VOICE ACTIVITY DETECTION BASED ON A STATISTICAL SEMIPARAMETRIC TEST

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ABSTRACT

This paper adresses the voice activity detection problem within a semiparametric hypothesis testing framework. Semiparametric detection consists in combining the statistical optimality of a parametric test with the robustness regarding the learning data of a nonparametric test. The proposed semiparametric approach splits the frame vector into two parts such that the first part has a known statistical distribution. The second part is processed by a non-parametric detector producing a binary decision. A likelihood ratio test, based on the first part and the nonparametric binary decision, is then applied to classify the frame as either speech or non-speech. The statistical performance of the resulting fusion test is analytically established and validated using real speech signals.

***Index Terms*—** Voice activity detection, Semiparametric test, Nonparametric test, Likelihood ratio test, Fusion test.

1. INTRODUCTION

Voice Activity Detection (VAD) refers to the classical problem of distinguishing active speech from nonspeech. Recently, numerous approaches have been developed for improving the performance of VAD schemes in noisy environments. They can be divided into two categories: parametric (derived from a model) and nonparametric (learned from data) approaches. In the parametric approaches, often called statistical model-based VAD algorithms, the distributions of both noisy speech and noise spectra are assumed to follow a particular parametric model such as a Gaussian or Laplacian distribution. Based on the assumed distributions, the likelihood ratio is calculated and thresholded to form a decision rule [1, 2, 3, 4]. Nonparametric approaches incorporate machine learning techniques such as minimum classification error methods [5, 6] and Support Vector Machine (SVM) schemes [7, 8] to exploit prior knowledge.

Recently, there is a considerable interest in semiparametric tests, which combine the advantages of each approach. There are two main categories of approaches in the literature. The first, e.g. [9, 10], involve parametric models which incorporate an unknown non-parametric (often infinite dimensional) part. The main difficulty involves the estimatation of the nonparametric part of the model [11] and the design of an appropriate speech/non-speech test. The second category, e.g.[12, 13, 14, 15], involves the combination or fusion of multiple statistical tests in speech/non-speech classification. The individual tests are generally both parametric and nonparametric. The resulting test is called the fusion test. This paper is concerned with the second category.

The contributions in this paper are three-fold. First, we propose an optimal fusion rule combining one parametric detector and one nonparametric detector. Due to this combination, the proposed approach is referred to as semiparametric detection. This combination is nevertheless parametric in essence since it is based on the well-known Likelihood Ratio Test (LRT). Second, the theoretical statistical performance of the proposed detector is theoretically established. Finally, the theory is validated with practical experiments with real speech signals.

The remainder of this paper is organized as follows. Section 2 describes the VAD semiparametric decision problem. Section 3 derives the optimal semiparametric test. It also establishes likely performance according to various statistical approximations. Section 4 reports an evaluation of the proposed test with real speech signals. Our conclusion are presented in Section 5.

2. PROBLEM STATEMENT

The VAD problem can be modeled as a binary hypothesis test between the two hypotheses $\mathcal{H}_0$: {noise alone} and $\mathcal{H}_1$: {speech plus noise}. In this section we show how a frame of speech $\boldsymbol{x}_i$ can be split into two statistically independent subvectors $\boldsymbol{x}_i^p$ and $\boldsymbol{x}_i^q$. The subvector $\boldsymbol{x}_i^q$ is processed independently with a nonparametric detector which produces a decision value 0 or 1 consequently to $\mathcal{H}_0$ and $\mathcal{H}_1$ respectively. The subvector $\boldsymbol{x}_i^p$ is used in combination with the decision value of the nonparametric test to form a couple whose distribution is known under each hypothesis.

2.1. Noisy Speech Measurement Model

The speech signal is first segmented into contiguous frames. The i-th analyzed frame is denoted $\boldsymbol{x}_i \in \mathbb{R}^n$ where n is frame length and $i \in \{1, \dots, F\}$ with F the number of frames.

