

# Rate optimal power policies in underlay cognitive radios with limited channel feedback

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**Abstract**—In this paper, two new power policies (PPs), namely the Rate Maximization Policy (RMP) and the Rate-Bound Maximization Policy (RBMP), for Cognitive Radio (CR) systems are developed based on an expected rate maximization criterion. Both policies are designed such as to satisfy a constraint related to the average interference caused by the CR network to primary communication. The key novelty here is that the proposed PP takes into account the existence of limited (hybrid) channel feedback, where the direct secondary channel is known instantaneously while the interference caused by the primary transmission to the secondary receiver is only known statistically. The optimal policy (RMP) is characterized, and a low complexity algorithm is presented that allows for its efficient and accurate implementation. The two policies are compared and RMP is shown to lead to substantial energy consumption savings at equal rate performance.

**Index Terms**—Underlay Cognitive Radio, power policy, ergodic rate maximization, average interference constraint.

## I. INTRODUCTION

Underlay CR-Spectrum Sharing networks [1] are considered a promising solution to the spectrum scarcity problem. Such networks are mostly limited by Quality of Service (QoS) requirements for primary communication, which are usually translated to constraints on the average and/or peak interference (PI) caused by the transmission of the secondary transmitter (STx) to the primary receiver (PRx) [2]. The design of rate optimal PP for underlay CR schemes subject to such primary communication QoS constraints has been studied by several researchers. In [3], considering an average STx-PRx interference constraint, the expected CR rate maximizing policy is derived, while in [4], this policy is compared with CR PPs that are based on PI constraints. Recently, in [5], rate maximizing PPs under peak or average transmit power constraints and imperfect channel-state information (CSI) at STx were studied. For the derivation of these PPs several interference related constraints have been considered. The techniques presented in [3]-[5] are applicable to the so-called “Z” channel model, that neglects the interference caused by the primary transmitter (PTx) to the secondary receiver (SRx). Such a channel model arises in cases that either the PTx causes no interference to the SRx or when the PTx-SRx interference is strong enough so that the SRx can decode and cancel this interference fully [6].

In this paper, we consider a more general scenario where interference caused by the primary transmitter cannot be

neglected due to typical propagation effects or cancelled due to complexity limitation at the receiver side. In this case, the policies presented in [3]-[5] are inappropriate. For such scenarios, the rate optimal policy subject to average STx-PRx interference and average PRx transmit power constraints has been examined in [7], [8], but with the assumption that exact instantaneous CSI knowledge for the PTx-SRx link is available at the secondary transmitter which would necessitate an accurate SRx-STx feedback link and may not always be feasible.

Instead, we focus on CR networks limited by both noise, PTx-SRx interference, and its feedback capabilities. Unlike [7] and [8], a limited feedback scenario is studied, where SRx provides as feedback to STx only the statistics of the PTx-SRx link, and PPs are derived aiming to maximize cognitive communication expected rate. Specifically, the key contributions of the paper can be summarized as follows: *a)* The optimal (with respect to an expected rate maximization criterion) PP, namely RMP, for CR networks operating based on an average STx-PRx interference constraint is derived. Unlike most previously derived policies, this novel PP takes into account both noise and the statistics of PTx-SRx interference. *b)* A low complexity algorithm is suggested that provides highly accurate, sharp approximations to the optimal PP. *c)* A suboptimum PP, namely RBMP, is derived by properly tuning the PP presented in [3] such as to also account for the PTx-SRx interference. *d)* The two policies are compared in terms of achievable rates and energy efficiency for a Rayleigh fading channel scenario and it is shown that both policies achieve similar performance in terms of expected rate. Nevertheless, the optimal PP is proven to be superior in terms of energy efficiency.

The paper is structured as follows. In Section II, the system model is presented. In Section III, the optimal power policy determination problem for this system is posed and solved. Section IV presents a low complexity approximation to the optimal PP, i.e. RMP, that is based on deriving a simple method for approximating the inverse of a function. Section V presents the derivation of the suboptimal PP, i.e., RBMP. Section VI illustrates the application of both RMP and RBMP in a Rayleigh fading scenario. In Section VII simulations results obtained by applying these new PPs are presented. Finally, Section VIII concludes the paper.

