Coordinated Beamforming in Multicell Networks with Channel State Information Exchange Delays

Bruhtesfa E. Godana
Dept. of Electronics and Telecommunications
Norwegian University of Science and Technology
Trondheim, Norway
Email: godana@iet.ntnu.no

David Gesbert
Mobile Communications Department
EURECOM, Campus SophiaTech
Sophia-Antipolis, France
Email: gesbert@eurecom.fr

***Abstract*—Coordinated beamforming is an important technique for dealing with interference channels arising in multicell systems. However, the performance of coordinated beamforming suffers significantly from Channel State Information (CSI) feedback delay and back-haul delay caused by exchanging CSI through the back-haul link. Hence, designing a beamforming scheme that takes into consideration the different delays incurred by the local and interfering channel links is an important problem. In this paper, average Virtual SINR based criterion is used to design an optimal combination of Maximum Ratio Transmission (MRT) and Intercell Interference Cancellation (IIC) beamformers as a function of the feedback and back-haul delays. Although it is well known that MRT is close to optimality at low SNR and IIC better at high SNR, we demonstrate how the optimal solution is modified when taking feedback delays into account. In particular, our results suggest the egoistic strategy is much more robust with respect to delays, thus affecting the shape of the optimal beamformer and giving extra performance for a wide range of scenarios.**

I. INTRODUCTION

The performance of cellular systems can be enhanced by using multicell cooperation [1], [2]. Various levels of base station cooperation can be considered in multicell systems. In this paper, we focus on coordinated beamforming, in which only Channel State Information (CSI) is shared among base stations. Coordinated beamforming can be used to improve the performance of cellular systems with limited back-haul capacity [3].

Base stations can behave in a so-called ”selfish” manner and maximize the useful signal power to their in-cell user using Maximum Ratio Transmission (MRT). On the other extreme, altruism-minded base stations can use Intercell Interference Cancellation (IIC) to nullify the interference caused to users in neighboring cells [4]. A possible approach to exploiting both the altruistic and egoistic concepts consists in establishing a switching policy from one to another. In [5], an optimal switch policy is presented that takes into account the signal-to-noise ratio (SNR) and channel state feedback delays. However, it is proven that beamforming solutions that achieve Pareto optimal rate points, points on the capacity region of the multi-input single-output interference channel (MISO-IC), are actually linear combinations of these two extreme beamforming schemes as opposed to a simple switch [6], [7]. However, this analysis was done under perfect CSI conditions in which the availability of full and centralized CSI of all channel links is assumed. In practical standards (such as LTE, LTE-A), base stations have knowledge of only local channel state information. That means, each base stations receives information about its channel links to all users, but not the channel links of other base stations. In presence of such local CSI, some beamforming algorithms have been suggested in the literature [8]–[11]. The adaptive method in [8] is iterative and requires feedback from users. The method of virtual signal-to-interference-and-noise ratio (Virtual SINR), proposed in [9] in the MISO-IC context and subsequently applied in [10]–[12] among others, is a distributed algorithm that allows each base station to compute its own beamforming vector using only local channel information and its optimality is proven. Even though local CSI is used in these works, it is assumed that there is no delay in obtaining the information. In practice, base stations receive a delayed CSI due to propagation, signal processing or information routing delays. Cooperative multicell systems which exchange CSI over the back-haul network suffer the most due to an extra back-haul latency [13], [14]. In presence of channel state information delay, it is not clear how the egoistic and altruistic beamforming schemes can be combined to maximize performance. The fact that the direct and interfering channels incur different delays makes the problem even more challenging. We address such problems in this paper.

In this paper, an average Virtual SINR based beamforming design is used to combine egoistic and altruistic beamformers in order to maximize performance in presence of delay. We show that the optimal beamformer is a function of the feedback and back-haul delays. Unlike most previous works, the beamformer design in this paper is derived for any number of cooperating cells (cluster size). The paper is organized as follows. Section II outlines the system and channel model used in the analysis. In Section III, MRT and IIC beamforming schemes and their optimal combination is reviewed. In Section IV, the beamforming design problem is formulated considering feedback and back-haul delays and the optimal solution is obtained. Section V demonstrates the performance of the proposed beamforming design using various simulations. Section

D. Gesbert acknowledges the partial support of EU FP7 funded project ”HARP”.

VI concludes the paper.

II. SYSTEM MODEL

Consider a downlink cellular MISO system consisting of K cooperating cells, each having a base station with N_t transmit antennas and a single antenna user. Assume that each base station serves only a single user per resource block. The channel between base station (BS) i and user k is denoted by a $1 \times N_t$ row vector $\mathbf{h}_{ik}$.

Each user estimates the direct channel ("local channel") from its serving BS and interfering channels ("cross channels") from other base stations using downlink pilot symbols and feedbacks both the local and cross channels to its serving base station. Then, the base stations (BSs) exchange the cross channels information through the back-haul link. This feedback scheme is compliant to single-cell systems and the LTE framework [15]. Note that the base stations are not assumed to exchange the user data packets, which precludes the use of so-called joint processing CoMP or Network MIMO.