A complete parametric model of the frame is out of reach due to the high variability of speech signals but parametric models based on a linear transformation of the frame samples are possible. This paper is based on the statistical Perceptual Ephraim-Malah (PEM) model discussed in [1]. Let $\boldsymbol{x}_i^p$ be the vector of PEM coefficients, which are obtained from $\boldsymbol{x}_i$ as

$$\boldsymbol{x}_i^p = \boldsymbol{M}\boldsymbol{D}\,\boldsymbol{x}_i = \boldsymbol{P}\boldsymbol{x}_i \tag{1}$$

where $p \times n$ matrix $\boldsymbol{M}$ and $n \times n$ matrix $\boldsymbol{D}$ represent the Mel frequency filter bank and discrete cosine transform (DCT), respectively. The PEM model accommodates the critical-band phenomenon where the matrix product $\boldsymbol{P} = \boldsymbol{M}\boldsymbol{D}$ represents a perceptual transform, and thus $\boldsymbol{x}_i^p$ represents the perceptual spectrum of frame $\boldsymbol{x}_i$. It is assumed that $\boldsymbol{M}$ is a full-row rank matrix such that $\text{rank}\,(\boldsymbol{P}) = p$. Let $\boldsymbol{R}$ be the $n \times n$ matrix defined by

$$\boldsymbol{R} = \begin{pmatrix} \boldsymbol{P} \\ \boldsymbol{Q} \end{pmatrix}$$

where the $q \times n$ matrix $\boldsymbol{Q}$ is chosen such that $\boldsymbol{R}$ is a regular matrix of size n (thus, $q = n - p$). Without loss of generality, it is supposed that the rows of $\boldsymbol{Q}$ form an orthonormal set of vectors. It is sufficient to take an orthonormal basis of the orthogonal complement of the space spanned by the rows of $\boldsymbol{P}$ in $\mathbb{R}^n$. Hence, $\boldsymbol{x}_i$ is statistically equivalent to $\boldsymbol{R}\boldsymbol{x}_i$ given by

$$\boldsymbol{R}\boldsymbol{x}_i = \begin{pmatrix} \boldsymbol{x}_i^p \\ \boldsymbol{x}_i^q \end{pmatrix} = \begin{pmatrix} \boldsymbol{P}\boldsymbol{x}_i \\ \boldsymbol{Q}\boldsymbol{x}_i \end{pmatrix}. \tag{2}$$

It is straightforward to verify that $\boldsymbol{P}\boldsymbol{x}_i$ and $\boldsymbol{Q}\boldsymbol{x}_i$ are uncorrelated. It is therefore assumed that they are also independent.

2.2. Parametric Model

It is numerically shown in [1] that the statistical distribution of PEM coefficients is well approximated by a zero-mean Gaussian distribution with a known covariance matrix $\boldsymbol{\Sigma}_k$ depending on the hypothesis, i.e.,

$$\boldsymbol{x}_i^p \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma}_k) \text{ under } \mathcal{H}_k \tag{3}$$

where $\boldsymbol{\Sigma}_k = \text{diag}(\sigma_{k,1}^2, \ldots, \sigma_{k,n}^2)$ is a positive definite, diagonal $p \times p$ matrix under hypothesis $\mathcal{H}_k$ and $\mathcal{N}(0, \boldsymbol{\Sigma}_k)$ denotes the zero-mean normal distribution with covariance matrix $\boldsymbol{\Sigma}_k$. It is assumed that $\sigma_{1,j}^2 \geq \sigma_{0,j}^2$ for all $1 \leq j \leq n$. In other words, speech has a higher variance than none. This assumption is generally satisfied with real data. In practice, $\boldsymbol{\Sigma}_k$ is replaced by an estimate which is rarely diagonal, but the diagonal elements are generally dominant and non-diagonal elements could be neglected.

