Degrees of Freedom of Time-Correlated Broadcast Channels with Delayed CSIT: The MIMO Case

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***Abstract*—The two-user Multiple-Input Multiple-Output (MIMO) broadcast channel (BC) with arbitrary antenna configuration is considered, in which the transmitter obtains (i) *delayed* channel state information (CSI) from a latency-prone feedback channel as well as (ii) imperfect *current* CSI, e.g., from prediction based on these past channel samples. The degrees of freedom (DoF) region under such a setting is fully characterized as a function of a prediction quality exponent. This work extends prior work, previously limited to MISO, to fully general antenna settings. An intriguing by-product of our results is to reveal the benefits of dealing with an asymmetric multi-user MIMO configuration (i.e., one in which terminals do not have the same number of antennas) in the case of non-perfect CSIT (e.g., caused by feedback delays or limited preciseness).** [1]

I. Introduction

The capacity region of the two-user Multiple-Input Multiple-Output (MIMO) broadcast channel (BC) with perfect channel state information at the transmitter (CSIT) was established in [1], which suggests that the sum degrees of freedom (DoF) can be as many as the rank of the overall channel with *both* receivers considered. On the other extreme, in the absence of CSIT, the DoF region largely collapses and is constrained by the number of antennas at *either* receiver [2], [3]. The large gap between perfect instantaneous CSIT and no CSIT cases indicates the importance of the quality of CSI both in terms of preciseness and timeliness. In practice, however, the acquisition of perfect and instantaneous CSI at the transmitter is difficult, if not impossible, especially for fast fading channels. While the preciseness of CSIT has been widely investigated [4], [5] (and references therein), the timeliness of available CSIT was relatively less exposed. The feedback delay renders the available CSIT possibly uncorrelated with the current true channel if it exceeds the channel coherence time. It would seem intuitively that the benefits of such feedback information (referred to as "delayed CSIT") are not exploitable.

Recently, this commonly accepted viewpoint was challenged by an interesting work [6], in which a novel scheme (termed here as "MAT") was proposed for the MISO BC to demonstrate even the completely outdated channel feedback is still useful. By establishing the usefulness of even completely outdated CSI, strictly better DoF than what is obtained without any CSIT are achieved. Most recently, generalizations to the MIMO BC [7] settings, among others, were also addressed, where the DoF region is fully characterized with arbitrary antenna configurations, again establishing DoF strictly beyond the ones obtained without CSIT. Note that other recent interesting lines of work combining instantaneous and delayed forms of feedback were reported in [8], [9].

Although fascinating from a conceptual point of view, these works made a pessimistic assumption that the channel is independent and identically distributed (i.i.d.) across time, where the delayed CSIT bears no correlation with the current channel realization. On the contrary, during the coherence time, the past channel realizations are somehow temporally correlated to the current one, and hence can provide *some* information about current CSIT, albeit imperfect. Together with the delayed CSIT, the benefit of such imperfect current CSIT was first exploited in [10] for the MISO BC whereby a novel transmission scheme achieves a strictly larger DoF than the i.i.d. delayed CSIT case in [6]. The full characterization of the optimal DoF was later reported in [11], [12]. The key idea behind the schemes (referred to as "α-MAT") in [10]–[12] relies on a combination of MAT alignment together with the use of approximate zero-forcing (ZF) precoders, and the forwarding of the residual interferences in a *compressed* fashion.

However, the generalization to the case of multiple receive antennas is unfortunately not a trivial step. The main challenge lies in two aspects: (a) the additional spatial dimension at the receiver side enables to cancel a certain amount of residual interference (generated by the impreciseness of transmit precoding) and thus intuitively is expected to enhance the achievable DoF, whereas it also increases the probability to generate more interferences at unintended receiver, and (b) the asymmetry of receive antenna configurations, which results in the discrepancy of common-message-decoding capability at different receivers. In particular, the total number of streams that can be delivered as common messages to both receivers is inevitably limited by the weaker one (i.e., with fewer antennas). Such a constraint prevents the system from achieving the optimal DoF of the asymmetric case by simply extending the previous schemes we found in [13].

To counter this challenge, in this paper, we develop new strategies balancing the discrepancy of common-message-decoding capability at two receivers. We hereby fully charac-

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$$d_1 \leq \min\{M, N_1\}, \quad d_2 \leq \min\{M, N_2\}, \quad d_1 + d_2 \leq \min\{M, N_1 + N_2\}, \tag{1a}$$

$$\frac{d_1}{\min\{M, N_1\}} + \frac{d_2}{\min\{M, N_1 + N_2\}} \leq 1 + \frac{\min\{M, N_1 + N_2\} - \min\{M, N_1\}}{\min\{M, N_1 + N_2\}} \alpha, \tag{1b}$$

$$\frac{d_1}{\min\{M, N_1 + N_2\}} + \frac{d_2}{\min\{M, N_2\}} \leq 1 + \frac{\min\{M, N_1 + N_2\} - \min\{M, N_2\}}{\min\{M, N_1 + N_2\}} \alpha, \tag{1c}$$

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terize the DoF region of MIMO BC, by a unified framework under the concept of *block-Markov encoding*. This concept was first introduced in [14] for the relay channels, and later revisited in multiple access channels [15], and recently in MISO BC with "mixed" CSIT of asymmetric current CSIT qualities [16]. Generally speaking, in the block-Markov encoding process, the symbols sent in each block are functions of a new message in the current block as well as the past messages in previous blocks.