In practice, there is a delay before the BS acquires the necessary channel state information for beamforming. First, there is a feedback delay until each BS receives CSI from the local user, which affects all channel vectors. Then, there is a back-haul delay while exchanging CSI through the back-haul, which affects only the cross-channels. Hence, the delay D_{ik} incurred by each channel vector $\mathbf{h}_{ik}$ is given by:

$$D_{ik} = \begin{cases} D_f & \text{if } i = k \\ D_f + D_b & \text{if } i \neq k \end{cases}$$

where D_f and D_b denote the feedback and back-haul delays respectively. Lets denote $D_f + D_b$ by D_{f+b} for notational simplicity. This feedback scheme is shown in Fig.1 for a 3-cell system.

The importance of this work lies in the fact that a degradation of the information on the channel vectors, received from other base stations through the back-haul, is accounted for as this will affect the overall cooperation strategy. It is assumed in this paper that there is a high quality quantization of the magnitude and direction of the channels (quantization error is ignored).

A. Autoregressive Order one Channel Model

Autoregressive order one (AR1) channel model can be used to model the temporal variation of channels with sufficient accuracy [16]. Using this model, the channel vector $\mathbf{h}_{ik}[n]$ at time index n can be expressed as,

$$\mathbf{h}_{ik}[n] = \rho_{ik}\mathbf{h}_{ik}[n - D_{ik}] + \sqrt{1-\rho_{ik}^2}\,\mathbf{e}_{ik}[n] \qquad (1)$$

where $\mathbf{e}_{ik}[n]$ is a process noise vector having independent entries with complex Gaussian distribution of zero mean and unit variance. The temporal correlation ρ_{ik} is equal to either a local channel correlation ρ_l for local channels $(i = k)$ or a cross channel correlation ρ_c for cross channels $(i \neq k)$ with typically $\rho_c < \rho_l$ due to the fact that the interference related channel state information is obtained with a larger delay. In presence of isotropic scattering and reception, the local and cross channel correlations can be given by $\rho_l = J_0(2\pi f_d \tau D_f)$ and $\rho_c = J_0(2\pi f_d \tau D_{f+b})$ respectively, where f_d is the Doppler frequency, τ is the channel update time interval and $J_0(\cdot)$ is the zeroth order Bessel function of the first kind. The temporal correlation is determined by the product $f_d\tau$, which is referred to as normalized Doppler frequency.

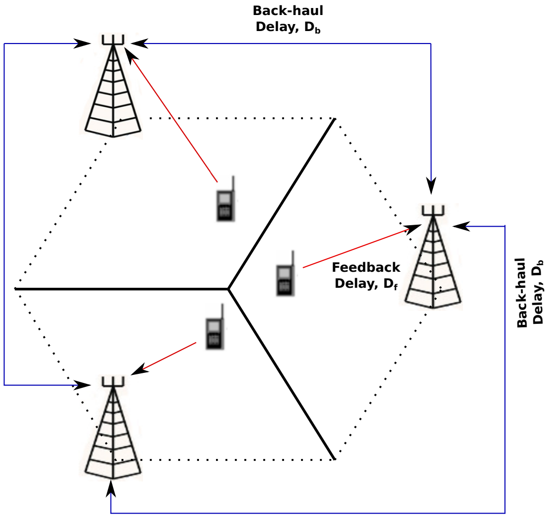

Fig. 1. 3-cell Setup

B. Input-Output Model

Assume that base station k transmits a symbol s_k to its local user using a beamformer $\mathbf{w}_k$. Let the power transmitted from BS_k be denoted by P_k, which satisfies a per base station power constraint. In addition, let the path loss from BS_i to user k be L_{ik}. Thus, the signal received by user k is then given by:

$$\begin{aligned} y_k[n] &= \sqrt{\frac{P_k}{L_{kk}}}\mathbf{h}_{kk}[n]\mathbf{w}_k[n]s_k[n] \\ &\quad + \sum_{i=1, i\neq k}^{K}\sqrt{\frac{P_i}{L_{ik}}}\mathbf{h}_{ik}[n]\mathbf{w}_i[n]s_i[n] + v_k[n] \end{aligned}$$

where, $v_k[n]$ denotes the additive complex Gaussian noise. Hence, the SINR and sum rate R are given by,

$$\begin{aligned} \text{SINR}_k[n] &= \frac{|\mathbf{h}_{kk}[n]\mathbf{w}_k[n]|^2}{\frac{1}{\gamma_k} + \sum_{i=1, i\neq k}^{K}\alpha_{ik}^2|\mathbf{h}_{ik}[n]\mathbf{w}_i[n]|^2} \\ R[n] &= \sum_{k=1}^{K}\mathbb{E}\{\log(1 + \text{SINR}_k[n])\} \end{aligned}$$

where, γ_k represents the SNR at user k and α_{ik}^2 represents the ratio of the power from an interfering base station to the useful signal power from local base station, $\alpha_{ik}^2 = \left(\frac{P_i}{P_k}\right)\left(\frac{L_{kk}}{L_{ik}}\right)$. Assume that all BSs transmit with equal power and this results in a cell-edge SNR denoted by γ_E. In addition, denoting the cell radius by r and the distance of user k from base station i by d_{ik}, γ_k and α_{ik}^2 can be expressed as:

$$\gamma_k = \gamma_E \left(\frac{r}{d_{kk}}\right)^{\mu}; \ \ \alpha_{ik}^2 = \left(\frac{d_{kk}}{d_{ik}}\right)^{\mu}$$

where, μ is the path-loss exponent.

