Minimizing the Effect of Feedback Delay in a Multi-user System through Adaptive Feedback Scheduling

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***Abstract*—In this paper, we analyze the downlink performance of a multi-user system consisting of mobile users with wide range of velocity. We categorize these users into different groups on the basis of their velocity range and study the impact of feedback delay on each group for closed-loop transmit beamforming. Based on this analysis, we propose an adaptive feedback-scheduling algorithm to minimize the effect of feedback delay on the performance of the system. We derive generalized performance measuring expressions for the proposed algorithm and carry out simulations to validate the analysis.**

***Keywords*: Closed-loop transmit beamforming, Feedback Delay, High-velocity User, Pedestrian User, Scheduler, Urban User.**

I. Introduction

Closed-loop transmit beamforming provides gain due to the feedback information given by the receiver (user) to the transmitter (base station (BS)) [1]. However, when the channel is time-variant, the feedback provided by the user might get outdated by the time it is actually used for beamforming [2]. The rate of channel variation is directly related to the velocity of the user with respect to BS [3]. Therefore the system performance is dependent on user's feedback delay and velocity [4]. In [4], the effect of feedback delay on closed-loop transmit beamforming is measured in terms of SNR gain for varying user-velocity. It shows that the sensitivity to feedback delay increases with increasing velocity, which results in performance degradation. However, the analysis there is restricted only to a single user system. In a multi-user system, a large number of users exists with wide range of velocity. These users can experience different feedback delay depending up on their order of scheduling in time division multiple access (TDMA). Therefore, the fundamental problem is to analyze the combined effect of user-velocity and different feedback delays for all the users in a system and devise an optimal feedback scheduler.

A. Motivation and Contribution

The primary motivation is to minimize the effect of feedback delay in a multi-user system while taking into account the feedback overhead constraint. Based on this motivation, the contribution of the paper can be described in two stages. In the first stage we categorize users on the basis of their velocity: pedestrian, urban and high-velocity user groups and perform feedback delay analysis of these groups. This categorization is generally followed in several wireless standards [5]. The main reason for categorizing the users for our study is to reduce feedback overhead while implementing our adaptive feedback-scheduler in a practical system. We analyze the feedback delay effect on each group by calculating the average SNR gains and upper capacity bound for the system. With this analysis, the tolerance to feedback delay is known for each group and it can be utilized to devise scheduling algorithms. Therefore, in the second stage, we propose optimal scheduling of feedback from different users based on their velocity group. The main idea behind this algorithm is to adjust users feedback delay according to its velocity. We compare the overall systems performance of the proposed algorithm with random scheduling. The performance measuring parameters are average SNR gain and the upper bound for system capacity. We observe significant gains using our proposed algorithm.

B. Organization

The rest of the paper is organized as follows: Section II gives a brief description of the system model. Section III and IV summarize the velocity-based user groups and the analysis of the feedback delay effects on each of the user groups respectively. Section V describes the proposed feedback user scheduling algorithm and Section VI shows comparative results for the scheduler. The paper is concluded with Section VII.

II. System Description

A. System Model

We consider a system with a stationary BS having 2 transmit antennas and K mobile users with single receive antenna. Closed-loop transmit beamforming is applied at the downlink transmission and only one user is served in the same time-frequency resource. We use frequency division duplex (FDD) configuration and therefore we have a separate feedback channel on the uplink that provides the side information for optimal beamforming at the transmitter. For our analysis, we use single path Rayleigh fading channel model. Therefore the received signal at the user is

$$r(t) = (\mathbf{h}(t)\mathbf{w})s(t) + n(t) \tag{1}$$

where $s(t)$ is the transmitted signal, $n(t)$ is zero-mean Gaussian noise, $\mathbf{w} = (w_1, w_2)^T$ consist of transmit weights for the two antennas which are selected based on the feedback and $\mathbf{h}(t) = (h_1(t), h_2(t))$ consist the channel vectors which are samples of zero-mean Gaussian process with common variance $\sigma^2 = \frac{1}{2}$. It is assumed that channels are constant during each time slot and they vary from slot to slot depending upon user's velocity. This time-correlation of the channel is given by Jakes model [6].