More specifically, we obtain the following key results:

- We establish outer bounds on the DoF region for the two-user MIMO BC under this setting, as a function of the current CSIT quality exponent. In addition to the genie-aided bounding techniques and the application of the extremal inequality in [11], we develop a set of upper and lower bounds of ergodic capacity for MIMO channels, which is essential for the MIMO case but not extendable from MISO.
- We propose a unified framework with a set of multi-slot transmission protocols which achieves the key vertices of the outer bound region. The proposed schemes still rely on a combination of MAT alignment and the approximate ZF precoding. This principle is a building stone in several previous works [10]–[12], however, here this idea is complemented with block-Markov encoding and backward decoding that is made necessarily by the configuration asymmetry.

It is worth noting that our results embrace the previously reported particular cases: the perfect CSIT setting [1], the pure delayed CSIT setting [7], and the special MISO case [10]–[12], [16].

II. System Model and Main Result

For a two-user (M, N_1, N_2) MIMO broadcast channel with M antennas at the transmitter (Tx) and N_1, N_2 antennas at two receivers (Rxs), respectively, the discrete time signal model is

$$\boldsymbol{y}(t) = \boldsymbol{H}(t)\boldsymbol{x}(t) + \boldsymbol{e}(t), \tag{2}$$

$$\boldsymbol{z}(t) = \boldsymbol{G}(t)\boldsymbol{x}(t) + \boldsymbol{b}(t) \tag{3}$$

for any time instant t, where $\boldsymbol{H}(t) \in \mathbb{C}^{N_1 \times M}$ and $\boldsymbol{G}(t) \in \mathbb{C}^{N_2 \times M}$ are the channel matrices for two Rxs; $\boldsymbol{e}(t), \boldsymbol{b}(t) \sim \mathcal{N}_{\mathbb{C}}(0, \mathbf{I})$ are the normalized noise vectors at the respective Rxs and are independent of channel matrices and transmitted signals; the coded input signal $\boldsymbol{x}(t) \in \mathbb{C}^{M \times 1}$ is subject to the power constraint $\mathbb{E}\left(\|\boldsymbol{x}(t)\|^2\right) \leq P$, $\forall t$.

Define $\boldsymbol{S}(t) \triangleq \begin{bmatrix} \boldsymbol{H}(t) \\ \boldsymbol{G}(t) \end{bmatrix}$ as the overall channel matrices. The element of channel matrix is assumed to be drawn from a stationary and ergodic random process. At each time instant t, we assume Tx knows perfectly the delayed CSI with unit delay, i.e., $\boldsymbol{S}^{t-1}$, and obtains an imperfect estimate of the current CSI $\hat{\boldsymbol{S}}(t)$. This current CSIT estimate can be modeled as [11]

$$\boldsymbol{H}(t) = \hat{\boldsymbol{H}}(t) + \tilde{\boldsymbol{H}}(t), \tag{4}$$

$$\boldsymbol{G}(t) = \hat{\boldsymbol{G}}(t) + \tilde{\boldsymbol{G}}(t) \tag{5}$$

where the estimate $\hat{\boldsymbol{H}}(t)$ (resp. $\hat{\boldsymbol{G}}(t)$) and estimation error $\tilde{\boldsymbol{H}}(t)$ (resp. $\tilde{\boldsymbol{G}}(t)$) are independent, and $\tilde{\boldsymbol{H}}(t)$ (resp. $\tilde{\boldsymbol{G}}(t)$) is independent and identically distributed (i.i.d.) with each entry being assumed to be a zero-mean Gaussian variable with variance σ^2 ($0 \leq \sigma^2 \leq 1$). Further, we assume the following Markov chain

$$(\boldsymbol{S}^{t-1}, \hat{\boldsymbol{S}}^{t-1}) \rightarrow \hat{\boldsymbol{S}}^t \rightarrow \boldsymbol{S}^t, \quad \forall t, \tag{6}$$

which means $\boldsymbol{S}^t$ is independent of $(\boldsymbol{S}^{t-1}, \hat{\boldsymbol{S}}^{t-1})$ conditionally on $\hat{\boldsymbol{S}}^t$. At the receiver side, both Rxs are assumed to have access to $\boldsymbol{S}(t)$ and $\hat{\boldsymbol{S}}(t)$ at each slot t.

As it was established in previous works [10], [11], the imperfect current CSIT has beneficial value (in terms of improving the DoF) only if the CSIT error decays at least exponentially with the SNR or faster. Thus it is reasonable to study the regime by which the CSIT quality can be parameterized by an indicator α such that:

