Recent experimental studies largely diverge in their conclusions about the gains of delayed offloading [6], [7]. Additionally, the exact amount of delay a flow can tolerate is expected to depend heavily on (a) the user, and (b) the application type. For example, a study performed in [12] suggests that “more than 50% of the interviewed users would wait up to 10 minutes to stream YouTube videos and 3-5 hours for file downloads”. More importantly, *the amount of patience will also depend on the potential gains for the user*. As a result, two interesting questions arise in the context of delayed offloading:

- *If deadlines are externally defined (e.g. by the user or application), what kind of performance gains for the user/operator should one expect from delayed offloading and what parameters do these depend on?*
- *If an algorithm can choose the deadline(s) to achieve different performance-cost trade offs, how should these deadlines be optimally chosen?*

The main contributions of this paper can be summarized as follows: (i) We propose a queueing analytic model for the problem of delayed offloading, based on two-dimensional Markov chains, and derive expressions for the average delay, and other performance metrics as a function of the deadlines, and key system parameters; we also give closed-form approximations for different regimes of interest; (Section II) (ii) We validate our results extensively, using also scenarios and parameters observed in real measurement traces that depart from the assumptions made in our model; (Section III) (iv) We formulate and solve basic cost-performance optimization problems, and derive the achievable tradeoff regions as a function of the network parameters (WiFi availability, user load, etc.) in hand (Section IV).

## II. ANALYSIS OF DELAYED OFFLOADING

In this section, we formulate the delayed offloading problem, and derive analytical expressions for key metrics (e.g. mean per flow delay). We consider a mobile user that enters and leaves zones with WiFi coverage, with a rate that depends on the user's mobility (e.g. pedestrian, vehicular) and the environment (e.g. rural, urban). Without loss of generality, we assume that there is always cellular network coverage. We also assume that the user generates flows over time (different sizes, different applications, etc.) that need to be transmitted (uploaded or downloaded) over the network<sup>1</sup>. Whenever there is coverage by some WiFi AP, all traffic will be transmitted through WiFi, assuming for simplicity a First Come First Served (FCFS) queueing discipline. When the WiFi connectivity is lost, we assume that flows waiting in the queue and new flows arriving can be delayed until there is WiFi coverage again. However, each flow has a maximum delay it can wait for (a *deadline*), which might differ between flows and users [12]. If the deadline expires before the flow can be transmitted over some WiFi AP, then it is sent over the cellular network<sup>2</sup>.

To facilitate the analysis of the above system, we make the following assumptions. We model the WiFi network availability as an ON-OFF alternating renewal process [13]  $(T_{ON}^{(i)}, T_{OFF}^{(i)})$ ,  $i \geq 1$ , as shown in Fig. 1. The duration of each ON period (WiFi connectivity),  $T_{ON}^{(i)}$ , is assumed to be an exponentially distributed random variable with rate  $\eta$ , and independent of the duration of other ON or OFF periods. During such ON periods data can be transmitted over the WiFi network with a rate equal to  $\mu$ . Similarly, all OFF periods (Cellular connectivity only) are assumed to be independent and exponentially distributed with rate  $\gamma$ , and a data rate that is lower than the WiFi rate<sup>3</sup>. We further assume that traffic arrives as a Poisson process with rate  $\lambda$ , and file sizes are exponentially distributed. Finally, to capture the fact that each file or flow may have a different deadline assigned to it, we assume that deadlines are also random variables that are exponentially distributed with rate  $\xi$ .

<sup>1</sup>We will use the terms “flow”, “file”, and “packet” interchangeably throughout the paper, as the most appropriate term often depends on the application and the level at which offloading is implemented.

<sup>2</sup>In practice the switch in connectivity might sometimes occur while some flow is running. Without loss of generality, we will assume that the transmission is resumed from the point it was interrupted when WiFi was lost. It might continue over the cellular network (vertical handover) or paused until WiFi becomes available again or the deadline expires.

<sup>3</sup>Although this might not *always* be the case, everyday experience as well as a number of measurements [6] suggest this to be the case, on average.

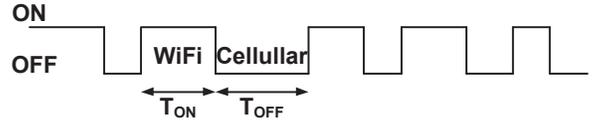


Fig. 1: The WiFi network availability model.

The above model is flexible enough to describe a large number of interesting settings: high vs. low WiFi availability (by manipulating  $\frac{\gamma}{\gamma+\eta}$ ), low vs. high speed users (low  $\gamma$ ,  $\eta$  vs. high  $\gamma$ ,  $\eta$ , respectively), low utilization vs. congested scenarios (via  $\lambda$  and  $\mu$ ), etc. However, the assumptions of exponentiality, while necessary to proceed with any meaningful analysis (as it will be soon made evident), might “hide” the effect of second order statistics (e.g. variability of ON/OFF periods, flow sizes, etc.). To address this, in Section III we relax most of these assumptions, and validate our results in scenarios with generic ON/OFF periods, generic flow size distributions, and non-exponential deadlines.

Our goal is to analyze this system to answer the following questions: (i) if the deadlines are given (e.g. defined “externally” by the user or application), what is the expected performance as a function of network parameters like WiFi availability statistics, and user traffic load? (ii) if the deadlines are “flexible”, i.e. the user would like to choose these deadlines in order to optimize his overall performance (e.g. trading off some delay, waiting for WiFi, to avoid the often higher energy and monetary cost of cellular transmission), how should they be chosen?

We will answer the first question in the remainder of this section, and use the derived expressions to provide some answers to the second question, in Section IV. Before proceeding, we summarize in Table I some useful notation. Also, the total time a file spends in the system (queueing+ service time) will be referred to as the *system time* or *transmission delay*.

TABLE I: Variables and Shorthand Notation.