2.3. Nonparametric Model

Contrary to $\boldsymbol{x}_i^p$, the random vector $\boldsymbol{x}_i^q$ does not admit a known statistical model. Hence, it is not possible to know or have an accurate approximation of its statistical distribution. For this reason, a nonparametric model is generally used or, equivalently, a learning-based detector. According to the splitting of the frame (2) into two independent subvectors $\boldsymbol{x}_i^p$ and $\boldsymbol{x}_i^q$, the nonparametric approach is based on the learning data set

$$\mathcal{S}^q = \left\{ (\boldsymbol{x}_{(1)}^q, \ell_1), (\boldsymbol{x}_{(2)}^q, \ell_2), \ldots, (\boldsymbol{x}_{(N)}^q, \ell_N) \right\}$$

composed of N independent and identically distributed couples of values $(\boldsymbol{x}_{(i)}^q, \ell_i)$. The integer $\ell_i \in \{0, 1\}$ is the label of the subvector $\boldsymbol{x}_{(i)}^q$, i.e., $\ell_i = 0$ if the subvector $\boldsymbol{x}_{(i)}^q$ contains noise only and $\ell_i = 1$ otherwise.

A binary statistical test, also called a detector, applied to $\boldsymbol{x}_i^q$ is a Borel measurable function $\delta^* : \mathbb{R}^q \to \{0, 1\}$ mapping the space $\mathbb{R}^q$ to class labels. In standard classification, the performance of δ^* is measured by the probability of two errors, namely the probability of false alarms α^* and the probability of correct detections β^* given by

$$\alpha^* = \text{Pr}_0\big(\delta^*(\boldsymbol{x}_i^q) = 1\big) \text{ and } \beta^* = \text{Pr}_1\big(\delta^*(\boldsymbol{x}_i^q) = 1\big). \tag{4}$$

Here, the notation $\text{Pr}_k(A)$ represents the probability of the event A when hypothesis $\mathcal{H}_k$ is true. Let $\mathcal{C}$ be a class of detectors and let Ω^q be the set of all possible learning sets $\mathcal{S}^q$. A learning detector is a mapping $\delta_q : \Omega^q \to \mathcal{C}$. In other words, the learning detector δ_q is a rule for selecting a detector based on a training. The class $\mathcal{C}$ of detectors used in this paper is the class of Support Vector Machines (SVM) detectors. Other class of detection may also be used. It is for example possible to use a nonparametric regression as a detector (like the well-known logistic regression). In this case, the theoretical study should be more difficult because a nonparametric regression takes more than two output values (the probability of each output must be known to use the approach proposed in this paper). Let $\delta_q(\boldsymbol{x}_i^q)$ be the binary decision (0 or 1) provided by the SVM detector. Details on SVM detectors $\delta_q(\boldsymbol{x}_i^q)$ are given in [16].

The false alarm probability of $\delta_q(\boldsymbol{x}_i^q)$ is α_q and the probability of correct detection is β_q. These two probabilities are assumed to be known or well estimated. Consequently, under $\mathcal{H}_0$, the random variable $\delta_q(\boldsymbol{x}_i^q)$ follows the Bernoulli distribution $\mathcal{B}(\alpha_q)$: it takes the value 1 with probability α_q and the value 0 with probability $1 - \alpha_q$. Under $\mathcal{H}_1$, $\delta_q(\boldsymbol{x}_i^q)$ follows the Bernoulli distribution $\mathcal{B}(\beta_q)$.

2.4. Semiparametric Detection Problem

The proposed semiparametric approach consist of solving the decision problem

$$\begin{aligned} \mathcal{H}_0^s &: \left\{ \boldsymbol{x}_i^p \sim \mathcal{N}(0, \boldsymbol{\Sigma}_0),\; \delta_q(\boldsymbol{x}_i^q) \sim \mathcal{B}(\alpha_q) \right\}, \\ \mathcal{H}_1^s &: \left\{ \boldsymbol{x}_i^p \sim \mathcal{N}(0, \boldsymbol{\Sigma}_1),\; \delta_q(\boldsymbol{x}_i^q) \sim \mathcal{B}(\beta_q) \right\}. \end{aligned} \tag{5}$$