$$\alpha \triangleq -\lim_{P \to \infty} \frac{\log \sigma^2}{\log P}. \tag{7}$$

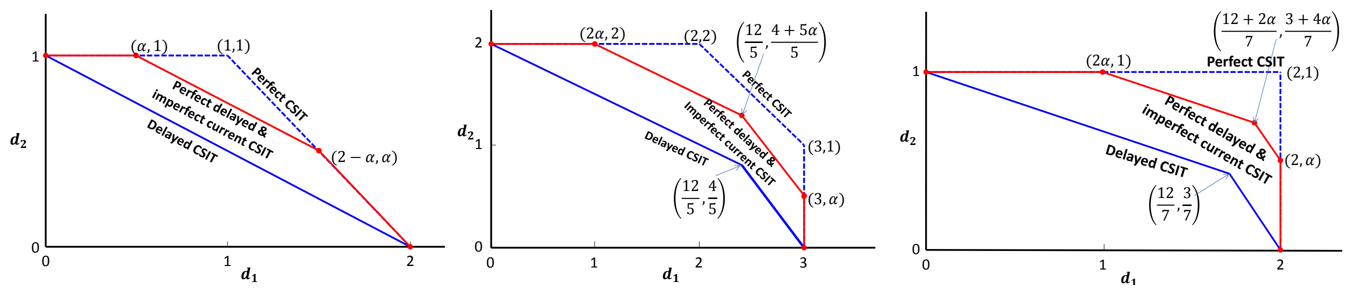
with $\alpha \geq 0$. This α indicates the quality of current CSIT at high SNR. While $\alpha = 0$ reflects the case with no current CSIT, $\alpha \to \infty$ corresponds to that with perfect instantaneous CSIT. Here, we focus on the case $\alpha \in [0, 1]$ since the quality of the imperfect current CSIT is sufficient to avoid the DoF loss when $\alpha \geq 1$. The connections between the above model and the linear prediction over existing temporally-correlated channel models with prescribed user mobility are highlighted in [10], [11].

The capacity region $\mathcal{C}$ is defined as the set of all achievable rate pairs and the DoF region is defined as the set of all pairs of achievable DoF $(d_1, d_2) \in \mathbb{R}_+^2$, where $d_i = \limsup_{P \to \infty} \sup_{(R_1, R_2) \in \mathcal{C}} \frac{R_i}{\log P}$.

Our main result is given by the following theorem.

Theorem 1. *For the two-user (M, N_1, N_2) MIMO BC with delayed and current imperfect CSIT, the DoF region $\{(d_1, d_2) | (d_1, d_2) \in \mathbb{R}_+^2\}$ is characterized by eq-(1), where $\alpha \in [0, 1]$ indicates the current CSIT quality exponent.*

Proof: See Appendix for a sketch of the converse proof, the next section for toy examples of achievability proof, and



(a) Case $N_1 \geq M > N_2$: $(2,2,1)$ (b) Case $N_1+N_2 > M > N_1 \geq N_2$: $(4,3,2)$ (c) Case $M \geq N_1+N_2$: $(3,2,1)$

Fig. 1: The DoF region for the two-user MIMO BC.

[13], [17] for full details on the more general settings of MIMO BC with arbitrary antenna configurations. ■

Example 1. *The DoF regions of some special examples are provided in Fig. 1 together with those with pure perfect delayed CSIT, and with perfect and instantaneous CSIT.*

A. Benefits of Asymmetric MIMO Settings

The above results suggest some interesting benefits arising from asymmetry across the number of antennas at the terminal side. From these results, it follows that the asymmetric antenna configuration achieves larger sum DoF than its symmetric counterpart. More specifically, when $M \geq N_1 + N_2$, given total N receive antennas, there is a non-negligible gain to concentrate $N-1$ antennas to one receiver especially when the imperfect current channel quality α is poor. The reason why an asymmetric multi-user configuration is preferable to a symmetric one is due to the fact that asymmetry draws the multi-user network closer to a single-user one (in terms of how the DoF are split across the users, with the many-antenna user carrying most of the data rate). Since single-user MIMO systems are insensitive to CSIT quality from a DoF point of view, an asymmetric multi-user network also exhibits a greater robustness with respect to feedback quality.

III. Achievability

Due to the page limit, we take $(3,2,1)$ MIMO BC for the example, and describe the achievability of the corner point $\left(\frac{12+2\alpha}{7},\frac{3+\alpha}{7}\right)$ alone in detail. For other cases, please refer to [17] for details. Before proceeding further, we define

$$\boldsymbol{V}(t) \in \mathcal{R}\{\hat{\boldsymbol{H}}(t)\} \in \mathbb{C}^{3\times 2},\ \boldsymbol{V}^{\perp}(t) \in \mathcal{N}\{\hat{\boldsymbol{H}}(t)\} \in \mathbb{C}^{3\times 1}$$
$$\boldsymbol{U}(t) \in \mathcal{R}\{\hat{\boldsymbol{G}}(t)\} \in \mathbb{C}^{3\times 1},\ \boldsymbol{U}^{\perp}(t) \in \mathcal{N}\{\hat{\boldsymbol{G}}(t)\} \in \mathbb{C}^{3\times 2}$$

where for example $\mathbb{E}\left(\|\boldsymbol{H}(t)\boldsymbol{V}^{\perp}(t)\|^2\right) \sim P^{-\alpha}$. Note that $\mathcal{R}\{\cdot\}$ and $\mathcal{N}\{\cdot\}$ are defined as the range and null spaces spanned by column vectors. Let $\boldsymbol{g}^{\mathsf{H}}$ denote $\boldsymbol{G} \in \mathbb{C}^{1\times 2}$ for conciseness. We reserve $\boldsymbol{u}(t) = [u_1(t)\ u_2(t)\ u_3(t)]^{\mathsf{T}}$ and $\boldsymbol{v}(t)$ [or $v(t)$] to denote the private messages intended to Rx-1 and Rx-2, respectively. We also denote for example the last two elements of $\boldsymbol{u}(t)$ by $\boldsymbol{u}_{[2:3]}(t)$.