Variable	Definition/Description
$T_{ON}$	Duration of ON (WiFi) periods
$T_{OFF}$	Duration of periods (OFF) without WiFi connectivity
$\lambda$	Average packet (file) arrival rate at the mobile user
$\pi_{i,c}$	Stationary probability of finding $i$ files in cellular state
$\pi_{i,w}$	Stationary probability of finding $i$ files in WiFi state
$\pi_c$	Probability of finding the system under cellular coverage only
$\pi_w$	Probability of finding the system under WiFi coverage
$p_r$	Probability of renegeing
$\eta$	The rate of leaving the WiFi state
$\gamma$	The rate of leaving the cellular state
$\mu$	The service rate while in WiFi state
$\xi$	The renegeing rate
$E[S]$	The average service time
$E[T]$	The average system (transmission) time
$T_d$	The deadline time
$\rho = \lambda E[S]$	Average user utilization ratio

### A. Performance of WiFi queue

All files arriving to the system are by default sent to the WiFi interface with a deadline assigned (drawn from an exponential distribution). Files are queued (in FCFS order) if there is another file already in service (i.e. being transmitted) or if there is no WiFi connectivity at the moment, until their deadline expires. If the deadline for a file expires (either while

queued or while at the head of the queue, but waiting for WiFi), the file *abandons* the WiFi queue and is transmitted through the cellular network. These kind of systems are known as queueing systems with *impatient* customers [14] or with *reneging* [15]. Throughout our analysis, we'll assume that files will abandon the queue only during periods without WiFi connectivity<sup>4</sup>. Nevertheless, in Section III we consider also the deterministic deadlines, i.e. a file will be sent over the cellular network. Our focus here will be on the WiFi queue for two reasons: First, this is the place where files accumulate most of the delay. Second, this is the point where a decision can be made, which will be relevant to the deadline optimization (Section IV). For the moment, we can assume that a file sent back to the cellular interface will incur a fixed delay (this might also include some mean queueing delay) that is larger, in general, than the service time over WiFi (i.e.  $\frac{\text{file-size}}{\mu}$ ).

Given the previously stated assumptions, the WiFi queue can be modeled with a 2D Markov chain, as shown in Figure 2. States with WiFi connectivity are denoted with  $\{i, w\}$ , and states with cellular connectivity only with  $\{i, c\}$ .  $i$  corresponds to the number of customers in the system (service+queue). During WiFi states, the system empties at rate  $\mu$  (since files are transmitted 1-by-1) and during cellular states the system empties at rate  $i \cdot \xi$  since any of the  $i$  queued packets can renege. The following theorem uses probability generating functions (PGF) to derive the mean system time for this queue. The use of PGFs in 2D Markov chains is known for quite a long time [16], [17], [18].

**Theorem 1.** *The mean system time for the WiFi queue when delayed mobile data offloading is performed is*

$$E[T] = \frac{1}{\lambda} \left[ \left(1 + \frac{\gamma}{\eta}\right) \frac{\lambda - \mu(\pi_w - \pi_{0,w})}{\xi} + \frac{(\lambda - \mu)\pi_w + \mu\pi_{0,w}}{\eta} \right]. \quad (1)$$

*Proof:* Let  $\pi_{i,c}$  and  $\pi_{i,w}$  denote the stationary probability of finding  $i$  files when there is only cellular network coverage, or WiFi coverage, respectively.

Writing the balance equations for the cellular and WiFi state gives

$$(\lambda + \gamma)\pi_{0,c} = \eta\pi_{0,w} + \xi\pi_{1,c} \quad (2)$$

$$(\lambda + \gamma + i\xi)\pi_{i,c} = \eta\pi_{i,w} + (i+1)\xi\pi_{i+1,c} + \lambda\pi_{i-1,c} \quad (3)$$

$$(\lambda + \eta)\pi_{0,w} = \gamma\pi_{0,c} + \mu\pi_{1,w} \quad (4)$$

$$(\lambda + \eta + \mu)\pi_{i,w} = \gamma\pi_{i,c} + \mu\pi_{i+1,w} + \lambda\pi_{i-1,w} \quad (5)$$

The long term probabilities of finding the system in cellular or WiFi state are  $\pi_c = \frac{\eta}{\eta+\gamma}$  and  $\pi_w = \frac{\gamma}{\eta+\gamma}$ , respectively.

We define the probability generating functions for both the cellular and WiFi states as  $G_c(z) = \sum_{i=0}^{\infty} \pi_{i,c} z^i$ , and  $G_w(z) = \sum_{i=0}^{\infty} \pi_{i,w} z^i$ ,  $|z| \leq 1$ . After multiplying Eq.(3) with  $z^i$  and adding to Eq.(2) we obtain

$$(\lambda + \gamma)G_c(z) + \xi(1 - \frac{1}{z}) \sum_{i=1}^{\infty} i\pi_{i,c} z^i = \eta G_w(z) + \lambda z G_c(z). \quad (6)$$

The summation in the above equation gives  $\sum_{i=1}^{\infty} i\pi_{i,c} z^i = zG'_c(z)$ . Hence, after some rearrangements in Eq.(6) we obtain

$$\xi(1-z)G'_c(z) = (\lambda(1-z) + \gamma)G_c(z) - \eta G_w(z). \quad (7)$$

Repeating the same procedure for Eq.(4)-(5) we get

$$(\lambda + \eta)G_w(z) = \gamma G_c(z) + \lambda z G_w(z) + \mu(\frac{1}{z} - 1)(G_w(z) - \pi_{0,w}),$$

which after some rearrangements yields to

$$((\lambda z - \mu)(1-z) + \eta z)G_w(z) = \gamma z G_c(z) - \mu(1-z)\pi_{0,w}.$$

Next, we make two replacements  $\alpha(z) = \lambda(1-z) + \gamma$ , and  $\beta(z) = (\lambda z - \mu)(1-z) + \eta z$ . Now, we have the system of equations

$$G_w(z) = \frac{\gamma z G_c(z) - \mu(1-z)\pi_{0,w}}{\beta(z)}, \quad (8)$$

$$G'_c(z) - \frac{\alpha(z)\beta(z) - \eta\gamma z}{\xi(1-z)\beta(z)}G_c(z) = \frac{\eta\mu\pi_{0,w}}{\xi\beta(z)}. \quad (9)$$

The roots of  $\beta(z)$  are

$$z_{1,2} = \frac{\lambda + \mu + \eta \mp \sqrt{(\lambda + \mu + \eta)^2 - 4\lambda\mu}}{2\lambda}. \quad (10)$$

It can easily be shown that these roots satisfy the relation  $0 < z_1 < 1 < z_2$ . We introduce the function  $f(z) = -\frac{\alpha(z)\beta(z) - \eta\gamma z}{\xi(1-z)\beta(z)}$ , as the multiplying factor of  $G_c(z)$  in the differential equation of Eq.(9). Performing some simple calculus operations, the above function transforms into

$$f(z) = -\frac{\lambda}{\xi} + \frac{\gamma}{\xi(1-z)} \left( \frac{\eta z}{\beta(z)} - 1 \right). \quad (11)$$

After some algebra and applying the *partial fraction expansion* the function  $f(z)$  becomes

$$f(z) = -\frac{\lambda}{\xi} + \frac{\gamma}{\xi} \left( \frac{M}{z - z_1} + \frac{N}{z_2 - z} \right). \quad (12)$$

We determine the coefficients  $M$  and  $N$  in the standard way as  $M = \frac{\lambda - z}{z_2 - z_1} \Big|_{z=z_1} = \frac{\lambda - z_1}{z_2 - z_1} = \frac{z_1 z_2 - z_1}{z_2 - z_1} > 0$ , and  $N = \frac{\lambda - z}{z - z_1} \Big|_{z=z_2} = \frac{\lambda - z_2}{z_2 - z_1} < 0$ .