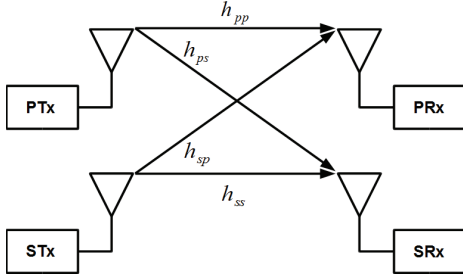


Fig. 1. The presumed Cognitive Radio system model

*Notation:* We use  $\exp(\cdot)$  and  $E_1(\cdot)$  to denote the exponential and exponential integral [9, eq. (5.1.1)] respectively.  $\ln$  and  $\log_2$  represent natural and base-2 logarithms, and  $(\cdot)^+$  stands for  $\max\{\cdot, 0\}$ . The probability density function (PDF) of a random variable (RV)  $u$  is denoted by  $f_u(u)$ , while  $E[\cdot]$  is the expectation operator. The complex circularly symmetric Gaussian distribution with mean  $\mu$  and variance  $\sigma^2$  is denoted as  $\mathcal{CN}(\mu, \sigma^2)$  and  $\sim$  denotes equality in terms of distribution.

## II. SYSTEM MODEL

The system model shown in Fig. 1 is considered where communication between STx and SRx takes place in the presence of a primary users pair (PTx and PRx) that communicate with each other. Unlike the ‘‘Z’’ channel model, we assume that secondary communication is subject to interference caused by the primary users communication. Hence, the signals  $y_p, y_s$  reaching PRx and SRx respectively, are written as

$$\begin{aligned} y_p &= h_{pp}\sqrt{P}x_p + h_{sp}\sqrt{P}x_s + n_p \\ y_s &= h_{ss}\sqrt{P}x_s + h_{ps}\sqrt{P}x_p + n_s \end{aligned} \quad (1)$$

where  $x_p$  and  $x_s$  are the signals transmitted by PTx and STx respectively,  $h_{pp}, h_{sp}$  are the PTx-PRx and STx-PRx fading channels,  $h_{ss}, h_{ps}$  are the STx-SRx and PTx-SRx channels and  $n_p, n_s \sim \mathcal{CN}(0, N_0)$  denote the complex additive white Gaussian Noise (AWGN) at PRx and SRx respectively with noise density  $N_0$  assumed to be common for both receivers. Moreover,  $P$  denotes the fixed transmit power of PTx, known to STx, while  $P$  denotes the instantaneous power used by STx for its transmissions. We assume that  $P$  is not fixed and depends on the specific channel realizations. In what follows, we make no assumption for the statistics of fading channels  $h_{pp}, h_{sp}, h_{ss}$  other than the fact that they are independent. Nevertheless we will assume that  $h_{ps}$  follows a Rayleigh fading channel model, i.e.  $h_{ps} \sim \mathcal{CN}(0, \sigma_{ps}^2)$ , and is independent of the rest of the fading channels of the system. The average power gain  $\sigma_{ps}^2$  of this channel is also assumed to be known at STx.

Regarding the transmitted signals, it is assumed that Gaussian codebooks are used, i.e.,  $x_p, x_s \sim \mathcal{CN}(0, 1)$ . By treating interference as noise, the instantaneous rate on the STx-SRx link, measured in bits/sec/Hz is then expressed as

$$C_{inst,s}(P) = \log_2 \left( 1 + \frac{gP(g, \eta, w)}{N_0 + w} \right), \quad (2)$$