The notation $\mathcal{H}_0^s$ instead of $\mathcal{H}_0$ (and also $\mathcal{H}_1^s$ instead of $\mathcal{H}_1$) underlines the fact that $\mathcal{H}_0^s$ is not strictly equivalent to $\mathcal{H}_0$. It is a new formulation of the statistical decision problem $\mathcal{H}_0$ against $\mathcal{H}_1$ which is based on the frame statistical models derived from the decomposition (2). It must be noted that each hypothesis (5) is simple [17], which means that the statistical distribution of the observation vectors under each hypothesis is perfectly known.

3. OPTIMAL SEMIPARAMETRIC DETECTION

The likelihood ratio test is calculated from the couple of random variables identified in the previous section. This yields the semiparametric test.

3.1. Semiparametric Test

The decision problem (5) can be solved by the well-known log-likelihood ratio test [17] given by

$$\delta_s(\boldsymbol{x}_i) = \delta_s(\boldsymbol{x}_i^p, \boldsymbol{x}_i^q) = \begin{cases} 0 \text{ if } \Lambda_s(\boldsymbol{x}_i) \leq \lambda_s, \\ 1 \text{ else,} \end{cases} \tag{6}$$

with the decision function $\Lambda_s(\boldsymbol{x}_i)$:

$$\Lambda_s(\boldsymbol{x}_i) = \Lambda_s(\boldsymbol{x}_i^p, \delta_q(\boldsymbol{x}_i^q)) = \log \frac{f_1(\boldsymbol{x}_i^p)\, b_{\beta_q}(\boldsymbol{x}_i^q)}{f_0(\boldsymbol{x}_i^p)\, b_{\alpha_q}(\boldsymbol{x}_i^q)}. \tag{7}$$

The threshold λ_s^* is chosen to satisfy a prescribed false alarm probability α, i.e. $\alpha_s = \alpha_s(\lambda_s^*) = \alpha$ where the notation $\alpha_s(\lambda_s^*)$ underlines the fact that the false alarm probability of $\delta_s(\boldsymbol{x}_i)$ depends on λ_s^*. In (7), $b_a(x)$ denotes the probability density function (pdf) of the Bernouilli distribution $\mathcal{B}(a)$,

$$b_a(x) = a^x\,(1-a)^{1-x},\ x \in \{0, 1\}, \tag{8}$$

and $f_k(\boldsymbol{x})$ is the pdf of the normal distribution with mean 0 and covariance matrix $\boldsymbol{\Sigma}_k$,

$$f_k(\boldsymbol{x}) = \frac{1}{\sqrt{2\pi \det(\boldsymbol{\Sigma}_k)}} e^{-\frac{1}{2}\boldsymbol{x}^\top \boldsymbol{\Sigma}_k^{-1}\boldsymbol{x}},\ \boldsymbol{x} \in \mathbb{R}^p, \tag{9}$$

where $\det(\boldsymbol{\Sigma}_k)$ is the determinant of matrix $\boldsymbol{\Sigma}_k$.

A straightforward calculation shows that the test (6) is equivalent to the comparison of the decision function $\Lambda_s^*(\boldsymbol{x}_i)$ to a threshold λ_s^* where $\Lambda_s^*(\boldsymbol{x}_i)$ is given by:

$$\begin{aligned} \Lambda_s^*(\boldsymbol{x}_i) &= (\boldsymbol{x}_i^p)^\top (\boldsymbol{\Sigma}_0^{-1} - \boldsymbol{\Sigma}_1^{-1})\boldsymbol{x}_i^p + 2\,\gamma_q\,\delta_q(\boldsymbol{x}_i^q) \\ &= \sum_{j=1}^{p} c_j\,(x_{i,j}^p)^2 + 2\,\gamma_q\,\delta_q(\boldsymbol{x}_i^q). \end{aligned} \tag{10}$$