In order to solve the differential equation in system Eq.(9) we can multiply it by  $e^{\int f(z) dz}$ . Hence, we get

$$G'_c(z) e^{\int f(z) dz} + f(z) G_c(z) e^{\int f(z) dz} = \frac{\eta\mu\pi_{0,w}}{\xi\beta(z)} e^{\int f(z) dz}. \quad (13)$$

We thus need to integrate the function in Eq.(13):

$$\int f(z) dz = -\frac{\lambda}{\xi} z + \frac{\gamma M}{\xi} \ln|z - z_1| - \frac{\gamma N}{\xi} \ln(z_2 - z). \quad (14)$$

The constant normally needed on the right-hand side of Eq.(14) can be ignored in our case. We next raise Eq.(14) to the power of  $e$  to get

$$e^{\int f(z) dz} = e^{-\frac{\lambda}{\xi} z} |z - z_1|^{\frac{\gamma M}{\xi}} (z_2 - z)^{-\frac{\gamma N}{\xi}}. \quad (15)$$

Now, Eq.(13) is equivalent to

$$\frac{d}{dz} \left( e^{-\frac{\lambda}{\xi} z} |z - z_1|^{\frac{\gamma M}{\xi}} (z_2 - z)^{-\frac{\gamma N}{\xi}} G_c(z) \right) = \frac{\eta\mu\pi_{0,w}}{\xi\beta(z)} e^{\int f(z) dz} \quad (16)$$

We define  $k_1(z)$  and  $k_2(z)$  as

$$k_1(z) = e^{-\frac{\lambda}{\xi} z} (z_1 - z)^{\frac{\gamma M}{\xi}} (z_2 - z)^{-\frac{\gamma N}{\xi}}, \quad z \leq z_1, \quad (17)$$

$$k_2(z) = e^{-\frac{\lambda}{\xi} z} (z - z_1)^{\frac{\gamma M}{\xi}} (z_2 - z)^{-\frac{\gamma N}{\xi}}, \quad z \geq z_1. \quad (18)$$

Eq.(16) now becomes

<sup>4</sup>In this manner, abandonments are plausibly associated with the accumulated "opportunity cost", i.e. the time spent waiting for WiFi connectivity (the "non-standard" option for transmission). Instead, if WiFi is available, but there are some files in front, it might make no sense to abandon, as queueing delays might also occur in the cellular interface.

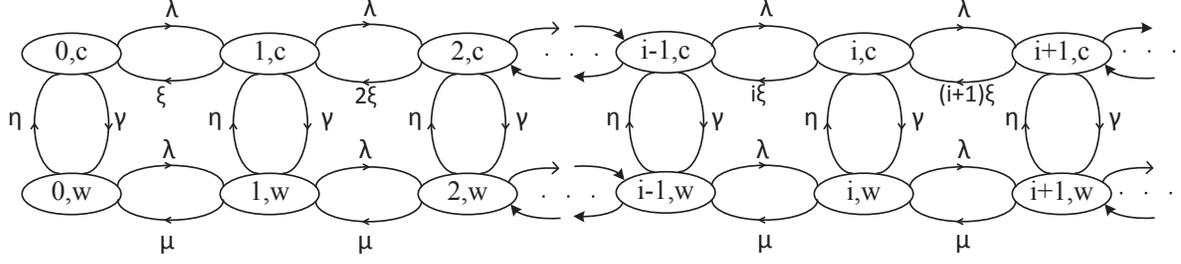


Fig. 2: The 2D Markov chain for the WiFi queue in delayed offloading.

$$\frac{d}{dz} (k_1(z)G_c(z)) = \frac{\eta\mu\pi_{0,w}}{\xi\beta(z)}k_1(z), z \leq z_1, \quad (19)$$

$$\frac{d}{dz} (k_2(z)G_c(z)) = \frac{\eta\mu\pi_{0,w}}{\xi\beta(z)}k_2(z), z \geq z_1, \quad (20)$$

and after integrating we obtain

$$k_1(z)G_c(z) = \frac{\eta\mu\pi_{0,w}}{\xi} \int_0^z \frac{k_1(x)}{\beta(x)} dx + C_1, z \leq z_1 \quad (21)$$

$$k_2(z)G_c(z) = \frac{\eta\mu\pi_{0,w}}{\xi} \int_{z_1}^z \frac{k_2(x)}{\beta(x)} dx + C_2, z \geq z_1. \quad (22)$$

The bounds of the integrals in Eq.(21) and Eq.(22) come from the defining region of  $z$  in Eq.(19)-(20). We need to determine the coefficients  $C_1$  and  $C_2$  in Eq.(21) and Eq.(22). We take  $z = 0$  in Eq.(21). We have  $k_1(0) = z_1^{\frac{\gamma M}{\xi} - \frac{\gamma N}{\xi} - z_2}$ , and knowing that  $G_c(0) = \pi_{0,c}$ , we get for  $C_1 = \pi_{0,c} z_1^{\frac{\gamma M}{\xi} - \frac{\gamma N}{\xi} - z_2}$ . In a similar fashion we get for  $C_2 = 0$ .

Finally, for the PGF in the cellular state we have

$$G_c(z) = \frac{\eta\mu\pi_{0,w} \int_0^z \frac{k_1(x)}{\beta(x)} dx + \xi\pi_{0,c} z_1^{\frac{\gamma M}{\xi} - \frac{\gamma N}{\xi} - z_2}}{\xi k_1(z)}, z \leq z_1, \quad (23)$$

$$G_c(z) = \frac{\eta\mu\pi_{0,w} \int_{z_1}^z \frac{k_2(x)}{\beta(x)} dx}{\xi k_2(z)}, z \geq z_1. \quad (24)$$

In the last equation, the 'zero probabilities'  $\pi_{0,c}$  and  $\pi_{0,w}$  are unknown. We can find them in the following way: We know

that  $\pi_c = \frac{\eta}{\eta + \gamma} = G_c(1) = \frac{\eta\mu\pi_{0,w} \int_{z_1}^1 \frac{k_2(x)}{\beta(x)} dx}{\xi k_2(1)}$ . From this we have

$$\frac{\xi k_2(1)}{\eta + \gamma} = \mu\pi_{0,w} \int_{z_1}^1 \frac{k_2(x)}{\beta(x)} dx. \quad (25)$$

Similarly, from the boundary conditions in Eq.(23) for  $z \leq z_1$ , we get

$$\eta\mu\pi_{0,w} \int_0^{z_1} \frac{k_1(x)}{\beta(x)} dx + \xi\pi_{0,c} z_1^{\frac{\gamma M}{\xi} - \frac{\gamma N}{\xi} - z_2} = 0. \quad (26)$$

After solving the system of equations Eq.(25) and Eq.(26), for the 'zero probabilities' we obtain

$$\pi_{0,w} = \frac{\xi k_2(1)}{(\eta + \gamma)\mu \int_{z_1}^1 \frac{k_2(x)}{\beta(x)} dx}, \text{ and} \quad (27)$$

$$\pi_{0,c} = - \frac{\eta k_2(1) \int_0^{z_1} \frac{k_1(x)}{\beta(x)} dx}{(\eta + \gamma) z_1^{\frac{\gamma M}{\xi} - \frac{\gamma N}{\xi} - z_2} \int_{z_1}^1 \frac{k_2(x)}{\beta(x)} dx}. \quad (28)$$

The value of the integral  $\int \frac{k_1(x)}{\beta(x)} dx$  is always negative, hence  $\pi_{0,c}$  is always positive.