III. PARETO-OPTIMAL BEAMFORMING

In MISO-IC beamforming, there are two extreme beamforming strategies: namely Maximum Ratio Transmission (MRT) and Inter-Cell Interference Cancellation (IIC) [4]. In MRT, each BS ignores the inter-cell interference it causes to other cells and aims to maximize the received signal power at its intra-cell user. Thus, the beamforming vector for BS k with perfect CSI is given by,

$$\mathbf{w}_{k,eg}[n] = \frac{\mathbf{h}_{kk}^H[n]}{\|\mathbf{h}_{kk}[n]\|} \tag{2}$$

where, the subscript eg refers to the beamformer being egoistic (competitive). In IIC, the focus of each BS is to avoid interference caused to other cells. Hence, base stations use their degree of freedom to form a null in the direction of users in other cells. The IIC beamformer can be expressed as,

$$\mathbf{w}_{k,al}[n] = \frac{\mathbf{\Pi}_k^{\perp}[n]\,\mathbf{h}_{kk}^H[n]}{\left\|\mathbf{\Pi}_k^{\perp}[n]\,\mathbf{h}_{kk}^H[n]\right\|} \tag{3}$$

where $\mathbf{\Pi}_k^{\perp}[n]$ is a projection matrix onto the subspace orthogonal to the channels being interfered and the subscript al denotes the beamformer being altruistic (competitive).

It is proven in [6] that Pareto optimal rate points, points on the capacity region of the MISO interference channel, can be expressed as a linear combination of the egoistic and altruistic beamformers. Rate points on the Pareto boundary can be obtained using a beamformer $\mathbf{w}_{k,O}$ which can be expressed as [6]:

$$\mathbf{w}_{k,O}[n] = \frac{\lambda_k \mathbf{w}_{k,eg}[n] + (1-\lambda_k)\,\mathbf{w}_{k,al}[n]}{\|\lambda_k \mathbf{w}_{k,eg}[n] + (1-\lambda_k)\,\mathbf{w}_{k,al}[n]\|}$$

where the parameter λ_k satisfies $0 \leq \lambda_k \leq 1$. The Pareto optimal beamformer can also be expressed as a linear combination of the IIC beamformer $\mathbf{w}_{k,al}$ and a beamformer obtained by projecting the local channel $\mathbf{h}_{kk}$ on the interference subspace (denoted by $\mathbf{\Pi}_k$). Thus, $\mathbf{w}_{k,O}$ can be expressed as [6]:

$$\mathbf{w}_{k,O}[n] \ = \ \sqrt{\mu_k}\mathbf{w}_{k,al}^{\perp}[n] + \sqrt{1-\mu_k}\mathbf{w}_{k,al}[n] \tag{4}$$

where, $0 \leq \mu_k < 1$. $\mathbf{w}_{k,al}^{\perp}$ is vector orthogonal to $\mathbf{w}_{k,al}$ and given by:

$$\mathbf{w}_{k,al}^{\perp} = \frac{\mathbf{\Pi}_k[n]\,\mathbf{h}_{kk}[n]^H}{\left\|\mathbf{\Pi}_k[n]\,\mathbf{h}_{kk}[n]^H\right\|}$$

The formulation in (4) and the orthogonality of $\mathbf{w}_{k,al}^{\perp}$ and $\mathbf{w}_{k,al}$ ensures that $\mathbf{w}_{k,O}[n]$ is a unit norm vector.

Determining the optimal combining factor μ_k in (4) that maximizes the sum rate is an important problem. However, the problem of maximization of the sum rate (or even the individual rate) involves coupled variables $\mu_1, \cdots, \mu_K$ of all base stations. Therefore, this can be solved only with centralized knowledge of all channel state information or with iterative information exchange between base stations. In order to determine an optimal beamformer using localized channel state information only, a Virtual SINR based method is proposed in [9]. This virtual SINR method is inspired by the powerful and well established idea of uplink-downlink duality in downlink beamforming [17], [18].

IV. BEAMFORMING DESIGN IN PRESENCE OF DELAY

In practice, coordinated beamforming is affected by channel state information delay. The local and cross channel links incur different delays as discussed in Section II; moreover, MRT and IIC have different levels of sensitivity to delay. As a result, the optimal beamformer must be a function of the feedback and back-haul delays.