B. Feedback Delay Model

In closed-loop transmit beamforming, the transmitter (BS) transmits the pilot symbols to the receiver (user) and the user estimates the channel. Then the user selects the optimum transmit weight and this information is sent as N bits of quantized feedback to the BS via feedback channel which is utilized for beamforming. Now this information is received by the BS only after certain feedback delay τ which consists of round-trip and processing delay. Figure 1 shows the feedback delay involved in closed-loop transmit beamforming for a single user.

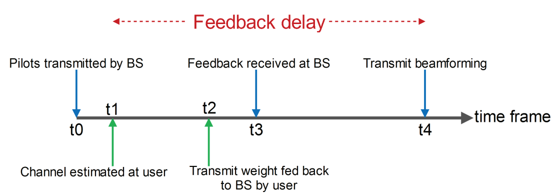

Fig. 1. Feedback Model

III. Velocity-based User Categorization

Users can be categorized into different groups depending on one or several parameters. Our analysis mainly concentrates on the effects of feedback delay and since the velocity of the user affects the time-variation of the channel, therefore we distinguish users on the basis of their velocity. We define users into three groups: pedestrian, urban and high-velocity. Pedestrian users have a decreasing velocity distribution function given by $f(v) = a + vb$, where $a > 0$ and $b < 0$. Urban users are equally distributed over the entire velocity range which is given as $f(v) = a$, where $a > 0$ and high-velocity users have an increasing type function given by $f(v) = av$, where $a > 0$. In all the expressions v is the respective velocity in each group and $f(v)$ is the probability of user at velocity v. The velocity distribution for the three groups are shown in Figure 2 along with their minimum and maximum velocity constraints.

IV. Feedback Delay Analysis

Generally for closed-loop transmit diversity beamforming, the problem of finding the optimal transmit weight $\mathbf{w}$ at each time instant t is

$$\text{Find } \mathbf{w_0} \in \mathrm{W}: \quad |\mathbf{h}(\mathrm{t})\mathbf{w_0}| = \max_{\mathbf{w} \in W} |\mathbf{h}(t)\mathbf{w}| \tag{2}$$

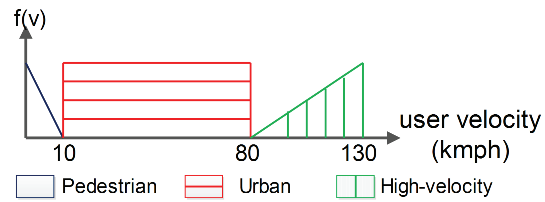

Fig. 2. Velocity-based User Groups

where $\mathrm{W} = \left\{\mathbf{w} = (\mathrm{w}_1, \mathrm{w}_2)^{\mathrm{T}} : \mathrm{w}_{\mathrm{m}} \in \mathrm{C}\right\}$ and it is assumed that $\|\mathbf{w}\| = 1$.

For our feedback method, the first antenna is the reference and therefore, the transmit weight for second antenna is given in terms of first antenna channel. The transmit weight for second antenna at time $t + \tau$ is written as [7]

$$|\mathrm{h}_1(\mathrm{t}) + \widehat{\mathrm{w}}_2\mathrm{h}_2(\mathrm{t})| = \max_{\mathrm{w}_2 \in \mathrm{W}_\mathrm{N}} |\mathrm{h}_1(\mathrm{t}) + \mathrm{w}_2\mathrm{h}_2(\mathrm{t})| \tag{3}$$

where $W_N = \left\{e^{-j2\pi(n-1)/2^N}/\sqrt{2} : n = 1, 2, ..., 2^N\right\}$.