where for the ease of presentation we have introduced the RVs  $g = |h_{ss}|^2$ ,  $w = |h_{ps}|^2 P$ ,  $\eta = |h_{sp}|^2$ . In (2),  $P(g, \eta, w)$  stands for the applied PP. Based on (2), and assuming exact instantaneous CSI regarding  $g, \eta$ , and  $w$ , STx can design the rate optimal PP subject to average and/or peak STx-PRx interference constraints as a function of this CSI as in [7], [8]. The demand for exact knowledge of  $g$  and  $\eta$  at STx can be satisfied by means of feedback mechanisms or channel reciprocity as described in [7]. Nevertheless, the need for exact knowledge of  $w$  introduces further complexity, inter-network coordination and feedback requirements. Hence, in our analysis we assume that STx has only statistical knowledge regarding the PTx-SRx link (i.e. knowledge of  $\sigma_{ps}^2$ ) as well as exact and statistical knowledge of  $g$  and  $\eta$ . With this assumption, STx cannot calculate the instantaneous rate in (2) nor adjust its PP to instantaneous values of  $w$ . Nevertheless, by taking into account the fact that  $h_{ps}$  follows a Rayleigh fading model, and dropping the dependence of the PP on  $w$ , STx can calculate the expected, with respect to the interference, rate for the STx-SRx link as

$$C_s(P, g, \eta) = \frac{1}{\bar{w}} \int_0^\infty \log_2 \left( 1 + \frac{gP(g, \eta)}{N_0 + w} \right) \exp\left(-\frac{w}{\bar{w}}\right) dw. \quad (3)$$

Using [10, eq. (4.337.1)] and [11, eq. 3.1.3], (3) is written as

$$C_s(P, g, \eta) = \frac{\ln \left( 1 + \frac{gP(g, \eta)}{N_0} \right) + U(gP(g, \eta)) - U(0)}{\ln 2} \quad (4)$$

$$\text{where } U(x) = \exp\left(\frac{N_0 + x}{\bar{w}}\right) E_1\left(\frac{N_0 + x}{\bar{w}}\right), x \geq 0. \quad (5)$$

We are interested on the design of the optimal STx PP  $P^*(g, \eta)$  based on the criterion of the maximization of the expected rate  $E[C_s(P, g, \eta)]$ , subject to (s.t.) a constraint on the average STx-PRx interference. This problem is mathematically formulated and solved in the following section.

## III. DERIVATION OF THE OPTIMAL POWER POLICY

Using (4), the problem of finding the optimal PP that maximizes  $E[C_s(P, g, \eta)]$  s.t. a constraint on the average STx-PRx interference can be mathematically formulated as<sup>1</sup>

$$\text{maximize: } \int_0^\infty \int_0^\infty C_s(P, g, \eta) f_g(g) f_\eta(\eta) dg d\eta \quad (6a)$$

$$\text{s.t.: } E[\eta P] \leq Q, \text{ and } P \geq 0. \quad (6b)$$

where  $C_s(P, g, \eta)$  is concave. Optimization problem (6) is then equivalent to the following optimization problem

$$\begin{aligned} &\text{minimize:} \\ & - \int_0^\infty \int_0^\infty \underbrace{\left( U(gP) + \ln \left( 1 + \frac{gP}{N_0} \right) \right)}_{W(P)} f_g(g) f_\eta(\eta) dg d\eta \\ & \text{s.t. } E[\eta P] \leq Q, \text{ and } P \geq 0. \end{aligned} \quad (7)$$

<sup>1</sup>For the ease of presentation, in the remaining equations the dependence of  $P$  on  $g$  and  $\eta$  is not written explicitly. The readers should keep in mind though that  $P$  varies as a function of  $g$  and  $\eta$ .