where $\boldsymbol{x}_i^p = (x_{i,1}^p, \ldots, x_{i,p}^p)$, $c_j = \sigma_{0,j}^{-2} - \sigma_{1,j}^{-2} \geq 0$ and

$$\gamma_q = \log\left(\frac{\beta_q(1-\alpha_q)}{\alpha_q(1-\beta_q)}\right).$$

From (10), the semiparametric test is shown to be a sum of two terms. The first corresponds to an energy detector; this is the parametric part. The second corresponds to the nonparametric decision $\delta_q(\boldsymbol{x}_i^q)$. The parameter γ_q is a tradeoff between the two terms. It measures the global performance of the nonparametric test $\delta_q(\boldsymbol{x}_i^q)$.

3.2. Statistical Performance

The calculation of the statistical performance, namely the false alarm probability α_s and the probability of correct detection β_s, of the test $\delta_s(\boldsymbol{x}_i)$ is not straightforward due to the complexity of $\Lambda_s^*(\boldsymbol{x}_i)$. For this reason, α_s and β_s are approximated.

From (10), it is straightforward to obtain

$$\begin{aligned} \alpha_s &\overset{\text{def.}}{=} \Pr\nolimits_0(\Lambda_s^* \geq \lambda_s^*) \\ &= \overline{F}_0(\lambda_s^*)(1-\alpha_q) + \overline{F}_0\,(\lambda_s^* - 2\gamma_q)\alpha_q, \end{aligned} \tag{11}$$

$$\begin{aligned} \beta_s &\overset{\text{def.}}{=} \Pr\nolimits_1(\Lambda_s^* \geq \lambda_s^*) \\ &= \overline{F}_1(\lambda_s^*)(1-\beta_q) + \overline{F}_1\,(\lambda_s^* - 2\gamma_q)\beta_q \end{aligned} \tag{12}$$

where $F_k(\cdot)$ is the Cumulative Distribution Function (cdf) of the random variable

$$T(\boldsymbol{x}_i^p) \overset{\text{def.}}{=} \sum_{j=1}^{p} c_j\,(x_{i,j}^p)^2 \tag{13}$$

under hypothesis $\mathcal{H}_k$ and $\overline{F}_k(x) = 1 - F_k(x)$ for $k = 0, 1$. The theoretical calculation of the threshold λ_s^* is also complex but a numerical computation is manageable. For this, it is necessary to calculate $\overline{F}_k(x)$. Since its exact calculation is difficult, another approximation is used. Let us note that

$$T(\boldsymbol{x}_i^p) = \sum_{j=1}^{p} d_{k,j}\,u_{k,j}^2 \quad \text{for} \quad k = 0, 1,$$

where $d_{k,j} = c_j \sigma_{k,j}$ and $u_{k,j} = x_{i,j}^p / \sigma_{k,j}$ for $j = 1, \ldots, p$. Under $\mathcal{H}_k$, $x_{i,j}^p \sim \mathcal{N}(0, \sigma_{k,j}^2)$, hence $u_{k,j} \sim \mathcal{N}(0, 1)$. Consequently, $T(\boldsymbol{x}_i^p)$ is a weighted sum of chi-square variables whatever the hypothesis. One of the common approximations of $F_k(x)$ is a gamma distribution having the same first two moments as that of $T(\boldsymbol{x}_i^p)$ (see details in [18]). Let $g_{(a,b)}(x)$ be the pdf of the gamma distribution with parameters (a, b):

$$g_{(a,b)}(x) = \frac{a^b}{\Gamma(b)} e^{-at} t^{b-1},\ x \geq 0. \tag{14}$$

Under $\mathcal{H}_k$, this approximation is obtained for the parameters:

$$a_k^* = \frac{1}{2}\frac{\sum_{j=1}^p d_{k,j}}{\sum_{j=1}^p d_{k,j}^2} \quad \text{and} \quad b_k^* = \frac{1}{2}\frac{\left(\sum_{j=1}^p d_{k,j}\right)^2}{\sum_{j=1}^p d_{k,j}^2}.$$