The performance of the algorithm in the presence of feedback delay is given by the expected SNR gain $\gamma = E\left\{|\mathrm{h}_1(\mathrm{t}) + \widehat{\mathrm{w}}_2\mathrm{h}_2(\mathrm{t})|^2\right\}$, which provides the upper bound for the system capacity. The expected SNR gain for a single user is calculated in [4] as

$$\gamma = 1 + \frac{\pi c_N}{4} J_0(\frac{2\pi v \tau}{\lambda})^2 \tag{4}$$

where $c_N = \frac{2^N}{\pi} sin \frac{\pi}{2^N}$, $J_0(.)$ is the Bessel function of zero order and v, λ and τ refer to the user velocity, carrier wavelength and feedback delay respectively.

Now in our analysis, we are interested in calculating the average SNR gain for the entire user group. In simple terms, we derive the average SNR averaged over velocity range of a particular group. Mathematically, the average SNR for a user group can be written as

$$\gamma_{group} = \int_{v_{\min}}^{v_{\max}} f(v)(1 + \frac{\pi c_N}{4} J_0(\frac{2\pi v \tau}{\lambda})^2) dv \tag{5}$$

where $v_{\min}$ and $v_{\max}$ are the minimum and maximum velocity respectively of that particular user group. Utilizing the velocity distributions for the three user groups give in section III, we derive the average SNR gain expressions for each group.

A. Pedestrian User Group

Proposition A.1: The average SNR gain for the pedestrian user group is given by

$$\begin{aligned}\gamma_{\text{pedestrian}} =& a v_{\text{max,ped}} + \frac{b}{2} v_{\text{max,ped}}^2 + \frac{\pi c_N b}{8} v_{\text{max,ped}}^2 \\ & \left[J_0(\frac{2\pi\tau}{\lambda} v_{\text{max,ped}})^2 + J_1(\frac{2\pi\tau}{\lambda} v_{\text{max,ped}})^2 \right] + \\ & \frac{a\pi c_N}{4} v_{\text{max,ped}} \, 2F3(0.5, 0.5; 1, 1.5, 1; \\ & -(\frac{2\pi\tau}{\lambda} v_{\text{max,ped}})^2)\end{aligned} \tag{6}$$

where $J_1(.)$ is Bessel function of first order and $2F3(\cdot)$ is mathematical function called hypergeometric function [8].

Proof: For pedestrian user group, Equation 8 can be written as

$$\begin{aligned}\gamma_{\text{pedestrian}} =& \int_0^{v_{\max,\text{ped}}} (a+vb)(1+\frac{\pi c_N}{4}J_0(\frac{2\pi v\tau}{\lambda})^2)dv \\ =& a\int_0^{v_{\max,\text{ped}}} dv + b\int_0^{v_{\max,\text{ped}}} vdv+ \\ & an^1\int_0^{v_{\max,\text{ped}}} J_0(m^1 v)^2 dv+ \\ & bn^1\int_0^{v_{\max,\text{ped}}} vJ_0(m^1 v)^2 dv\end{aligned}$$

where $n^1 = \frac{\pi c_N}{4}$ and $m^1 = \frac{2\pi\tau}{\lambda}$. Now definite integration gives the required expression:

$$\begin{aligned}\gamma_{\text{pedestrian}} =& av_{\max,\text{ped}} + \frac{b}{2}v_{\max,\text{ped}}^2 + \frac{\pi c_N b}{8}v_{\max,\text{ped}}^2 \\ & \left[J_0(\frac{2\pi\tau}{\lambda}v_{\max,\text{ped}})^2 + J_1(\frac{2\pi\tau}{\lambda}v_{\max,\text{ped}})^2\right]+ \\ & \frac{a\pi c_N}{4}v_{\max,\text{ped}}\, 2F3(0.5,0.5;1,1.5,1; \\ & -(\frac{2\pi\tau}{\lambda}v_{\max,\text{ped}})^2)\end{aligned}$$