In the presence of delay, the beamformer combination $\mathbf{w}_{k,O}$ in (4) is expressed as:

$$\mathbf{w}_{k,O}[n] \ = \ \sqrt{\mu_k}\hat{\mathbf{w}}_{k,al}^{\perp}[n] + \sqrt{1-\mu_k}\hat{\mathbf{w}}_{k,al}[n] \tag{5}$$

where,

$$\begin{aligned}\hat{\mathbf{w}}_{k,al}^{\perp}[n] \ &= \ \frac{\mathbf{\Pi}_k[n-D_{f+b}]\,\mathbf{h}_{kk}[n-D_f]^H}{\left\|\mathbf{\Pi}_k[n-D_{f+b}]\,\mathbf{h}_{kk}[n-D_f]^H\right\|}\\ \hat{\mathbf{w}}_{k,al}[n] \ &= \ \frac{\mathbf{\Pi}_k^{\perp}[n-D_{f+b}]\,\mathbf{h}_{kk}[n-D_f]^H}{\left\|\mathbf{\Pi}_k^{\perp}[n-D_{f+b}]\,\mathbf{h}_{kk}[n-D_f]^H\right\|}\end{aligned}$$

In order to determine the optimal combining factor μ_k, the Virtual SINR based method is adopted. Virtual SINR is the ratio of the useful signal transmitted by a base station to the interference plus noise caused to users in neighboring cells. In a TDD reciprocal channel, the Virtual SINR happens to coincide with the uplink SINR at the base station. In [9], maximizing the virtual SINR was proved to be Pareto optimal for the downlink (i.e. for FDD scenarios) as well. The Virtual SINR of BS k can be expressed as:

$$\mathrm{SINR}_k^{Virtual}[n] \ = \ \frac{|\mathbf{h}_{kk}[n]\,\mathbf{w}_{O,k}[n]|^2}{\frac{1}{\gamma_k} + \sum_{i=1,i\neq k}^{K} \alpha_{ki}^2\,|\mathbf{h}_{ki}[n]\,\mathbf{w}_{O,k}[n]|^2}$$

Note that the Virtual SINR expression, unlike the SINR, contains only the beamformer of a single base station. Now, the problem we would like to pose is the following.

Given the delayed channel vectors from base station k to all users, find the optimal combining factor μ_k that maximizes the Virtual SINR. Thus, the maximization is obtained by averaging the Virtual SINR over the channel process noise vectors using the following problem.

$$\max_{\mu_k} \ \mathbb{E}\left\{\mathrm{SINR}_k^{Virtual}[n]\right\} |_{\mathbf{h}_{kk}[n-D_f],\mathbf{h}_{ki}[n-D_{f+b}],\forall i\neq k}$$

where, $\mathbf{h}_{kk}[n-D_f]$ is the delayed channel from BS k to its local user and $\mathbf{h}_{ki}[n-D_{f+b}], \forall i \neq k$ represents the delayed cross channels from BS k to users in other cells. The solution to this problem is stated in the following theorem.

Theorem 1. *The beamformer combining factor μ_k that maximizes the Virtual SINR in the presence of feedback and back-haul delays is given by the solution of the following maximization problem,*

$$\mu_k = arg\,\underset{\mu}{max}\ \frac{\rho_l^2\left(\sqrt{\mu\, a_k}+\sqrt{(1-\mu)b_k}\right)^2+1-\rho_l^2}{\frac{1}{\gamma_k}+\mu\left(\rho_c^2\sum_{i=1,i\neq k}^{K}\alpha_{ki}^2\frac{c_{ki}}{a_k}\right)+(1-\rho_c^2)\left(\sum_{i=1,i\neq k}^{K}\alpha_{ki}^2\right)} \tag{6}$$

where,

$$\begin{aligned} a_k &= \left\|\mathbf{\Pi}_k[n-D_{f+b}]\mathbf{h}_{kk}[n-D_f]^H\right\|^2 \\ b_k &= \|\mathbf{h}_{kk}[n-D_f]\|^2-a_k \\ c_{ki} &= \left|\mathbf{h}_{ki}[n-D_{f+b}]\mathbf{h}_{kk}[n-D_f]^H\right|^2 \end{aligned}$$

and $\rho_l = J_0\left(2\pi f_d \tau D_f\right)$ and $\rho_c = J_0\left(2\pi f_d \tau D_{f+b}\right)$ are the local and cross channel correlations. The proof is shown in Appendix A.

From Theorem 1, the following limiting scenarious can be deduced.