The partial Lagrangian for this problem is expressed as

$$\begin{aligned} \mathcal{L}(P, \lambda) = & - \int_0^\infty \int_0^\infty W(P) dg d\eta \\ & + \lambda \int_0^\infty \int_0^\infty \eta P f_g(g) f_\eta(\eta) dg d\eta. \end{aligned} \quad (8)$$

Applying dual decomposition as in [12], the initial problem then reduces to solving the following series of sub-problems, one for each channel state  $(g, \eta)$

$$\text{minimize: } - \left( U(gP) + \ln \left( 1 + \frac{gP}{N_0} \right) \right) + \lambda \eta P \text{ s.t. } P \geq 0. \quad (9)$$

The above problem can be solved by introducing a Lagrange Multiplier  $\mu(g, \eta)$  for the non negative transmit power constraint and applying KKT conditions. The resulting power policy is then found to be given as

$$P^* = \begin{cases} 0, & \text{if } u > \frac{U(0)}{\lambda \bar{w}} \\ \frac{1}{g} V(\lambda^* u \bar{w}), & \text{otherwise} \end{cases} \quad (10)$$

where  $u = \eta/g$ ,  $V(y)$  is defined as the inverse of  $U(x)$ , i.e.,  $V(y = U(x)) = x$  and  $\lambda^*$  is the optimal value of the multiplier  $\lambda$ , i.e., it is chosen such that the constraint (6b) is active, i.e.,  $E[\eta P] = Q$ , or equivalently, by using (10), and the definition of RV  $u$ , such that

$$\underbrace{\int_0^{\frac{U(0)}{\lambda^* \bar{w}}} u V(\lambda^* \bar{w} u) f_u(u) du}_{\mathcal{I}_{opt}(\lambda^*)} = Q. \quad (11)$$

By inspecting (11) and exploiting the fact that  $U(\cdot)$  is a strictly decreasing function, it is easy to see that  $\mathcal{I}_{opt}(\lambda)$  is strictly decreasing with respect to  $\lambda$ . Therefore, the unique  $\lambda^*$  that satisfies (11) can be found by using iterative root finding methods along with a numerical method, e.g. the trapezoidal rule, for the calculation of the integral  $\mathcal{I}_{opt}(\cdot)$ . Moreover, since  $\lambda^*$  depends only on channel statistics, its calculation can be performed offline for different values of the channel parameters so as to create a lookup table that will be used in practice. Due to the fact that this policy achieves the maximum rate for a given constraint  $Q$ , for the rest of the paper we will refer to this policy as the **Rate Maximization Policy (RMP)**.

Going back to (10), the evaluation of  $V(y)$  can be done by using any iterative root finding algorithm to solve the equation

$$y = U(x) \quad (12)$$

with respect to  $x$ . Nevertheless, since the argument of  $V(\cdot)$  depends on instantaneous CSI, the use of such an iterative root finding algorithm would lead to substantially increased computational complexity. To cope with this complexity, in the following section a sharp approximation to  $V(y)$  is derived which is then used for providing a low complexity, accurate approximation to RMP.

#### IV. ACCURATE LOW COMPLEXITY APPROXIMATION FOR THE OPTIMAL POWER POLICY

In order to derive an approximation to  $P^*$ , one needs to approximate  $V(y)$ , the inverse of  $U(x)$ , or equivalently, derive a close estimate to the solution of (12) for any given  $y$ . The starting point for deriving such an approximate solution to (12) is the remark that  $U(x)$  is strictly monotonic and more specifically, strictly decreasing. In what follows, we present a novel framework for approximating the inverse of such functions. This framework is then applied to our problem.

##### A. A novel method for the approximate inversion of composite functions

Given lower and upper bounds  $U_{low}(x)$  and  $U_{up}(x)$  for any strictly decreasing  $U(x)$ , it is easy to prove that if  $x_{low}$  and  $x_{up}$  are such that

$$U_{low}(x_{low}) = y, \text{ and } U_{up}(x_{up}) = y \quad (13)$$

then  $x_{low} \leq x_{up}$ , and the root  $x$  of (12) is bounded in the interval  $[x_{low}, x_{up}]$ . Thus, provided that interval  $[x_{low}, x_{up}]$  is tight enough, one can approximate the inverse of  $U(\cdot)$ , i.e. the solution of (12), by substituting  $U(x)$  in (12) by its first order Taylor series approximation around the midpoint  $x_0 = (x_{low} + x_{up})/2$ <sup>2</sup> and solving the resulting linear equation. In the sequel, this generic approach is further illustrated for the  $U(x)$  under investigation.