Proposition A.2: The upper bound of the average capacity for the pedestrian user group can be calculated using the average SNR as

$$C_{\text{pedestrian}} = log(1+\gamma_{\text{pedestrian}}) \tag{7}$$

B. Urban User Group

Proposition B.1: The average SNR gain for the urban user group is given by

$$\begin{aligned}\gamma_{urban} =& [av_{\max,\text{urb}} + \frac{a\pi c_N}{4}v_{\max,\text{urb}} \\ & 2F3(0.5,0.5;1,1.5,1;-(\frac{2\pi\tau}{\lambda}v_{\max,\text{urb}})^2)] \\ & -[av_{\min,\text{urb}} + \frac{a\pi c_N}{4}v_{\min,\text{urb}} \\ & 2F3(0.5,0.5;1,1.5,1;-(\frac{2\pi\tau}{\lambda}v_{\min,\text{urb}})^2)]\end{aligned} \tag{8}$$

Proof: For urban user group, Equation 8 can be written as

$$\begin{aligned}\gamma_{\text{urban}} =& \int_{v_{\min,\text{urb}}}^{v_{\max,\text{urb}}} a(1+\frac{\pi c_N}{4}J_0(\frac{2\pi v\tau}{\lambda})^2)dv \\ =& a\int_{v_{\min,\text{urb}}}^{v_{\max,\text{urb}}} dv+ \\ & an^1\int_{v_{\min,\text{urb}}}^{v_{\max,\text{urb}}} J_0(m^1 v)^2 dv\end{aligned}$$

Definite integration gives the required expression:

$$\begin{aligned}\gamma_{\text{urban}} =& [av_{\max,\text{urb}} + \frac{a\pi c_N}{4}v_{\max,\text{urb}} \\ & 2F3(0.5,0.5;1,1.5,1;-(\frac{2\pi\tau}{\lambda}v_{\max,\text{urb}})^2)] \\ & -[av_{\min,\text{urb}} + \frac{a\pi c_N}{4}v_{\min,\text{urb}} \\ & 2F3(0.5,0.5;1,1.5,1;-(\frac{2\pi\tau}{\lambda}v_{\min,\text{urb}})^2)]\end{aligned}$$

Proposition B.2: The upper bound of the average capacity for the urban user group can be calculate using the average SNR as

$$C_{\text{urban}} = log(1+\gamma_{\text{urban}}) \tag{9}$$

C. High-velocity User Group

Proposition C.1: The average SNR gain for the high-velocity user group is given by

$$\begin{aligned}\gamma_{\text{high}} =& [\frac{a}{2}v_{\max,\text{high}}^2 + \frac{\pi c_N a}{8}v_{\max,\text{high}}^2 \\ & \left[J_0(\frac{2\pi\tau}{\lambda}v_{\max,\text{high}})^2 + J_1(\frac{2\pi\tau}{\lambda}v_{\max,\text{high}})^2\right]] \\ & -[\frac{a}{2}v_{\min,\text{high}}^2 + \frac{\pi c_N a}{8}v_{\min,\text{high}}^2 \\ & \left[J_0(\frac{2\pi\tau}{\lambda}v_{\min,\text{high}})^2 + J_1(\frac{2\pi\tau}{\lambda}v_{\min,\text{high}})^2\right]]\end{aligned} \tag{10}$$

Proof: For high-velocity user group, Equation 8 can be written as

$$\begin{aligned}\gamma_{\text{high}} =& \int_{v_{\min,\text{high}}}^{v_{\max,\text{high}}} av(1+\frac{\pi c_N}{4}J_0(\frac{2\pi v\tau}{\lambda})^2)dv \\ =& a\int_{v_{\min,\text{high}}}^{v_{\max,\text{high}}} vdv+ \\ & an^1\int_{v_{\min,\text{high}}}^{v_{\max,\text{high}}} J_0(m^1 v)^2 dv\end{aligned}$$