With pure delayed CSIT, it takes 7 time slots to achieve DoF pair $(\frac{12}{7},\frac{3}{7})$ [7]. Together with imperfect current CSIT, we propose a novel scheme to achieve DoF pair $\left(\frac{12+2\alpha}{7},\frac{3+\alpha}{7}\right)$, with expanded number of time slots from 7 to $7B$, where B is defined as a large enough positive integer. The transmission protocol consists of three phases and is detailed as follows.

Phase-1: This phase includes B time slots. Specifically, in Slot-t $(1 \leq t \leq B)$, we transmit

$$\boldsymbol{x}(t) = \begin{bmatrix}\boldsymbol{U}(t) & \boldsymbol{U}^{\perp}(t)\end{bmatrix}\boldsymbol{u}(t) + \begin{bmatrix}\boldsymbol{V}(t) & \boldsymbol{V}^{\perp}(t)\end{bmatrix}\boldsymbol{v}(t) \tag{8}$$

where $\boldsymbol{u}(t), \boldsymbol{v}(t) \in \mathbb{C}^{3\times 1}$ and thus receive

$$\boldsymbol{y}(t) = \boldsymbol{H}(t)\begin{bmatrix}\boldsymbol{U}(t) & \boldsymbol{U}^{\perp}(t)\end{bmatrix}\boldsymbol{u}(t) + \underbrace{\boldsymbol{H}(t)\begin{bmatrix}\boldsymbol{V}(t) & \boldsymbol{V}^{\perp}(t)\end{bmatrix}\boldsymbol{v}(t)}_{\boldsymbol{\eta}_1^{[1]}(t)\sim P^{1-\alpha}}$$
$$z(t) = \underbrace{\boldsymbol{g}^{\mathsf{H}}(t)\begin{bmatrix}\boldsymbol{U}(t) & \boldsymbol{U}^{\perp}(t)\end{bmatrix}\boldsymbol{u}(t)}_{\eta_2^{[1]}(t)\sim P^{1-\alpha}} + \boldsymbol{g}^{\mathsf{H}}(t)\begin{bmatrix}\boldsymbol{V}(t) & \boldsymbol{V}^{\perp}(t)\end{bmatrix}\boldsymbol{v}(t)$$

where $\boldsymbol{\eta}_1^{[1]}(t) \in \mathbb{C}^{2\times 1}$ and $\eta_2^{[1]}(t)$ are overheard interference by Rx-1 and Rx-2, respectively. With power allocation $\mathbb{E}\left(|u_1(t)|^2\right) = P^{1-\alpha}$, $\mathbb{E}\left(\|\boldsymbol{v}_{[1:2]}(t)\|^2\right) = 2P^{1-\alpha}$, $\mathbb{E}\left(\|\boldsymbol{u}_{[2:3]}(t)\|^2\right) = P$, $\mathbb{E}\left(|v_3(t)|^2\right) = P$, and rate $R_{u_1(t)} = (1-\alpha)\log P$, $R_{\boldsymbol{v}_{[1:2]}(t)} = 2(1-\alpha)\log P$, $R_{\boldsymbol{u}_{[2:3]}(t)} = 2\log P$, $R_{v_3(t)} = \log P$, we have the power of interferences $\mathbb{E}\left(\|\boldsymbol{\eta}_1^{[1]}(t)\|^2\right) \sim P^{1-\alpha}$ and $\mathbb{E}\left(|\eta_2^{[1]}(t)|^2\right) \sim P^{1-\alpha}$, and the efficient transmission rates $(3-\alpha)\log P$ and $(3-2\alpha)\log P$ for Rx-1 and Rx-2, respectively. Such a power allocation reduces the interference power and makes it possible to "compress" them into $(1-\alpha)\log P$ bits for each element, expecting less channel resource consumed during the retransmission. Instead of forwarding the interference directly as MAT alignment always did, we first quantize them into $(1-\alpha)\log P$ bits each using scalar quantization with negligible quantization error, encode the digitalized interference with Gaussian channel codes, and then transmit to both Rxs with reduced rate. With a slight abuse of notation, we use $\hat{\boldsymbol{\eta}}$ (or $\hat{\eta}$) to denote the channel coded version of the compress indices, not the compressed version of the interference itself. For description brevity, we omit this procedure of source and channel coding, and the details can be found in our previous paper [11].