##### B. Inversion of $U(x)$ in (5)

Focusing on the specific  $U(x)$  defined in (5), one can use [9, eq. (5.1.20)], to construct the following upper and lower bounds for  $U(x)$

$$\underbrace{\frac{1}{2} \ln \left( 1 + \frac{2\bar{w}}{N_0 + x} \right)}_{U_{low}(x)} < U(x) < \underbrace{\ln \left( 1 + \frac{\bar{w}}{N_0 + x} \right)}_{U_{up}(x)}. \quad (14)$$

Based on (14), the solution  $x$  of (12) is bounded in the interval  $[x_{low}, x_{up}]$ , where

$$x_{low} = \frac{2\bar{w}}{\exp(2y) - 1} - N_0, \text{ and } x_{up} = \frac{\bar{w}}{\exp(y) - 1} - N_0. \quad (15)$$

Therefore, by defining  $x_0 = ((x_{up} + x_{low})/2)^+$  and following the previous described method, the inverse of  $U(x)$  is approximated by solving the equation

$$U(x_0) + \left. \frac{dU(x)}{dx} \right|_{x_0} (x - x_0) = y \quad (16)$$

with respect to  $x$ , to obtain

$$x = \tilde{V}(y) = x_0 + \left. \frac{y - U(x_0)}{\frac{dU(x)}{dx}} \right|_{x_0} \approx V(y), \quad (17)$$

<sup>2</sup>In case that  $U(x)$  is defined solely for  $x \geq 0$ , instead of the midpoint, the point  $x_0 = \left( \frac{x_{low} + x_{up}}{2} \right)^+$  can be used.

where the derivative of  $U(\cdot)$  in (17) can be calculated as [9, eq. (5.1.27)]

$$\frac{dU(x)}{dx} = \frac{U(x)}{\bar{w}} - \frac{1}{N_0 + x}. \quad (18)$$

Using (17) the RMP can then be approximated by substituting  $V(\cdot)$  by  $\tilde{V}(\cdot)$  in (10). Instead of (17), one could also consider the inversion method developed in [13] for the evaluation of the spectral-efficiency energy-efficiency trade-off for communication over Rayleigh fading channels. Note however that the method in [13] is derived based on a different, interesting, but computationally expensive approach that involves complicated functions such as the Lambert W function.

## V. DERIVATION OF A SUBOPTIMUM POWER POLICY

A suboptimum PP can be derived by substituting  $C_s(P, g, \eta)$  in (6) by a different function that is easier to manipulate. Such a function can be found by noticing that (2) is convex with respect to  $w$ . Thus, by applying Jensen's inequality, [14],  $C_s(P, g, \eta)$  in (3) can be lower bounded by

$$C_{low,s}(P) = \log_2 \left( 1 + \frac{gP(g, \eta, w)}{N_0 + \bar{w}} \right) \leq C_s(P, g, \eta). \quad (19)$$

Therefore, by substituting  $C_s(P, g, \eta)$  by  $C_{low,s}(P)$  in (6), and following the analysis presented in [3], the following suboptimum policy can be derived:

$$P^0(g, \eta) = \left( \frac{1}{\lambda_0 \eta} - \frac{N_0 + \bar{w}}{g} \right)^+ \quad (20)$$

that is a simple extension of the method presented in [3]. Parameter  $\lambda_0$  in (20) can be found by solving the average interference constraint equation, i.e.

$$\int_0^\infty \left( \frac{1}{\lambda_0} - (N_0 + \bar{w})u \right)^+ f_u(u) du = Q. \quad (21)$$

In what follows, we will refer to this policy as the **Rate Bound Maximization Policy (RBMP)**. Unlike RMP, the derived suboptimal policy does not require the calculation of any complicated function such as  $V(\cdot)$ . Nevertheless, as it will become evident later on, the derived, simple estimate  $\tilde{V}(\cdot)$  is very tight, thus allowing for the calculation of a sharp, low complexity approximation to  $V(\cdot)$ . This fact effectively reduces the benefits offered by RBMP. In the following section, both the optimum and suboptimum PPs are further exemplified for the case of Rayleigh fading channels. Specific focus is given on the calculation of parameters  $\lambda^*$  and  $\lambda_0$ .