Definite integration gives the required expression:

$$\begin{aligned}\gamma_{\text{high}} =& [\frac{a}{2}v_{\max,\text{high}}^2 + \frac{\pi c_N a}{8}v_{\max,\text{high}}^2 \\ & \left[J_0(\frac{2\pi\tau}{\lambda}v_{\max,\text{high}})^2 + J_1(\frac{2\pi\tau}{\lambda}v_{\max,\text{high}})^2\right]] \\ & -[\frac{a}{2}v_{\min,\text{high}}^2 + \frac{\pi c_N a}{8}v_{\min,\text{high}}^2 \\ & \left[J_0(\frac{2\pi\tau}{\lambda}v_{\min,\text{high}})^2 + J_1(\frac{2\pi\tau}{\lambda}v_{\min,\text{high}})^2\right]]\end{aligned}$$

Proposition C.2: The upper bound of the average capacity for the high-velocity user group can be calculate using the average SNR as

$$C_{\text{high}} = log(1+\gamma_{\text{high}}) \tag{11}$$

We plot the analytical expressions derived here and compare them with the simulation results for validation.

In Figure 3, we plot the average SNR gain and upper capacity bound for each group with number of feedback bits $N = 2$, carrier frequency of 2MHz, the maximum velocity for pedestrian, urban and high-velocity groups are 10, 80, 130 kmph respectively and the feedback follows QPSK constellation. As mentioned earlier, we use Jake's model with finite number of oscillators to replicate the time-variation of the channel [6]. As can be seen in Figure 3, the analytical results seem to be overlapping with the simulation results except for some points. This is because of the Jake's model assumption of having infinite oscillators which is not the case in practice. For simulations, we have used 8 oscillators in our Jake's model. The second key observation is the behavior of

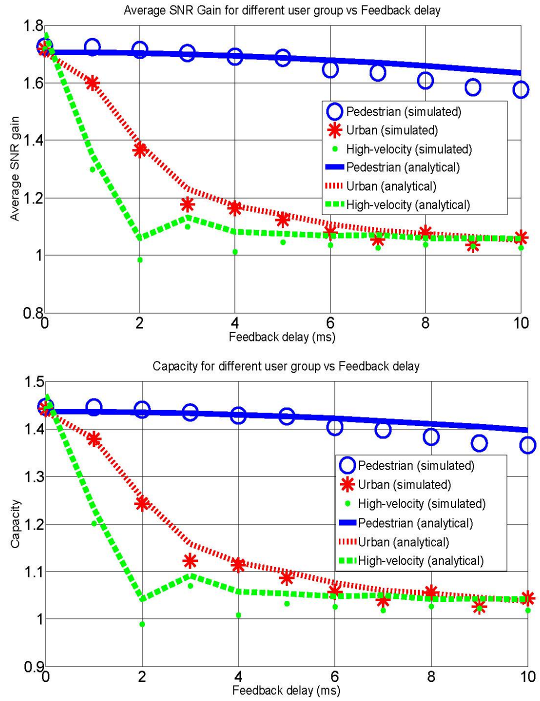

Fig. 3. Feedback Delay Analysis

different groups with respect to feedback delay. In the case of pedestrian user group, the gains are mostly unaffected and for the urban and high-velocity groups, the performance degrades significantly with feedback delay. The main explanation for this behavior is the rate of change of channel with time. The higher the velocity group, more fast is the variation. These results give us the main motivation to propose a scheduling algorithm that can schedule feedback users based on their group in order to adjust the feedback delay according to the velocity.