- At high SNR $\left(\frac{1}{\gamma_k}\to 0\right)$, the denominator is small, therefore Virtual SINR can be maximized using μ_k close to zero depending on the quality of CSI. When $\rho_c \to 1$, then $\mu_k = 0$ maximizes (6) and the optimal beamformer reduces to IIC . This is expected as the system is interference limited at high SNR and zero forcing maximizes performance.
- At low SNR $\left(\frac{1}{\gamma_k}\to\infty\right)$, the system in noise limited. Hence, (6) can be maximized by maximizing the signal power in the numerator. In this case, it can be shown that the value of μ_k that maximizes the signal power is $\mu_k = \frac{a_k}{a_k+b_k}$ irrespective of the quality of CSI. In this case, the optimal beamformer reduces to MRT.
- When the back-haul delay increases, which results in a very small ρ_c, again the optimal value of μ_k becomes $\mu_k = \frac{a_k}{a_k+b_k}$. Hence, the optimal beamformer is closer to MRT when there is large delay.

This shows that the beamformer combining factor μ_k is actually bounded in $\mu_k \in \left[0 : \frac{a_k}{a_k+b_k}\right]$.

Each base station solves the maximization problem in (6) independently, resulting in a distributed algorithm. Various optimization algorithms can be used to solve the problem in (6). Since the parameter range is bounded, one suitable method for solving this problem is Golden section search [19], where the the initial lower and upper bounds of the solution are set to 0 and 1 respectively and the interval is successively narrowed.

A. Beamforming in TDD Systems

In Time Division Duplex (TDD) systems, channel reciprocity can be used such that each base station obtains all its channel links from uplink pilot symbols. Therefore, base stations can obtain channel state information directly from the users and inter base station CSI exchange will not be required.

In order to apply Theorem 1 to TDD systems, the only modification needed is to set the back-haul delay to zero. Assuming there is a delay D_{tdd} between obtaining CSI in the uplink and using it in the dowlink beamforming, the beamforming solution in Theorem 1 can easily be modified by using $\rho_l = \rho_c = J_0\left(2\pi f_d \tau D_{tdd}\right)$.

V. RESULTS

In order to demonstrate the performance of the proposed beamforming scheme as compared to MRT and IIC, simulations are performed using the 3-cell system shown in Fig.1. The feedback and back-haul delays are set to one symbol time each. Each base station is assumed to have 4 transmit antennas.

In Fig.2, the average sum rate achieved by the Virtual SINR based beamforming (denoted ”VSINR combining”) at a cell-edge SNR of $5\,dB$ is compared with MRT and IIC when the Doppler frequency increases. The corresponding average combining factor μ_{aver} (averaged over time and base stations) is also shown in Fig.3. It is assumed that users are located at half the cell radius and a path loss exponent of 2 is used. Note that the combining factor μ_{aver} adapts based on the Doppler frequency considering the higher sensitivity of IIC to delay. At low Doppler frequency, μ_{aver} is very low, thus the optimal beamformer is closer to the IIC beamformer. As the Doppler increases, μ_{aver} increases and the optimal beamformer moves away from the IIC and towards the MRT beamformer. As a result, the optimal beamformer achieves a sum rate higher than both IIC and MRT at all Doppler frequencies as demonstrated in Fig.2. The value of μ_{aver} approaches 0.5 at high Doppler frequencies, at which the optimal beamformer becomes MRT.

Similarly, Fig.4 shows the sum rate as a function of cell-edge SNR at a moderate Doppler frequency of 0.05 for two different user locations assuming a path loss exponent of 2. This figure also demonstrates the limit analysis shown in Section IV that the optimal beamformer is almost equal to the MRT beamformer at low SNR irrespective of the Doppler frequency. At high SNR on the other hand, the optimal beamformer is a combination of MRT and IIC depending on the Doppler frequency. The optimal beamformer has in general less sensitivity to location of users; this is because the SNR decreases and interference from other cells increases with increase in distance of users from cell center. While decrease in SNR encourages the use of MRT, increase in interference means IIC is preferable. As a result, IIC is only marginally preferable with increase in distance as shown in Fig.4 and the optimal beamformer moves slightly towards IIC.

In general, it is demonstrated in this section that an average Virtual SINR based beamformer design is able to achieve a sum rate higher than both MRT and IIC at all SNRs and channel state information delays. By using the proposed beamformer design, it is possible to improve the achieved sum rate significantly as opposed to selecting MRT or IIC.

VI. CONCLUSION

In this paper, an optimal beamforming design based on a Virtual SINR maximization is proposed to combine MRT and IIC beamformers based on the SNR, feedback delay, back-haul delay and Doppler frequency. It is demonstrated that the design is able to adapt the beamformer combining factor