Phase-2: This phase consists of three subphases, each of which includes $(B+1)$ time slots. The transmitted signal in Slot-1, as the initialization, of subphase-k $(k=2,3,4)$ can be given by (with subphase indies omitted for conciseness)

$$\boldsymbol{x}(1) = \begin{bmatrix}\boldsymbol{U}(1) & \boldsymbol{U}^{\perp}(1)\end{bmatrix}\boldsymbol{u}(1) + \boldsymbol{V}^{\perp}(1)v(1) \tag{9}$$

where $\boldsymbol{u}(1) \in \mathbb{C}^{3\times 1}$ with power allocation $\mathbb{E}\left(|u_1(1)|^2\right) = P^{1-\alpha}$, $\mathbb{E}\left(\|\boldsymbol{u}_{[2:3]}(1)\|^2\right) = P$ and rate $R_{u_1(1)} = (1-\alpha)\log P$,

$R_{\boldsymbol{u}_{[2:3]}(1)} = 2\log P$, and $v(1)$ is a scalar with power allocation $\mathbb{E}\left(|v(1)|^2\right) = P^{\alpha}$ and rate $R_{v(1)} = \alpha \log P$. It gives the received signals as (with the terms at noise level omitted)

$$\begin{aligned} \boldsymbol{y}(1) &= \boldsymbol{H}(1)\begin{bmatrix}\boldsymbol{U}(1) & \boldsymbol{U}^{\perp}(1)\end{bmatrix}\boldsymbol{u}(1) \\ z(1) &= \underbrace{\boldsymbol{g}^{\mathsf{H}}(1)\begin{bmatrix}\boldsymbol{U}(1) & \boldsymbol{U}^{\perp}(1)\end{bmatrix}\boldsymbol{u}(1)}_{\eta_2^{[k]}(1)\sim P^{1-\alpha}} + \underbrace{\boldsymbol{g}^{\mathsf{H}}(1)\boldsymbol{V}^{\perp}(1)v(1)}_{P^{\alpha}}. \end{aligned}$$

Note that the efficient transmission rates of both Rxs are $(3-\alpha)\log P$ and $\alpha \log P$, respectively. Similarly, we quantize the overheard interference, and forward them as common messages with power P in a digital fashion in the next slot.

In Slot-t $(2 \le t \le B)$, the objective of transmission is to retransmit the quantized interference as a common message, together with which we deliver some more private messages to fully utilize the channel. To this end, we transmit

$$\boldsymbol{x}(t) = \boldsymbol{w}(t)\hat{\eta}_2^{[k]}(t-1) + \begin{bmatrix}\boldsymbol{U}(t) & \boldsymbol{U}^{\perp}(t)\end{bmatrix}\boldsymbol{u}(t) + \boldsymbol{V}^{\perp}(t)v(t) \tag{10}$$

with $R_{\hat{\eta}_2^{[k]}(t-1)} = (1-\alpha)\log P$, $R_{u_1(t)} = (1-\alpha)\log P$, $R_{\boldsymbol{u}_{[2:3]}(t)} = 2\log P$, $R_{v(t)} = \alpha \log P$ and power allocation $\mathbb{E}\left(|\hat{\eta}_2^{[k]}(t-1)|^2\right) = P$, $\mathbb{E}\left(|u_1(t)|^2\right) = P^{1-\alpha}$, $\mathbb{E}\left(\|\boldsymbol{u}_{[2:3]}(t)\|^2\right) = P$, $\mathbb{E}\left(|v(t)|^2\right) = P^{\alpha}$. It gives the received signal as

$$\begin{aligned} \boldsymbol{y}(t) &= \boldsymbol{H}(t)\boldsymbol{w}(t)\hat{\eta}_2^{[k]}(t-1) + \boldsymbol{H}(t)\begin{bmatrix}\boldsymbol{U}(t) & \boldsymbol{U}^{\perp}(t)\end{bmatrix}\boldsymbol{u}(t) \\ z(t) &= \underbrace{\boldsymbol{g}^{\mathsf{H}}(t)\boldsymbol{w}(t)\hat{\eta}_2^{[k]}(t-1)}_{P} + \underbrace{\boldsymbol{g}^{\mathsf{H}}(t)\begin{bmatrix}\boldsymbol{U}(t) & \boldsymbol{U}^{\perp}(t)\end{bmatrix}\boldsymbol{u}(t)}_{\eta_2^{[k]}(t)\sim P^{1-\alpha}} \\ &\quad + \underbrace{\boldsymbol{g}^{\mathsf{H}}(t)\boldsymbol{V}^{\perp}(t)v(t)}_{P^{\alpha}}. \end{aligned}$$

Let us imagine: if given $\hat{\eta}_2^{[k]}(t)$ (in turn $\eta_2^{[k]}(t)$ with negligible quantization error), Rx-2 can recover $\hat{\eta}_2^{[k]}(t-1)$ as the common message and $v(t)$ as the private message by successive decoding, while Rx-1 requires another one equation to recover 4 unknowns $\boldsymbol{u}(t)$ and $\hat{\eta}_2^{[k]}(t-1)$ from $\boldsymbol{y}(t)$ and $\eta_2^{[k]}(t)$.

The above procedure lasts for a certain time requiring the knowledge of the new generated interference [cf. $\eta_2^{[k]}(t)$] until the Slot-$(B+1)$. In Slot-$(B+1)$, as a terminating step, we lower down the transmission rate of the private message $\boldsymbol{u}(B+1)$, in order not to cause any additional overheard interference to the unintended Rx and to guarantee the retransmission of $\hat{\eta}_2^{[k]}(B)$ can be decoded here. To this end, we transmit

$$\begin{aligned} \boldsymbol{x}(B+1) &= \boldsymbol{w}(B+1)\hat{\eta}_2^{[k]}(B) + \boldsymbol{U}^{\perp}(B+1)\boldsymbol{u}_{[2:3]}(B+1) \\ &\quad + \boldsymbol{V}^{\perp}(B+1)v(B+1) \end{aligned} \tag{11}$$

with each private symbol being of rate $\alpha \log P$ and power P^{α}. Similarly to the MISO case [11], by successive decoding, both the common message $\hat{\eta}_2^{[k]}(B)$ and private messages are recoverable at interested Rxs.