By using a vertical cut between any two-pairs of neighboring states in Fig. 2 and writing balance equations we have

$$\lambda\pi_{i,c} + \lambda\pi_{i,w} = \mu\pi_{i+1,w} + (i+1)\xi\pi_{i+1,c}. \quad (29)$$

Summing over all  $i$  yields to

$$\lambda(\pi_c + \pi_w) = \mu(\pi_w - \pi_{0,w}) + \xi \sum_{i=0}^{\infty} (i+1)\pi_{i+1,c}. \quad (30)$$

The last equation, obviously reduces to

$$\lambda = \mu(\pi_w - \pi_{0,w}) + \xi E[N_c], \quad (31)$$

where  $E[N_c] = G'_c(1)$ , and  $E[N_w] = G'_w(1)$ . Eq.(31) yields

$$E[N_c] = \frac{\lambda - \mu(\pi_w - \pi_{0,w})}{\xi}. \quad (32)$$

So far, we have derived  $E[N_c]$  as the first derivative at  $z = 1$  of  $G_c(z)$ . In order to find the average number of files in the system, we need  $E[N_w]$  as well. We can get it by differentiating Eq.(8)

$$G'_w(z) = \frac{\beta(z) (\gamma G_c(z) + \gamma z G'_c(z) + \mu\pi_{0,w})}{\beta^2(z)} - \frac{\beta'(z) (\gamma z G_c(z) - \mu(1-z)\pi_{0,w})}{\beta^2(z)}, \quad (33)$$

and setting  $z = 1$ . After some calculus we obtain

$$E[N_w] = \frac{(\gamma E[N_c] + \mu\pi_{0,c})\eta - \gamma\pi_c(\mu - \lambda)}{\eta^2}. \quad (34)$$

Replacing Eq.(32) into Eq.(34) we get

$$E[N_w] = \frac{\gamma\lambda - \mu(\pi_w - \pi_{0,w})}{\eta\xi} + \frac{\mu\pi_{0,w}}{\eta} - \frac{\gamma\pi_c(\mu - \lambda)}{\eta^2}. \quad (35)$$

The average number of files in the system is

$$E[N] = E[N_c] + E[N_w]. \quad (36)$$

Finally, using the Little's law  $E[N] = \lambda E[T]$  [13], we obtain the average packet delay in delayed data offloading as in Eq.(1). ■

The above result gives the total expected delay that incoming flows experience in the WiFi queue. For flows that do get transmitted over WiFi (i.e. whose deadline does not expire) this amounts to their total delay. Flows that end up renegeing (deadline expires before transmission) must be transmitted through the cellular system and thus incur an additional delay  $\Delta$  (related to their transmission time over the cellular link, i.e.  $\frac{\text{packet\_size}}{\text{cellular\_rate}}$ , and possibly some queueing delay as well). The following Corollary gives the probability of renegeing for each.

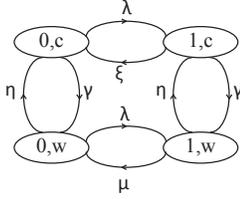


Fig. 3: The reduced Markov chain for  $\rho \rightarrow 0$ .

**Corollary 2.** *The probability that an arbitrary flow arriving to the WiFi queue will renege, i.e. its deadline will expire before it can be transmitted over a WiFi AP is*

$$p_r = \frac{\lambda - \mu(\pi_w - \pi_{0,w})}{\lambda}. \quad (37)$$

In other words, the rate of flows sent back to the cellular network is given by  $\lambda \cdot p_r$ . This must be equal to  $\xi \cdot E[N_c]$ , which is the average abandonment rate in Fig. 2, i.e.  $\lambda p_r = \xi E[N_c]$ . Replacing  $E[N_c]$  from Eq.(32) gives us the above result. This also gives us another important metric, the *offloading efficiency* of our system, namely the percentage of flows that get offloaded over some WiFi network, as  $E_{off} = 1 - p_r$ .

The above expressions can be used to predict the performance of a delayed offloading system, as a function of most parameters of interest, such as WiFi availability and performance, user traffic load, etc. As we shall see later, it does so with remarkable accuracy even in scenarios where many of the assumptions don't hold. However, Eq.(1) cannot easily be used to solve optimizations problems related to the deadline ( $\xi$ ), analytically, as the parameters  $\pi_{0,c}$  and  $\pi_{0,w}$  involve  $\xi$  in a non-trivial way. To this end, we propose next some closed-form approximations for the low and high utilization regimes. Due to space limitations, we present these without proof, and refer the interested reader to [19].

### B. Low utilization approximation

One interesting scenario is when resources are underloaded (e.g. nighttime, rural areas, or mostly low traffic users, etc) and/or traffic is relatively sparse (some examples are, background traffic from social and mailing applications, messaging, Machine-to-Machine communication, etc.). For very low utilization, the total system time essentially consists of the service time, as there is almost no queueing, so we can use a fraction of the Markov chain from Fig. 2 with only 4 states, as shown in Fig. 3 to derive  $E[T]$  and  $p_r$ .

**Low utilization approximation:** *The expected system time in the WiFi queue and the probability of renegeing for sparse input traffic can be approximated by*

$$E[T] = \frac{(\eta + \gamma)^2 + \gamma\xi + \eta\mu}{(\xi\mu + \xi\eta + \mu\gamma)(\gamma + \eta)}, \quad (38)$$

$$p_r = \frac{\theta_1 \xi}{\theta_2 \xi + \theta_3}, \quad (39)$$

where  $\theta_1 = \frac{\eta(\lambda + \eta + \gamma + \mu)}{\eta + \gamma}$ ,  $\theta_2 = \mu + \lambda + \eta$ , and  $\theta_3 = \mu\gamma + \lambda^2 + \lambda(\eta + \gamma + \mu)$ .