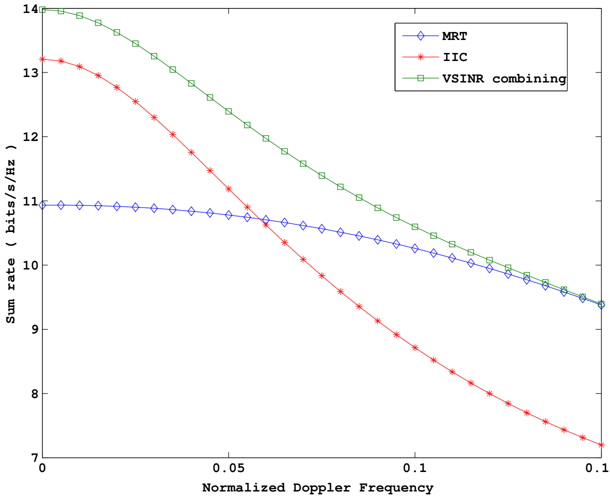

Fig. 2. Sum rate vs. Doppler frequency, at a cell-edge SNR of $5\,dB$

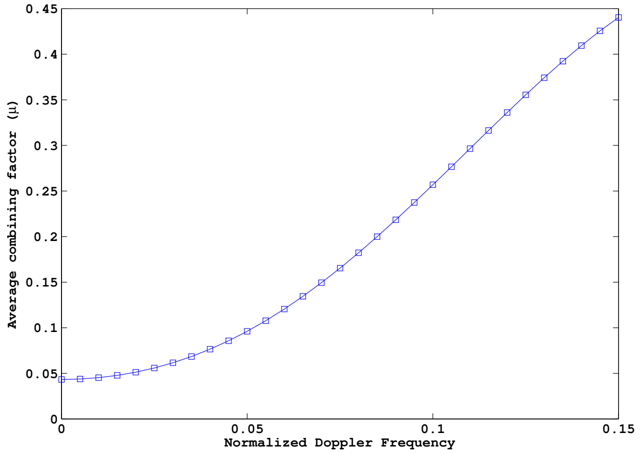

Fig. 3. Beamformer Combining Factor vs. Doppler frequency, at a cell-edge SNR of $5\,dB$

based on the quality of channel state information. It is shown that the optimal beamformer is closer to MRT at low SNR irrespective of the delay. In contrary, the optimal beamformer varies between MRT and IIC at high SNR depending on the channel state information delay.

APPENDIX

In order to maximize the average VSINR given the outdated channel vectors, first lets use an approximation. Using first order Taylor series approximation, the average VSINR can be approximated as follows:

$$\mathbb{E}\left\{\mathrm{SINR}_k^{Virtual}[n]\right\} \approx \frac{\mathbb{E}\left\{|\mathbf{h}_{kk}[n]\mathbf{w}_{k,O}[n]|^2\right\}}{\frac{1}{\gamma_k}+\sum_{i=1,i\neq k}^{K}\alpha_{ki}^2\,\mathbb{E}\left\{|\mathbf{h}_{ki}[n]\mathbf{w}_{k,O}[n]|^2\right\}} \tag{7}$$

Note that, the average values in this expression are expectations given the outdated channel vectors: $\mathbf{h}_{kk}[n-D_f]$ and $\mathbf{h}_{ki}[n-D_{f+b}], \forall i \neq k$. This is not indicated in the expressions only for notational brevity.

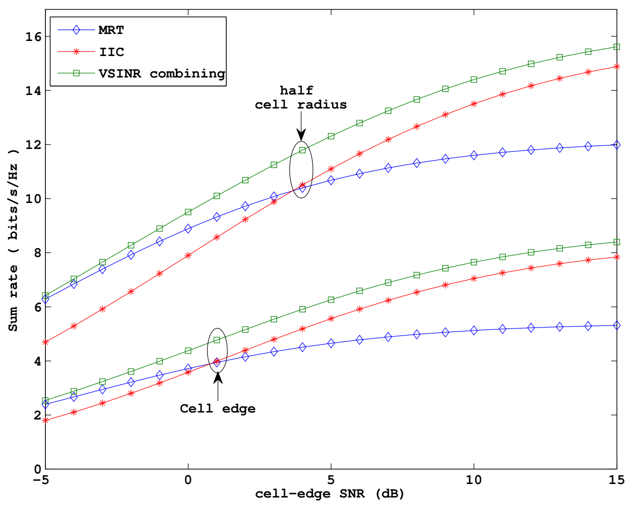

Fig. 4. Sum rate vs. cell-edge SNR, at a normalized Doppler frequency of 0.05 for two different user locations

The local channel $\mathbf{h}_{kk}[n]$ and the cross-channel $\mathbf{h}_{ki}[n]$ can be expressed using AR1 channel model as,

$$\begin{aligned}\mathbf{h}_{kk}[n] &= \rho_l\mathbf{h}_{kk}[n-D_f]+\sqrt{1-\rho_l^2}\,\mathbf{e}_{kk}[n]\\ \mathbf{h}_{ki}[n] &= \rho_c\mathbf{h}_{ki}[n-D_{f+b}]+\sqrt{1-\rho_c^2}\,\mathbf{e}_{ki}[n]\end{aligned}$$

Using this channel models and the beamformer in (5), each of the expectations in the Virtual SINR approximation can be evaluated. Lets denote: $a_k = \left\|\mathbf{\Pi}_k[n-D_{f+b}]\,\mathbf{h}_{kk}[n-D_f]^H\right\|^2$ and $b_k = \left\|\mathbf{\Pi}_k^{\perp}[n-D_{f+b}]\,\mathbf{h}_{kk}[n-D_f]^H\right\|^2$.