## VI. THE RAYLEIGH FADING CASE

Assuming Rayleigh fading for the STx-SRx and STx-PRx links, i.e., assuming that  $h_{sp} \sim \mathcal{CN}(0, \sigma_{sp}^2)$  and  $h_{ss} \sim \mathcal{CN}(0, \sigma_{ss}^2)$ , it is easy to show that RVs  $\eta$  and  $g$  are exponentially distributed. Thus, assuming independent fading, the PDF of  $u$  is expressed as [15, eq. (7.44)]

$$f_u(u) = \frac{\rho}{(1 + \rho u)^2} \quad (22)$$

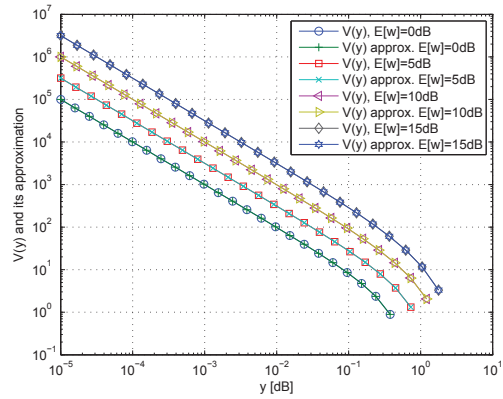


Fig. 2. Comparison of the exact  $V(y)$  and its approximation  $\tilde{V}(y)$ .

where  $\rho = \sigma_{ss}^2 / \sigma_{sp}^2$ . By employing (22), the calculation of the parameter  $\lambda^*$  can be performed using the methodology described in Section III. Moreover, by substituting (22) in (21) it is straightforward to show that  $\lambda_0$  is the solution of

$$\frac{1}{\lambda_0} - \frac{N_0 + \bar{w}}{\rho} \ln \left( 1 + \frac{\rho}{\lambda_0(N_0 + \bar{w})} \right) = Q. \quad (23)$$

## VII. NUMERICAL RESULTS AND DISCUSSION

In this section, numerical results related to the proposed PPs and their application to the Rayleigh fading channel model described in the previous section are shown. As a starting point, in Fig. 2 the approximate inverse  $\tilde{V}(\cdot)$  of  $U(\cdot)$  is plotted along with  $V(\cdot)$  for different values of  $\bar{w} = E[w]$  for a fixed value  $N_0 = 0dB$ . The calculation of the exact  $V(y)$  has been done by numerically solving (12) with a relative error tolerance of  $10^{-5}$ . The curves of  $\tilde{V}(y)$  and  $V(y)$  almost coincide, thus establishing that  $\tilde{V}(y)$  is a sharp approximation to  $V(y)$ . Similar curves have been obtained for smaller values of  $\bar{w}$ , illustrating that  $\tilde{V}(y)$  is a valid approximation for  $V(y)$  for most values of practical interest.

To deal with the fact that both PPs can result in arbitrarily large peak powers, for Figs. 3 and 4, we have modified the PPs by adding a maximum peak power constraint  $P_{max}$  in the initial problem to ensure that both PPs are characterized by finite means. This results in truncating the PP values in (10), (20) to  $P_{max}$  when this is exceeded. Then, the values of  $\lambda^*, \lambda_0$  that satisfy  $E[\eta P] = Q$ , need also to be recalculated while equations (11), (21) are not applicable. Provided that  $P_{max}$  is high enough, such a value of  $\lambda$  (or  $\lambda_0$ ) can be found. Alternatively  $\lambda$  (or  $\lambda_0$ ) should be set to zero. Due to space limitations, no further details are given for this straightforward procedure. Nevertheless, we should mention that this modification enables a comparison of the two PPs also in terms of power consumption. In Fig. 3, Monte Carlo simulations results are shown for the expected rate, i.e., the expectation  $E[C_{inst,s}(P, g, \eta)]$  with  $C_{inst,s}(\cdot, \cdot, \cdot)$  defined as in (2), as a function of the average interference constraint  $Q$  for different values of  $\bar{w}$  for the two policies. Regarding the RMP policy, due to the high accuracy of approximation  $\tilde{V}(\cdot)$  given in (17),