### C. High utilization approximation

Another interesting regime is that of high utilization. As explained earlier, wireless resources are often heavily loaded, especially in urban centers, due to the increasing use of smart

phones, tablets, and media-rich applications. Hence, it is of special interest to understand the average user performance in such scenarios. Here, we provide an approximation that corresponds to the region of high utilization ( $\rho \rightarrow 1$ ).

**High utilization approximation:** *The expected system time in the WiFi queue and the probability of renegeing for a user with heavy traffic can be approximated by*

$$E[T] = \frac{1}{\lambda} \left[ \left( 1 + \frac{\gamma}{\eta} \right) \frac{\lambda - \mu\pi_w}{\eta} + \frac{(\lambda - \mu)\pi_w}{\eta} \right], \quad (40)$$

$$p_r = \frac{\lambda - \mu\pi_w}{\lambda} + \frac{\mu}{\lambda} \pi_{0,w}, \quad (41)$$

where  $\pi_{0,w}$  is the first order Taylor series approximation of Eq.(27). The exact expression for  $\pi_{0,w}$  as well as the derivation for high utilization approximation can be found in [19].

## III. PERFORMANCE EVALUATION

In this section we will validate our theory against simulations for a wide range of traffic intensities, different values of file sizes, WiFi availability periods with different distributions, and different deadline times. We define the WiFi availability ratio as  $AR = \frac{E[T_{ON}]}{E[T_{ON}] + E[T_{OFF}]} = \frac{\gamma}{\eta + \gamma}$ . Unless otherwise stated the durations of WiFi availability and unavailability periods will be drawn from independent exponential distributions with rates  $\eta$  and  $\gamma$ , respectively. The deadlines are exponentially distributed with rate  $\xi$ , although we will simulate scenarios with deterministic deadlines as well. We mainly focus on two scenarios, related to the user's mobility. The first one considers mostly pedestrian users with data taken from [6]. Measurements in [6] report that the average duration of WiFi availability period is 122 min, while the average duration with only cellular network coverage is 41 min (we use these values to tune  $\eta$  and  $\gamma$ ). The availability ratio is thus 75 %. The second scenario corresponds to vehicular users, related to the measurement study of [7]. An availability ratio of 11 % has been reported in [7], although not all the details are mentioned there. For more details about the measurements we refer the interested reader to [6] and [7]. Finally, unless otherwise stated, file/flow sizes are exponentially distributed, and file arrival at the mobile user is a Poisson process with rate  $\lambda$ . Due to space limitations, we only depict here a representative sample of results. Additional results can be found in [19].

### A. Validation of main delay result

We first validate here our model and main delay result (Eq.(1)) against simulations for the two mobility scenarios mentioned (pedestrian and vehicular). The data rate for WiFi is assumed to be 1 Mbps. The mean packet size is assumed to be 7.5 MB for the pedestrian scenario and 125 kB for the vehicular scenario

TABLE II: Probability of renegeing for pedestrian and vehicular scenarios.

Scenario	Deadline	$\lambda = 0.1$	$\lambda = 0.5$	$\lambda = 1$	$\lambda = 1.5$
Pedestrian(Theory)	1 hour	0.103	0.109	0.252	0.501
Pedestrian(Simulation)	1 hour	0.1	0.117	0.239	0.508
Vehicular(Theory)	60 s	0.32	0.778	0.889	0.926
Vehicular(Simulation)	60 s	0.32	0.776	0.891	0.925

Fig. 4 shows the average file transmission delay (i.e. queueing + transmission) for the pedestrian scenario, for two different average deadline times of  $T_{d1} = 1$  hour ( $\xi_1 = 1/3600s^{-1}$ ) and  $T_{d2} = 2$  hours ( $\xi_2 = 1/7200s^{-1}$ ), respectively. The range

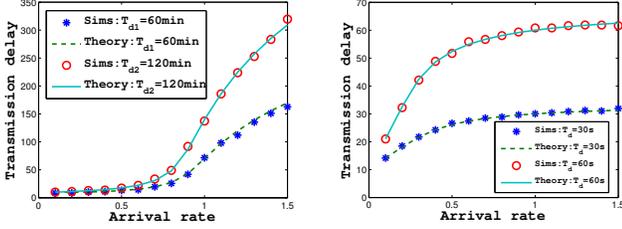


Fig. 4: The average delay for pedestrian users' scenarios.

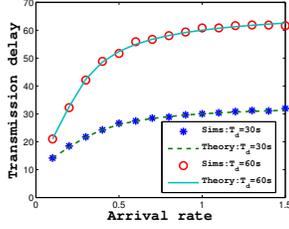


Fig. 5: The average delay for vehicular users' scenarios.

of arrival rates shown corresponds to a server utilization of 0-0.9. We can observe from Fig. 4 that there is a good match between theory and simulations. Furthermore, the average file transmission delay is increased by increasing the arrival rate, as expected, due to queueing effects. On the other hand, the average delay increases for higher deadlines, since flows with lower deadlines leave the WiFi queue earlier, leading to smaller queueing delays. Fig. 5 further illustrates the average file transmission delay for the vehicular scenario with average deadline times  $T_{d1} = 30s$  ( $\xi_1 = 1/30s^{-1}$ ) and  $T_{d2} = 60s$  ( $\xi_2 = 1/60s^{-1}$ ). Despite the differences of the vehicular scenario, similar conclusions can be drawn. Finally, Table II depicts the respective probabilities of reneging for the two scenarios. The percentage of flows that abandon the WiFi queue is higher in the vehicular scenario, since the availability ratio of the WiFi network is very small (11%), and deadlines are rather small. These observations agree with [7]. Nevertheless, our theory matches simulation values in all scenarios.

So far, we have assumed exponential distributions for ON and OFF periods, according to our model. While the actual distributions are subject to the user mobility pattern, a topic of intense research recently, initial measurement studies ([6], [7]) suggest these distributions to be "heavy-tailed". To this end, we consider a scenario with "heavy-tailed" ON/OFF distributions (Bounded Pareto). Due to space limitations, we focus on the vehicular scenario only. The shape parameters for the Bounded Pareto ON and OFF periods are  $\alpha = 0.59$  and  $\alpha = 0.64$ , respectively. The average deadline is 100s. Fig. 6 compares the average file delay against our theoretical prediction. Interestingly, our theory still offers a reasonable prediction accuracy, despite the considerably higher variability of ON/OFF periods in this scenario. While we cannot claim this to be a generic conclusion for any distribution and values, the results underline the utility of our model in practice.