V. Scheduling of Feedback Users

Let us consider a multi-group cellular environment employing closed-loop transmit beamforming. We have a single cell consisting of different users belonging to all three groups. We schedule all the feedback users in a TDMA way and serve them in the same order for downlink. Figure 4 shows our proposed scheduler for a practical system such as 3GPP. In most of the practical systems, the BS decides when the user must send the feedback [9]. Therefore as can be seen in Figure 4, our velocity-dependent scheduler is basically divided into 2 stages of feedback. In the first stage, the users transmit quantized information about their velocity group to the BS. Based on the velocity group, the BS prioritizes the users belonging to high-velocity group. Here we can realize the importance of user categorization in reducing the feedback overhead. When we define a user in particular velocity group, we do not need the exact information about the user velocity and thus no need for frequent update of their constantly changing velocity. The user only needs to update the BS with information about their group which would be less frequent than the specific velocity update. Therefore we propose velocity-group dependent feedback scheduling.

A. Velocity-group Dependent Feedback User Scheduling

We observed in Section IV that the performance of pedestrian user group is least affected by feedback delay and that of high-velocity group is most affected. Therefore we propose a feedback scheduler that is aware of the user group and schedules them in the following order:*high-velocity, urban and pedestrian users*. As a result the users belonging to the highest velocity group will experience least feedback delay and the users belonging to the lowest velocity group will have maximum feedback delay. Therefore, we can attain optimal system performance. Suppose, X users are pedestrian Y are urban and Z are high-velocity users, such that $X+Y+Z=K$.Then the average SNR gain using this velocity-group dependent feedback user algorithm can be given as

$$\gamma_{\text{average}} = \frac{\sum \gamma_{\text{pedestrian,x}} + \sum \gamma_{\text{urban,y}} + \sum \gamma_{\text{high-velocity,z}}}{K} \tag{12}$$

Where $\gamma_{\text{pedestrian,x}}$, $\gamma_{\text{urban,y}}$ and $\gamma_{\text{high-velocity,z}}$ can be derived from Equations 6, 8 and 10 respectively.

VI. Comparative Results

In this section, we compare the performance of our scheduling algorithm with the random feedback scheduling in terms of average SNR gain and the capacity bound for the system. Table 1 shows the simulation parameters. Figure 5 compares

Parameter	Value
User velocities (v_k in kmph)	2, 5, 10, 25, 36, 40, 55, 75, 94, 120
Number of pedestrian users	3
Number of urban users	5
Number of high-velocity users	2
User feedback delays (τ_k in ms)	1, 3, 5, 7, 9, 11, 13, 15, 17, 19
Carrier frequency	2MHz

TABLE I
Simulation Parameters

the performance in terms of average SNR gain and upper capacity bound against different value of feedback bits/user. As can be seen from Figure 5, we observe considerable gain in comparison to random scheduling. In addition, we have also compared the performance for each group independently when applying the velocity-group dependent scheduling. It can be observed that even though the pedestrian group is scheduled at the last, still it has the best performance. The urban user group's performance is better than that of random scheduling and high-velocity group performs almost same as random scheduling. These performance improvements are achieved as a result of this scheduling algorithm which adapts according to the user velocity groups.