Considering the signal power in the Virtual SINR,

$$\begin{aligned}&\mathbb{E}\left\{|\mathbf{h}_{kk}[n]\,\mathbf{w}_{k,O}[n]|^2\right\}\\ =\ &\mathbb{E}\left\{\left|\rho_l\sqrt{\mu_k a_k}+\rho_l\sqrt{(1-\mu_k)\,b_k}+\right.\right.\\ &\left.\left.\sqrt{1-\rho_l^2}\left(\sqrt{\mu_k}e_1[n]+\sqrt{1-\mu_k}e_2[n]\right)\right|^2\right\}\end{aligned}$$

where, $e_1[n]=\mathbf{e}_{kk}[n]\,\hat{\mathbf{w}}_{k,al}^{\perp}[n]$ and $e_2[n]=\mathbf{e}_{kk}[n]\,\hat{\mathbf{w}}_{k,al}[n]$. Since both $\hat{\mathbf{w}}_{k,al}^{\perp}[n]$ and $\hat{\mathbf{w}}_{k,al}[n]$ are unit vectors independent of $\mathbf{e}_{kk}[n]$, $e_1[n]$ and $e_2[n]$ are standard Gaussian random variables with zero mean and unit variance. Therefore, the expectation of these two variables and the cross-product between them is zero. a_k and b_k are treated as constants since the expectation is evaluated given the outdated channels. Therefore, $\mathbb{E}\left\{|\mathbf{h}_{kk}[n]\,\mathbf{w}_{O,k}[n]|^2\right\}$

reduces to:

$$\begin{aligned}
& \mathbb{E}\left\{\left|\mathbf{h}_{kk}\left[n\right]\mathbf{w}_{O,k}\left[n\right]\right|^2\right\} \\
= \ & \rho_l^2\left(\sqrt{\mu_k a_k}+\rho_l\sqrt{\left(1-\mu_k\right)b_k}\right)^2 + \\
& \left(1-\rho_l^2\right)\left\{\mu_k\mathbb{E}\left(\left|e_1\left[n\right]\right|^2\right)+\left(1-\mu_k\right)\mathbb{E}\left(\left|e_2\left[n\right]\right|^2\right)\right\} \\
= \ & \rho_l^2\left(\sqrt{\mu_k a_k}+\rho_l\sqrt{\left(1-\mu_k\right)b_k}\right)^2 + \\
& \left(1-\rho_l^2\right)\left(\mu_k\cdot 1+\left(1-\mu_k\right)\cdot 1\right) \\
= \ & \rho_l^2\left(\sqrt{\mu_k a_k}+\rho_l\sqrt{\left(1-\mu_k\right)b_k}\right)^2 + \left(1-\rho_l^2\right)
\end{aligned}$$

Similarly the interference power is expressed as follows considering the fact that $\mathbf{h}_{ki}\left[n-D_{f+b}\right]$ and $\hat{\mathbf{w}}_{k,al}\left[n\right]$ are orthogonal.

$$\begin{aligned}
\mathbb{E}\left\{\left|\mathbf{h}_{ki}\left[n\right]\mathbf{w}_{k,O}\left[n\right]\right|^2\right\} = \mathbb{E}\Bigg\{\Bigg|\rho_c\sqrt{\mu_k}\frac{\mathbf{h}_{ki}\left[n\right]\mathbf{h}_{kk}\left[n\right]^H}{\sqrt{a_k}} \\
+ \sqrt{1-\rho_c^2}\left(\sqrt{\mu_k}e_3\left[n\right]+\sqrt{1-\mu_k}e_4\left[n\right]\right)\Bigg|^2\Bigg\}
\end{aligned}$$

where, $e_3\left[n\right]=\mathbf{e}_{ki}\left[n\right]\hat{\mathbf{w}}_{k,al}^{\perp}\left[n\right]$ and $e_4\left[n\right]=\mathbf{e}_{ki}\left[n\right]\hat{\mathbf{w}}_{k,al}\left[n\right]$. Similarly, $e_3\left[n\right]$ and $e_4\left[n\right]$ are Gaussian distributed with zero mean and unit variance. Hence, the expectation simplifies to:

$$\mathbb{E}\left\{\left|\mathbf{h}_{ki}\left[n\right]\mathbf{w}_{O,k}\left[n\right]\right|^2\right\} \quad = \quad \rho_c^2\mu_k\frac{c_{ik}}{a_k}+\left(1-\rho_c^2\right)$$

where, $c_{ki}=\left|\mathbf{h}_{ki}\left[n-D_{f+b}\right]\mathbf{h}_{kk}\left[n-D_f\right]^H\right|^2$.