### B. Validation of approximations

We next validate the approximations we have proposed in Section II. We start with the low utilization approximation of Section II-B and consider the availability ratio to be 0.75 (similar accuracy levels have been obtained with other values) and with a deadline of 2min. Fig. 7 shows the packet delay for low arrival rates in the range 0.01 – 0.11, which correspond to a maximum utilization of up to 0.2. As  $\lambda$  increases, the difference between the approximated result and the actual value increases, since we have considered only the service time for this approximation.

Next, we consider the high utilization regime and respective approximation (Eq.(40)). We consider utilization values around

0.8. Fig. 8 shows the delay for high values of  $\lambda$ , and an availability ratio of 0.5. We can see there that our approximation is very close to the actual delay and should become exact as  $\rho$  gets larger.

### C. Variable WiFi rates and non-exponential parameters

While in our model we consider a fixed transmission rate for all WiFi hotspots, this is not realistic in practice. For this reason, we have also simulated scenarios where the WiFi rate varies uniformly in the range 0.4-1.6 Mbps. Fig. 9 shows the delay for for the vehicular scenario with a deadline of 10 minutes. Even in this case, our theory can give solid predictions for the incurred delay.

In all of the above scenarios, we have assumed variable deadlines for each file (drawn from an exponential distribution). In some cases, the user might choose the same deadline for many (or most) flows that can be delayed, which would be a measure of her patience. To this end, we simulate a scenario where the deadline is fixed for an arrival rate of 0.1. The other parameters are the same as for the vehicular scenario. In Fig. 10 we compare simulation results for this scenario against our theory (which assumes exponentially distributed deadlines with the same average). It is evident that even in this case, there is a reasonable match with our theory.

To conclude our validation, we finally drop the exponential packet assumption as well, and test our theoretical result vs. generic file size results. Fig. 11 does it for Bounded Pareto distributed files sizes (shape parameter  $\alpha = 1.2$  and  $c_v = 3$ ). In both scenarios, the deadline is  $T_d = 20s$ . Mean file size is in both cases 125KB, the availability ratio is 0.5, and the rest of the parameters correspond to the vehicular scenario. Our theoretical prediction remains reasonably close, despite higher size variability.

### D. Delayed offloading gains

In this last part, we will investigate the actual gains from data offloading, in terms of offloading efficiency. Higher offloading efficiency means better performance for both client and operator. We compare the offloading efficiencies for on-the-spot offloading [8] vs. delayed offloading for different deadline times ( $T_{d1} = 2min, T_{d2} = 1min$ ). Fig.12 illustrates the offloading efficiency vs. availability ratio for a moderate arrival rate of  $\lambda = 0.2$ . For comparison purposes we also depict the line  $x = y$  (offloading efficiency = availability ratio). First, as expected, we can observe that offloading efficiency increases with AR. However, this increase is not linear. More interestingly, the actual offloading efficiencies are always higher than the respective availability ratio. As expected, the delayed offloading provides higher offloading efficiencies compared to on-the-spot offloading, with higher deadlines leading to higher offloading efficiencies.

## IV. OPTIMIZING DELAYED OFFLOADING

The results considered so far allow us to predict the expected system delay when the deadlines are defined externally (e.g. by the user or the application). However, the user (or the device on her behalf) could choose the deadline in order to solve an optimization problem among additional (often conflicting) goals, such as the monetary *cost* for accessing

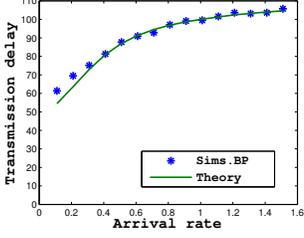


Fig. 6: The delay for BP ON-OFF periods vs. theory.

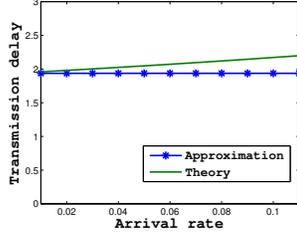


Fig. 7: Low utilization delay approximation for  $AR = 0.75$ .

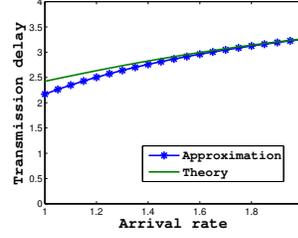


Fig. 8: High utilization delay approximation for  $AR = 0.5$ .

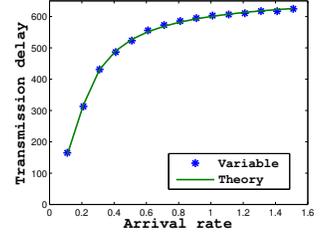


Fig. 9: Variable WiFi rates with the same average as theory.

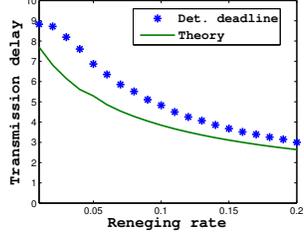


Fig. 10: The delay for deterministic deadlines vs. theory.

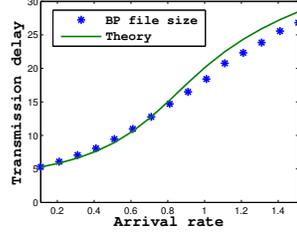


Fig. 11: The delay for BP packet sizes vs. theory.

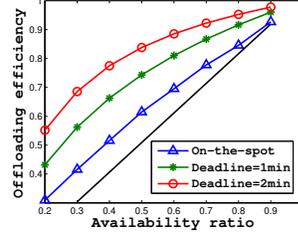


Fig. 12: Offloading gains for delayed vs. on-the-spot offloading.

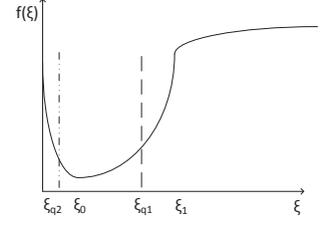


Fig. 13: The delay function for the optimization problem.

the Internet and the *energy consumption* of the device. For example, the user might want to minimize the delay subject to a maximum (energy or monetary) cost, or to minimize the cost subject to a maximum delay the user can tolerate.

To formulate and solve such optimization problems, we need analytical formulas for the average delay and the incurred cost. We already have such formulas for the delay of files sent over WiFi, where we will use the two approximations of Section II-B and II-C. Furthermore, we can assume that files transmitted over the cellular network incur a fixed delay  $\Delta$ , capturing both the service and queueing delays over the cellular interface<sup>5</sup>. To proceed, we need to also assume simple models for energy and cost, in order to get some initial intuition about the tradeoffs involved. We are aware that reality is more complex (for both energy and cost) and may differ based on technology (3G, LTE), provider, etc. We plan to extend our models in future work.