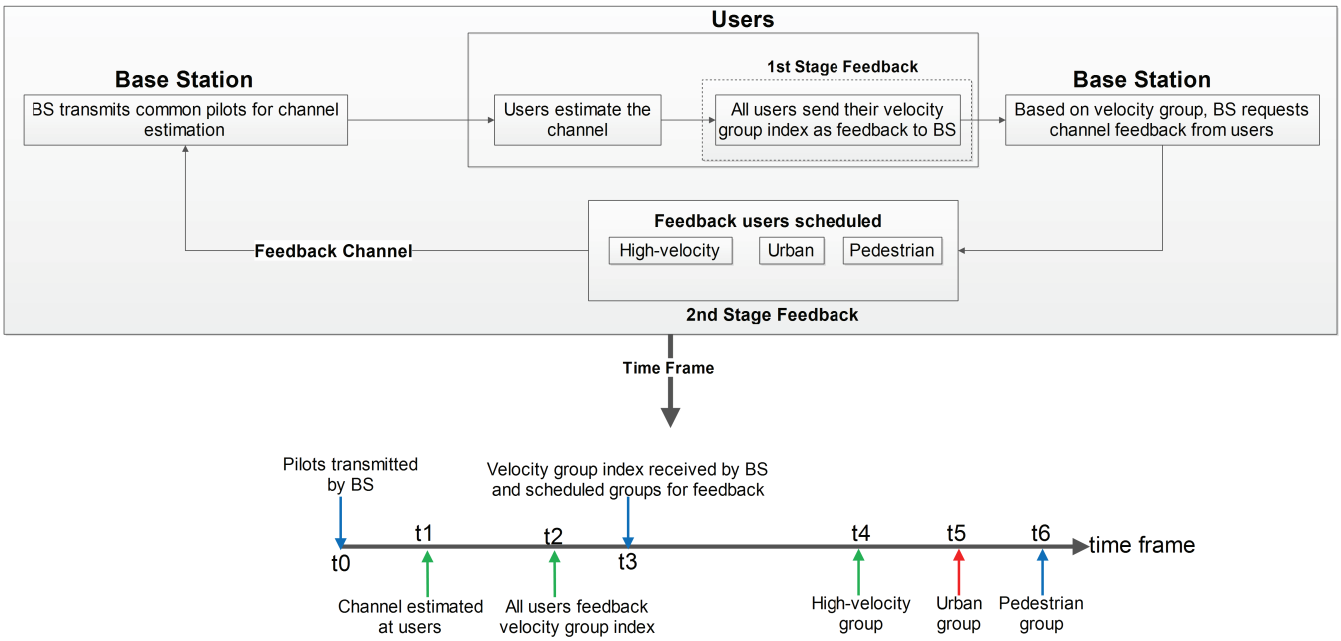

Fig. 4. Velocity-dependent Feedback Scheduling

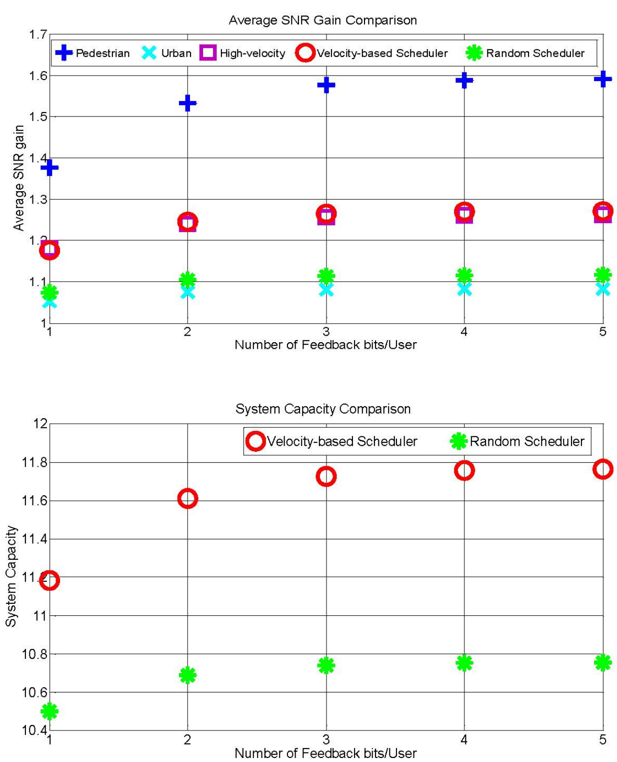

Fig. 5. Performance Comparison

VII. Conclusion

In this work, we utilized the analysis of feedback delay effect in closed-loop transmit beamforming and proposed a velocity-group dependent feedback scheduling algorithm for performance improvement in multi-user system. We adaptively schedule the users for sending feedback depending upon their velocity group to minimize the impact of feedback delay on the system and observe considerable gains in terms of average SNR and capacity upper bounds while taking into account the feedback overhead.

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