Going back to Slot-B, Rx-2 can recover $\hat{\eta}_2^{[k]}(B-1)$ and also the private message $v(B)$ by successive decoding, given the knowledge of $\hat{\eta}_2^{[k]}(B)$. Then, go back further and further, recursively. By such a recursive procedure, Rx-2 will have access to $\{\hat{\eta}_2^{[k]}(t), t = B, \cdots, 1\}$ and recover all its private messages, while this recursive procedure is interrupted in Slot-B at Rx-1, who requires another one equation that can be provided by retransmitting $\hat{\eta}_2^{[k]}(B-1)$. In other words, in Slot-t, if $\hat{\eta}_2^{[k]}(t-1)$ is also provided to Rx-1, which will be done in the next phase, then the recursive and joint decoding (or backward decoding) will continue.

Phase-3: This phase consists of 3 subphases, in each of which B time slots are consumed with two interference terms being forwarded to two Rxs in each slot. Before transmission, we concatenate the digitalized interference terms $\hat{\boldsymbol{\eta}}_1^{[1]}(t)$ and $\{\hat{\eta}_2^{[k]}(t), k = 1,2,3,4\}$, $t = 1, \cdots, B$ as

$$\bar{\boldsymbol{\eta}}_1(t) = \begin{bmatrix}\hat{\eta}_{11}^{[1]}(t) \\ \hat{\eta}_2^{[2]}(t)\end{bmatrix},\ \bar{\boldsymbol{\eta}}_2(t) = \begin{bmatrix}\hat{\eta}_{12}^{[1]}(t) \\ \hat{\eta}_2^{[3]}(t)\end{bmatrix},\ \bar{\boldsymbol{\eta}}_3(t) = \begin{bmatrix}\hat{\eta}_2^{[1]}(t) \\ \hat{\eta}_2^{[4]}(t)\end{bmatrix}$$

where $\{\hat{\eta}_2^{[k]}(t), k = 2,3,4\}$ are already known by Rx-2 according to Phase-2. In each slot, we transmit in Slot-t $(1 \le t \le B)$ of subphase-k $(k = 5,6,7)$

$$\boldsymbol{x}(t) = \boldsymbol{\Phi}(t)\bar{\boldsymbol{\eta}}_{k-4}(t) + \boldsymbol{U}^{\perp}(t)\boldsymbol{u}_{[2:3]}(t) + \boldsymbol{V}^{\perp}(t)v(t), \tag{12}$$

where $\boldsymbol{u}_{[2:3]}(t) \in \mathbb{C}^{2\times 1}$, $v(t)$ are with power P^{α} and rate $\alpha \log P$ for each element. By successive decoding, Rx-1 can recover $\bar{\boldsymbol{\eta}}_{k-4}(t)$ $(k = 5,6,7)$ and $\boldsymbol{u}_{[2:3]}(t)$ with two receive antennas whereas Rx-2 can recover $\bar{\boldsymbol{\eta}}_{k-4}(t)$ and $v(t)$ with only one receive antenna since $\hat{\eta}_2^{[k]}(t), k = 2,3,4$ is already known. Hence, both Rxs recover the interested private messages, from which obtain extra $6B\alpha$ and $3B\alpha$ DoF, respectively.

Going back to Phase-2, with side information $\hat{\eta}_2^{[k]}(t-1), k = 2,3,4$, Rx-1 formulates virtual MIMO channels to recover $\boldsymbol{u}(t)$ recursively. Similarly, going back to Phase-1, both Rxs can recover the desired messages with the knowledge of $\hat{\eta}_2^{[1]}(t)$ and $\hat{\boldsymbol{\eta}}_1^{[1]}(t)$. Thus, to sum up, we achieve asymptotically the following DoF pair

$$\begin{aligned} d_1 &= \frac{B(3-\alpha) + 3[B(3-\alpha)+2\alpha] + 6B\alpha}{7B} \overset{B\to\infty}{=} \frac{12+2\alpha}{7} \\ d_2 &= \frac{B(3-2\alpha) + 3B\alpha + 3B\alpha}{7B} \overset{B\to\infty}{=} \frac{3+4\alpha}{7}. \end{aligned}$$

IV. Conclusion

The optimal DoF region of the two-user time-correlated MIMO BC with arbitrary antenna configuration has been characterized in the presence of perfect delayed and imperfect current CSIT. The results further highlight the benefits from the exploitation of the channel time correlation, and also reveal that asymmetric antenna deployments are preferable to enhance the robustness for multi-user channels towards channel uncertainty. Interesting extensions should include the taking into account of training/feedback overhead in the performance enhancement as was recently done for the pure MAT setting [18].