Assume a user has to download or upload a total amount of data equal to  $L$ . On average  $p_r \cdot L$  data units will be transmitted through the cellular interface. Assume further that  $D_c$  and  $D_w$  denote the costs per transmitted data unit for a cellular and WiFi network, where  $D_w < D_c$  (often  $D_w = 0$ ). Finally, let  $c_c$  and  $c_w$  denote the transmission rates, and  $E_c$  and  $E_w$  energy spent per time unit during transmission over the cellular and WiFi network, respectively. It is normally the case that  $c_c < c_w$  as well as  $E_c \approx E_w$  [20]. It follows then that the total monetary and energy costs,  $D$ , and  $E$ , could be approximated by

$$D = (D_c - D_w)p_r + D_w \quad \text{and} \quad E = \left( \frac{E_c}{c_c} - \frac{E_w}{c_w} \right) p_r + \frac{E_w}{c_w}. \quad (42)$$

#### A. Optimization problems

Eq.(42) suggests that both the average power consumption and cost depend linearly on the probability of reneging,  $p_r$ ,

<sup>5</sup>We could also try to model the cellular queue as an M/M/1 or G/M/1 system, but we are more interested in the dynamics of the WiFi queue, since this is where the reneging decisions take place. To keep things simple, we defer this to future work.

which we have also derived in Section II, and which is a function of the system deadline  $\frac{1}{\xi}$ . The system delay is also a function of  $\xi$ . We can thus formulate optimization problems of the following form, for both the high and low utilization regimes, where  $\xi$  is the optimization parameter

$$\begin{aligned} \min_{\xi} \quad & E[T] + p_r \Delta \\ \text{s. t.} \quad & p_r \leq P_r^{max}, \end{aligned} \quad (43)$$

where  $E[T]$  is given by Eq.(38), and  $p_r$  by Eq.(39), for low utilization, and Eq.(40) and Eq.(41), for high utilization, respectively. Due to the linearity of Eq.(42), we can express the constraint directly for  $p_r$ , where  $P_r^{max}$  depends on whether we consider monetary cost, energy or a weighted sum of both, and the respective parameters. Finally, we can also exchange the optimization function with the constraint, to minimize the cost, subject to a maximum delay. This provides us with a large range of interesting optimization problems we can solve.

Due to space limitations, we will show here the methodology to solve the above problem in the low utilization regime. The solutions to other problems can be found in [19].

If we express the inequality constraint in Eq.(43) through  $\xi$ , we have the equivalent constraint  $\xi \leq \frac{\theta_3 P_r^{max}}{\theta_1 - \theta_2 P_r^{max}}$ . The probability of reneging from Eq.(39) is an increasing function of  $\xi$ , since  $p_r'(\xi) > 0$ . This implies that maximum  $p_r$  corresponds to maximum  $\xi$ . We denote by  $f(\xi)$  the total average delay of Eq.(43) (delay function from now on). Hence we have

$$f(\xi) = \frac{A_1 \xi + A_2}{B_1 \xi + B_2} + \frac{\theta_1 \xi}{\theta_2 \xi + \theta_3} \Delta, \quad (44)$$

where  $A_1 = \gamma, A_2 = (\eta + \gamma)^2 + \eta\mu, B_1 = (\mu + \eta)(\gamma + \eta), B_2 = \mu\gamma(\gamma + \eta)$ . In order to solve the optimization problem given by Eq.(43), we need to know the behavior of the delay function. For that purpose, we analyze the monotonicity and convexity of Eq.(44). To do that we need the first and second derivatives, which are

$$f'(\xi) = \frac{A_1 B_2 - A_2 B_1}{(B_1 \xi + B_2)^2} + \frac{\theta_1 \theta_3 \Delta}{(\theta_2 \xi + \theta_3)^2}, \quad \text{and}$$

$$f''(\xi) = \frac{2(A_2B_1 - A_1B_2)}{(B_1\xi + B_2)^3} - \frac{\theta_1\theta_2\theta_3\Delta}{(\theta_2\xi + \theta_3)^3}.$$

It is worth noting that  $A_1B_2 < A_2B_1$ . This prevents delay function being always concave. The delay function is decreasing in the interval for which  $f'(\xi) \leq 0$ . This happens when

$$\xi \leq \xi_0 = \frac{\theta_3 \sqrt[3]{\frac{A_2B_1 - A_1B_2}{\theta_1\theta_2\theta_3\Delta}} - B_2}{B_1 - \theta_2 \sqrt[3]{\frac{A_2B_1 - A_1B_2}{\theta_1\theta_2\theta_3\Delta}}}.$$

Hence, the delay function is decreasing in the interval  $(0, \xi_0)$ , and increasing in the rest, with  $\xi_0$  being a minimum. Further, the solution of  $f''(\xi) > 0$  gives the interval where the function is convex. This happens when

$$\xi \leq \xi_1 = \frac{\theta_3 \sqrt[3]{2 \frac{A_2B_1 - A_1B_2}{\theta_1\theta_2\theta_3\Delta}} - B_2}{B_1 - \theta_2 \sqrt[3]{2 \frac{A_2B_1 - A_1B_2}{\theta_1\theta_2\theta_3\Delta}}}. \quad (45)$$

It can be easily proven that  $\xi_0 < \xi_1$ .

Such constrained-optimization problems are often solved with the Lagrangian method and KKT conditions. However, the optimal solution for our problem can be found more easily. The delay function looks like in Fig. 13. The optimal deadline depends on the maximum cost, that is proportional to the probability of reneging. So, we can determine the optimal deadline based on the value of  $P_r^{max}$ . If this value of  $P_r^{max}$  is quite high, the corresponding reneging rate  $\xi_{q1}$  (dashed line in Fig. 13) will be higher than the global minimum  $\xi_0$ . Consequently, the global minimum of Eq.(45) is also the optimal reneging rate. On the other hand, if the maximum cost is quite low (low  $P_r^{max}$ ), the maximum reneging rate  $\xi_{q,2}$  (dotted line in Fig. 13) is lower than the global minimum. This implies that the minimum delay will be achieved for the maximum reneging rate of  $\xi_{q,2} = \frac{\theta_3 P_r^{max}}{\theta_1 - \theta_2 P_r^{max}}$ . In other words, the average deadline time that minimizes the delay for a given maximum cost is

$$T_{d,opt} = \frac{1}{\xi_{opt}} = \frac{1}{\min\left(\xi_0, \frac{\theta_3 P_r^{max}}{\theta_1 - \theta_2 P_r^{max}}\right)}. \quad (46)$$

Similar steps can be followed to solve the same optimization problem for high utilization, as well as other problems. These sometimes involve a quadratic constraint function [19].