Using the signal and interference terms, the first order approximation in (7) reduces to:

$$\frac{\rho_l^2\left(\sqrt{\mu_k a_k}+\rho_l\sqrt{\left(1-\mu_k\right)b_k}\right)^2+\left(1-\rho_l^2\right)}{\frac{1}{\gamma_k}+\sum_{i=1,i\neq k}^{K}\alpha_{ki}^2\left(\rho_c^2\mu_k\frac{c_{ik}}{a_k}+\left(1-\rho_c^2\right)\right)}$$

Note that, a_k and b_k are related by: $a_k+b_k=\left\|\mathbf{h}_{kk}\left[n-D_f\right]\right\|^2$. Using this relation, the maximization of the average Virtual SINR given the old channel estimates reduces to the maximization stated in Theorem 1.

REFERENCES

[1] S. Shamai and B. Zaidel, "Enhancing the cellular downlink capacity via co-processing at the transmitting end," in *Proc. IEEE Vehicular Technology Conference (VTC)*, vol. 3, May 2001, pp. 1745–1749.

[2] D. Gesbert, S. Hanly, H. Huang, S. Shamai Shitz, O. Simeone, and W. Yu, "Multi-cell MIMO cooperative networks: A new look at interference," *IEEE Journal on Selected Areas in Communications*, vol. 28, no. 9, pp. 1380–1408, Dec. 2010.

[3] N. Seifi, M. Viberg, R. Heath, J. Zhang, and M. Coldrey, "Coordinated single-cell vs multi-cell transmission with limited-capacity backhaul," in *Proc. Asilomar Conference on Signals, Systems and Computing*, Nov. 2010, pp. 1217–1221.

[4] E. Larsson and E. Jorswieck, "Competition versus cooperation on the MISO interference channel," *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 7, pp. 1059–1069, Sep. 2008.

[5] B. Godana and D. Gesbert, "Egoistic vs. altruistic beamforming in the presence of feedback and back-haul delays," *Submitted to EURASIP Journal on Wireless Communications and Networking*, 2013.

[6] E. Jorswieck and E. Larsson, "The MISO interference channel from a game-theoretic perspective: A combination of selfishness and altruism achieves Pareto optimality," in *Proc. IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, Mar. 31- Apr. 4 2008, pp. 5364–5367.

[7] E. Jorswieck, E. Larsson, and D. Danev, "Complete characterization of the pareto boundary for the MISO interference channel," *IEEE Transactions on Signal Processing*, vol. 56, no. 10, pp. 5292–5296, Oct. 2008.

[8] J. Lindblom and E. Karipidis, "Cooperative beamforming for the MISO interference channel," in *European Wireless Conference*, Apr. 2010, pp. 631–638.

[9] R. Zakhour and D. Gesbert, "Coordination on the MISO interference channel using the virtual SINR framework," in *Proc. International Workshop on Smart Antennas (WSA)*, Feb. 2009.

[10] R. Zakhour, Z. Ho, and D. Gesbert, "Distributed beamforming coordination in multicell MIMO channels," in *Proc. IEEE Vehicular Technology Conference (VTC)*, Apr. 2009, pp. 1–5.

[11] R. Zakhour and D. Gesbert, "Distributed multicell-MISO precoding using the layered virtual SINR framework," *IEEE Transactions on Wireless Communications*, vol. 9, no. 8, pp. 2444–2448, Aug. 2010.

[12] S.-H. Park, H. Park, H. Kong, and I. Lee, "New beamforming techniques based on virtual SINR maximization for coordinated multi-cell transmission," *IEEE Transactions on Wireless Communications*, vol. 11, no. 3, pp. 1034–1044, Mar. 2012.

[13] R. Fritzsche and G. Fettweis, "CSI distribution for joint processing in cooperative cellular networks," in *Proc. IEEE Vehicular Technology Conference (VTC)*, Sep. 2011, pp. 1–5.

[14] R. Bhagavatula and R. Heath, "Adaptive bit partitioning for multicell intercell interference nulling with delayed limited feedback," *IEEE Transactions on Signal Processing*, vol. 59, no. 8, pp. 3824–3836, Aug. 2011.

[15] I. F. Akyildiz, D. M. Gutierrez-Estevez, and E. C. Reyes, "The evolution to 4G cellular systems: LTE-Advanced," *Physical Communications*, vol. 3, no. 4, pp. 217–244, Aug. 2010.

[16] C. Tan and N. Beaulieu, "On first-order Markov modeling for the rayleigh fading channel," *IEEE Transactions on Communications*, vol. 48, no. 12, pp. 2032–2040, Dec. 2000.

[17] M. Bengtsson and B. Ottersten, "Optimal downlink beamforming using semidefinite optimization," in *Proc. Allerton Conference on Communication, Control, and Computing*, Sep. 1999, pp. 987–996.

[18] M. Schubert and H. Boche, "Solution of the multiuser downlink beamforming problem with individual SINR constraints," *IEEE Transactions on Vehicular Technology*, vol. 53, no. 1, pp. 18–28, Jan. 2004.

[19] W. Press, S. Taukolsky, W. Vetterling, and B. Flannery, *Numerical Recipes: The Art of Scientific Computing*. Cambridge University Press, 2007.