Appendix

To obtain the outer bounds, we adopt a genie-aided model reminiscened in [7], by assuming that (i) both Rxs know the CSI $\boldsymbol{S}(t)$ perfectly and instantaneously as well as the imperfect

$$\frac{1}{p}\sum_{t=1}^{n} h(\boldsymbol{y}(t),\boldsymbol{z}(t)|\mathcal{U}(t),\boldsymbol{S}(t)) - \frac{1}{q}\sum_{t=1}^{n} h(\boldsymbol{z}(t)|\mathcal{U}(t),\boldsymbol{S}(t)) \tag{13}$$

$$\leq \max_{\substack{\boldsymbol{C}\succeq 0,\\ \mathrm{tr}(\boldsymbol{C})\leq P}} \max_{\substack{p(\mathcal{U}(t)),\\ p(\boldsymbol{x}(t)|\mathcal{U}(t))\\ \boldsymbol{K}(t)\preceq \boldsymbol{C}}} \left(\frac{1}{p} h(\boldsymbol{y}(t),\boldsymbol{z}(t)|\mathcal{U}(t),\boldsymbol{S}(t)) - \frac{1}{q} h(\boldsymbol{z}(t)|\mathcal{U}(t),\boldsymbol{S}(t))\right) \tag{14}$$

$$\leq \mathbb{E}_{\hat{\boldsymbol{S}}(t)} \max_{\substack{\boldsymbol{C}\succeq 0,\\ \mathrm{tr}(\boldsymbol{C})\leq P}} \max_{\substack{p(\hat{\boldsymbol{S}}(t))\\ \boldsymbol{K}_*(t)\preceq \boldsymbol{C}}} \mathbb{E}_{\tilde{\boldsymbol{S}}(t)} \frac{1}{pq} \left(q \log\det(\mathbf{I}+\boldsymbol{S}(t)\boldsymbol{K}_*(t)\boldsymbol{S}^{\mathsf{H}}(t)) - p\log\det(\mathbf{I}+\boldsymbol{G}(t)\boldsymbol{K}_*(t)\boldsymbol{G}^{\mathsf{H}}(t))\right) \tag{15}$$

$$\leq \frac{\min\{M,N_1+N_2\}-\min\{M,N_2\}}{\min\{M,N_1+N_2\}}\alpha\log P + O(1) \tag{16}$$

current CSI $\hat{\boldsymbol{S}}(t)$, and (ii) the Rx-1 has the instantaneous knowledge of the Rx-2's received signal $\boldsymbol{z}(t)$.

With the above genie-aided model, we formulate a degraded BC $\boldsymbol{X}^n \to (\boldsymbol{Y}^n, \boldsymbol{Z}^n) \to \boldsymbol{Z}^n$, and therefore bound the achievable rates as stated in the following lemma:

Lemma 1 ([13], [17]). *We have*

$$n(R_1-\epsilon_n) \leq \sum_{t=1}^{n} h(\boldsymbol{y}(t),\boldsymbol{z}(t)|\mathcal{U}(t),\boldsymbol{S}(t))$$

$$n(R_2-\epsilon_n) \leq nN_2\log P - \sum_{t=1}^{n} h(\boldsymbol{z}(t)|\mathcal{U}(t),\boldsymbol{S}(t)) + n\cdot o(\log P)$$

where $\mathcal{U}(t)$ satisfies the Markov chain $\boldsymbol{S}(t)\to\mathcal{U}(t)\to\boldsymbol{x}(t)$.

To determine the weighted sum rate of two users, by letting $p=\min\{M,N_1+N_2\}$ and $q=\min\{M,N_2\}$, we have

$$n\left(\frac{R_1}{p}+\frac{R_2}{q}-2\epsilon_n\right) \leq \frac{N_2}{q} n\log P + n\cdot o(\log P)$$
$$+\, n\left(\frac{1}{p}\sum_{t=1}^{n} h(\boldsymbol{y}(t),\boldsymbol{z}(t)|\mathcal{U}(t),\boldsymbol{S}(t)) - \frac{1}{q}\sum_{t=1}^{n} h(\boldsymbol{z}(t)|\mathcal{U}(t),\boldsymbol{S}(t))\right).$$

We further bound the weighted difference of two differential entropies, as shown on the top of this page. Note that (15) is obtained because (i) Gaussian distribution maximizes the weighted difference of two differential entropies over all conditional distribution of $\boldsymbol{x}(t)$ with the same covariance matrix constraint, where $\boldsymbol{K}_*(t) = \max_{p(\mathcal{U}(t))}\boldsymbol{K}(t)$ if $p\geq q$ [11], [19] where $\boldsymbol{K}(t) \triangleq \mathbb{E}\{\boldsymbol{x}(t)\boldsymbol{x}^{\mathsf{H}}(t)|\mathcal{U}(t)\}$, and (ii) $\boldsymbol{z}(t)$ is a degraded version of $(\boldsymbol{y}(t),\boldsymbol{z}(t))$; the last inequality is obtained by the following lemma, where $L=N_1+N_2$ and $N=N_2$ and the details are provided in [13], [17].