### B. Optimization evaluation

We will now validate the solutions of the previous optimization problem for two different cases. In both of them the arrival rate is 0.1, and the maximum cost per data unit one can afford is 2.8 monetary units. The transmission of a data unit through WiFi costs 1, and through cellular 5 units. The choice of these values is simply for better visualizing the results; different values yield similar conclusions. Fig. 14 shows the delay vs. cost curve for cellular rate being  $2\times$  lower than WiFi rate. First thing that we can observe is that the minimum delay is achieved for the highest possible cost (2.8). The optimal average deadline is  $T_d = 1$ s. This is in agreement with the optimal value predicted from Eq.(46), and shown with an asterisk in Fig. 14. We replace Eq.(39) into Eq.(42) to get the relationship between the cost and the renege rate. We have shown in Eq.(42) that the cost is directly proportional to  $p_r$ , and the later one is an increasing function of  $\xi$ . This implies that the maximum cost is in fact the maximum  $\xi$  (minimum deadline). This practically means that in Eq.(43),

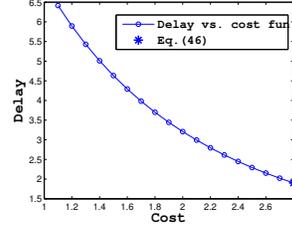


Fig. 14: The delay vs. cost curve for high cellular rate.

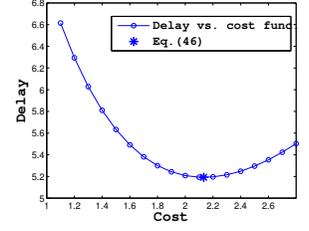


Fig. 15: The delay vs. cost curve for low cellular rate.

$\Delta$  is small and that the delay in the WiFi queue represents the largest component of the delay. As a consequence of that, it is better to redirect the files through the cellular interface as soon as possible. Hence, in these cases (when cellular rate is comparable to the WiFi), the optimum is to assign the shortest possible deadline constrained by the monetary cost.

Fig. 15 corresponds to a scenario with the same parameters as in Fig. 14, except that now the cellular rate is much lower ( $10\times$ ). In that case,  $\Delta$  is high, and  $p_r\Delta$  is the largest component of the delay function. As can be seen from Fig. 15, it is not the best option to leave as soon as possible from the WiFi queue, i.e. choose the smallest possible deadline. The optimum delay is achieved for  $T_d = 5$ s. This corresponds to an average cost of  $D = 2.1$  which is also very close to the theoretical solution of the problem. This is reasonable since for a large difference between the WiFi and cellular rates it is better to wait and then (possibly) be served with higher rate, than to move to a much slower interface (cellular).

Further, we use the solutions of the four optimization problems (the one solved here and the other three in [19]) for exponentially distributed deadline times to see how accurately our theory can predict the optimal deadline times, but for deterministic deadlines. The optimal policy essentially finds the optimal value for the *average* deadline (assuming these exponential). In practice, the chosen deadline will be assigned to all files, and will be deterministic. We consider four scenarios, one for each optimization problem. The costs are the same as before. The arrival rate for low utilization scenarios is 0.1, while for the high ones, 1.5. In Table III, we show the optimal deadlines by using our model (e.g. Eq.(46)), and the optimal deterministic deadlines by using simulations (delay vs. cost plots) with the same parameters as in theory. As can be seen from Table III, the error in determining the optimal deadline decreases for higher arrival rates. The error is in the range 10%-20%. This is reasonable, since the simulated scenarios are with deterministic deadlines and in our theory we use exponential deadlines. Another reason is that in optimization problems, we are only using the low and high utilization approximations and not the exact result (Eq.(1)).

TABLE III: Optimal deterministic deadline times vs theory.

Sc.	Constraint	$T_d$ (theory)	$T_d$ (deterministic)	Relative error
1	$D \leq 2.8$	2.71	2.2	18%
2	$T_{max} = 6.6$	1.56	1.22	22%
3	$D \leq 3.8$	1.73	1.55	11%
4	$T_{max} = 15$	7.97	6.82	15%

## V. RELATED WORK

Some recent influential work in offloading relates to measurements of WiFi availability [6], [7]. Authors in [6] have

tracked the behavior of 100 users (most of which were pedestrians) and their measurements reveal that during 75% of the time there is WiFi connectivity. In [7], measurements were conducted on users riding metropolitan area buses. In contrast to the previous study, the WiFi availability reported there is only around 10%. The mean duration of WiFi availability and non-availability periods is also different in the two studies, due to the difference in speeds between vehicular and pedestrian users. The most important difference between the two studies relates to the reported offloading efficiency, with [7] reporting values in the range from 20%-33% for different deadlines, and [6] reporting that offloading does not exceed 3%. We believe this is due to the different deadlines assumed together with the different availabilities.

The authors in [21] define a utility function related to delayed offloading to quantitatively describe the trade-offs between the user satisfaction in terms of the price that she has to pay and the experienced delay by waiting for WiFi connectivity. However, their analysis does not consider queueing effects. Such queueing effects may affect the performance significantly, especially in loaded systems (which are of most interest) or with long periods without WiFi. The work in [22] considers the traffic flow characteristics when deciding when to offload some data to the WiFi. However, there is no delay-related performance analysis. Modeling the cost factors is the focus of [23], which also shows where the offloading APs should be installed. A WiFi offloading system that takes into account a user's throughput-delay tradeoffs and cellular budget constraints is proposed in [24]. However, only heuristic algorithms are proposed, and queueing effects are ignored. Finally, in [25], an integrated architecture has been proposed based on opportunistic networking to switch the data traffic from the cellular to WiFi networks. Summarizing, in contrast to our work, these papers either perform no analysis or use simple models that ignore key system effects such as queueing.

To our best knowledge, the closest work in spirit to ours is [26]. The results in [26] are the extension of the results in [6] containing the analysis for delayed offloading. Authors there also use 2D Markov chains to model the state of the system. However, they use matrix-analytic methods to obtain a numerical solution for the offloading efficiency. Such numerical solutions unfortunately do not provide insights on the dependencies between different key parameters, and cannot be used to formulate and analytically solve optimization problems that include multiple metrics.

As a final note, in [8], we have proposed a queueing analytic model for on-the-spot mobile data offloading, and a closed form solution was derived for the average delay. While the model we propose here shares some similarities (ON/OFF availabilities, 2D Markov chain approach) with the basic model in [8], it is in fact considerably more difficult to solve.

## VI. CONCLUSION

In this paper, we have proposed a queueing analytic model for the performance of delayed mobile data offloading, and have validated it against realistic WiFi network availability statistics. We have also considered a number of scenarios where one or more of our model's assumptions do not hold, and have observed acceptable accuracy, in terms of predicting the system delay as a function of the user's patience. Finally,

we have also shown how to manipulate the maximum deadlines, in order to solve various optimization problems involving the system delay, monetary costs, and energy costs. In future work, we intend to consider more complex models for both the WiFi and cellular queues, as well as per-flow scheduling and dispatch policies.

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