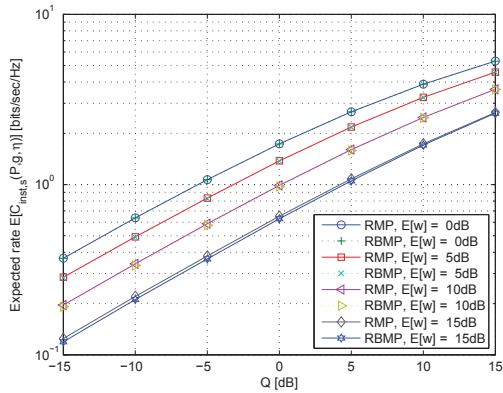


Fig. 3. Achievable expected rate for RMP and RBMP as a function of  $Q$ .

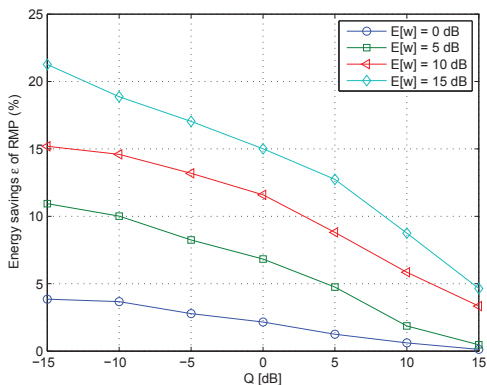


Fig. 4. Energy consumption comparison for RMP and RBMP.

that was also shown in Fig. 2, we have adopted  $\tilde{V}(\cdot)$  for its calculation. The depicted results were obtained by setting  $N_0 = 0dB$ . The ratio  $\rho = \sigma_{ss}^2/\sigma_{sp}^2$  was set equal to  $\rho = 3dB$  with  $\sigma_{sp}^2 = N_0$ . The maximum allowable peak transmit power  $P_{max}$  was set such that  $P_{max}/N_0 = 30dB$ . Both policies achieve similar results with RMP being slightly superior as the average PTx-SRx interference  $\bar{w}$  increases and/or the average STx-PRx interference constraint  $Q$  decreases.

While Fig. 3 illustrates the equivalence of the two developed policies in terms of achievable rate, it fails to capture the performance of the two methods in terms of energy consumption. Therefore, in Fig. 4 we compare RMP and RBMP by means of the percent energy savings achieved by RMP, defined as

$$\varepsilon = 100 \frac{E[P^0(g, \eta)] - E[P^*(g, \eta)]}{E[P^0(g, \eta)]} (\%), \quad (24)$$

for different values of the average interference constraint  $Q$ . The results concern four different cases for the value of  $\bar{w}$  while again we consider that  $N_0 = 0dB$  and  $\rho = 3dB$  with  $\sigma_{sp}^2 = N_0$ . By inspecting these results, it can be seen that RMP achieves higher energy efficiency, since in all cases, the energy consumption is less for RMP than for RBMP. We can thus deduce that on the high PTx-SRx interference and/or low STx-PRx interference regime, RMP achieves particularly notable

energy savings for the same rate performance as RBMP.

## VIII. CONCLUSION

An optimal in terms of expected rate maximization PP has been presented for underlay CR networks operating under an average STx-PRx interference constraint and an algorithm allowing for the efficient evaluation of this policy was developed. In addition, a second, suboptimum power policy has been introduced. Both PPs investigate a limited (hybrid) feedback scenario never studied before, where STx is assumed to have knowledge of the PTx-SRx interference statistics. By means of simulations, it is shown that both policies achieve similar performance with the optimal PP exhibiting better energy consumption characteristics.

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