Lemma 2. *For two random matrices $\boldsymbol{S}=\hat{\boldsymbol{S}}+\tilde{\boldsymbol{S}}\in\mathbb{C}^{L\times M}$ and $\boldsymbol{H}=\hat{\boldsymbol{H}}+\tilde{\boldsymbol{H}}\in\mathbb{C}^{N\times M}$ with $L\geq N$ and $\tilde{\boldsymbol{H}}\sim\mathcal{N}_{\mathbb{C}}(0,\sigma^2\mathbf{I})$, where $\tilde{\boldsymbol{S}}$, $\tilde{\boldsymbol{H}}$ are respectively independent of $\hat{\boldsymbol{S}}$, $\hat{\boldsymbol{H}}$, given any $\boldsymbol{K}\succeq 0$, we have*

$$\frac{1}{\min\{M,L\}}\mathbb{E}_{\tilde{\boldsymbol{S}}}\log\det(\mathbf{I}+\boldsymbol{S}\boldsymbol{K}\boldsymbol{S}^{\mathsf{H}}) - \frac{1}{\min\{M,N\}}\mathbb{E}_{\tilde{\boldsymbol{H}}}\log\det(\mathbf{I}+\boldsymbol{H}\boldsymbol{K}\boldsymbol{H}^{\mathsf{H}})$$
$$\leq -\frac{\min\{M,L\}-\min\{M,N\}}{\min\{M,L\}}\log(\sigma^2)+O(1) \tag{17}$$

as σ^2 goes to 0.

REFERENCES

[1] H. Weingarten, Y. Steinberg, and S. Shamai, "The capacity region of the Gaussian multiple-input multiple-output broadcast channel," *IEEE Trans. Inf. Theory*, vol. 52, no. 9, pp. 3936 –3964, Sept. 2006.

[2] C. Huang, S. Jafar, S. Shamai, and S. Vishwanath, "On degrees of freedom region of MIMO networks without channel state information at transmitters," *IEEE Trans. Inf. Theory*, vol. 58, no. 2, pp. 849–857, Feb. 2012.

[3] C. Vaze and M. Varanasi, "The degree-of-freedom regions of MIMO broadcast, interference, and cognitive radio channels with no CSIT," *IEEE Trans. Inf. Theory*, vol. 58, no. 8, pp. 5354–5374, Aug. 2012.

[4] N. Jindal, "MIMO broadcast channels with finite-rate feedback," *IEEE Trans. Inf. Theory*, vol. 52, no. 11, pp. 5045 –5060, Nov. 2006.

[5] D. Love, R. Heath, V. Lau, D. Gesbert, B. Rao, and M. Andrews, "An overview of limited feedback in wireless communication systems," *IEEE J. Sel. Areas in Commun.*, vol. 26, no. 8, pp. 1341 –1365, Oct. 2008.

[6] M. Maddah-Ali and D. Tse, "Completely stale transmitter channel state information is still very useful," *IEEE Trans. Inf. Theory*, vol. 58, no. 7, pp. 4418–4431, Jul. 2012.

[7] C. S. Vaze and M. K. Varanasi, "The degrees of freedom regions of two-user and certain three-user MIMO broadcast channels with delayed CSIT," *ArXiv:1101.0306*, Dec. 2011.

[8] R. Tandon, S. Jafar, S. Shamai, and H. Poor, "On the synergistic benefits of alternating CSIT for the MISO BC," *to appear in IEEE Trans. Inf. Theory, arXiv:1208.5071*, 2012.

[9] N. Lee and R. Heath Jr, "Not too delayed CSIT achieves the optimal degrees of freedom," in *Proc. Allerton 2012, arXiv:1207.2211*, 2012.

[10] M. Kobayashi, S. Yang, D. Gesbert, and X. Yi, "On the degrees of freedom of time correlated MISO broadcast channel with delayed CSIT," in *Proc. IEEE ISIT 2012*, Jul. 2012, pp. 2501–2505.

[11] S. Yang, M. Kobayashi, D. Gesbert, and X. Yi, "Degrees of freedom of time correlated MISO broadcast channel with delayed CSIT," *IEEE Trans. Inf. Theory*, vol. 59, no. 1, pp. 315–328, Jan. 2013.

[12] T. Gou and S. Jafar, "Optimal use of current and outdated channel state information: Degrees of freedom of the MISO BC with mixed CSIT," *IEEE Communications Letters*, vol. 16, no. 7, pp. 1084–1087, Jul. 2012.

[13] X. Yi, S. Yang, D. Gesbert, and M. Kobayashi, "The degrees of freedom region of temporally-correlated MIMO networks with delayed CSIT," *submitted to IEEE Trans. Inf. Theory, arXiv:1211.3322v1*, Oct. 2012.

[14] T. Cover and A. Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inf. Theory*, vol. 25, no. 5, pp. 572–584, 1979.

[15] F. Willems and E. van der Meulen, "The discrete memoryless multiple-access channel with cribbing encoders," *IEEE Trans. Inf. Theory*, vol. 31, no. 3, pp. 313 – 327, May 1985.

[16] J. Chen and P. Elia, "Degrees-of-freedom region of the MISO broadcast channel with general mixed-CSIT," *ITA 2013, arXiv:1205.3474*, 2012.

[17] X. Yi, D. Gesbert, S. Yang, and M. Kobayashi, "Degrees of freedom of time-correlated broadcast channels with delayed CSIT: The MIMO case," *ArXiv preprint*, 2013.

[18] A. Adhikary, H. Papadopoulos, S. Ramprashad, and G. Caire, "Multi-user MIMO with outdated CSI: training, feedback and scheduling," in *Proc. Allerton 2011*, 2011.

[19] T. Liu and P. Viswanath, "An extremal inequality motivated by multiterminal information-theoretic problems," *IEEE Trans. Inf. Theory*, vol. 53, no. 5, pp. 1839–1851, May 2007.