Let $G_{(a,b)}(x)$ be the cumulative distribution function of the gamma distribution with parameters (a, b) and $G_{(a,b)}^{-1}(x)$, its inverse. Then,

$$F_k(x) \approx G_{(a_k^*, b_k^*)}(x) \quad \text{and} \quad \overline{F}_k(x) \approx 1 - G_{(a_k^*, b_k^*)}(x). \tag{15}$$

Incorporating (15) into (11) and (12) yields approximations of α_s and β_s. These approximations can be easily evaluated numerically. Hence, the computation of the threshold λ_s^* satisfying $\alpha_s(\lambda_s^*) = \alpha$ is straightforward by using a root-finding algorithm and formula (11). Then, knowing λ_s^* immediately gives the power β_s from (12).

Fig. 1 shows the relevance of the theoretical approximations given in (15) for typical values of matrices $\mathbf{\Sigma}_0$ and $\mathbf{\Sigma}_1$, $\alpha_q = 0.1$ and $\beta_q = 0.8$. The theoretical Receiver Operating Characteristics (ROC) curve β_s of the semiparametric test is very close to the ROC curve $\widehat{\beta}_s$ estimated by using Monte-Carlo simulations.

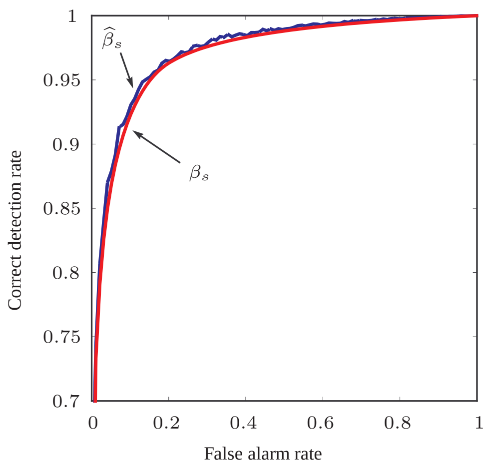

Fig. 1. Theoretical ROC curve β_s for $\delta_s(\boldsymbol{x}_i)$ and its estimation $\widehat{\beta}_s$ based on Monte-Carlo simulations.

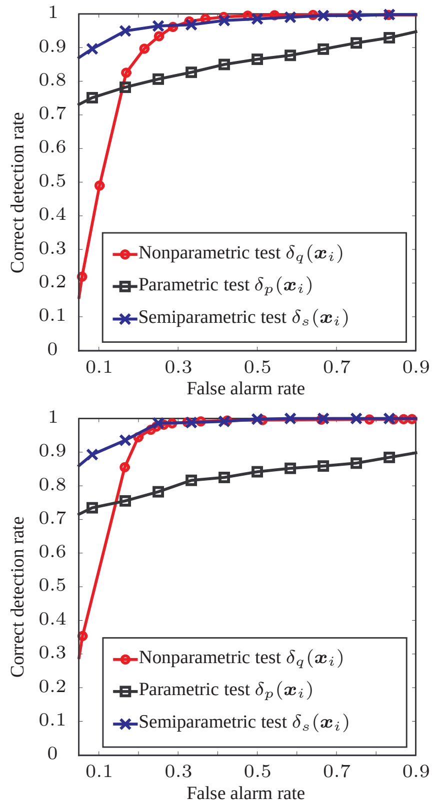

Fig. 2. ROC curves for $\delta_s(\boldsymbol{x}_i)$, $\delta_q(\boldsymbol{x}_i)$ and $\delta_p(\boldsymbol{x}_i)$ with white noise (up) and babble noise (down) at SNR = 5dB.

4. EXPERIMENTAL RESULTS

The performance of the proposed approach is evaluated on the TIMIT database. The test material consists of 480s long speech data which are split randomly into two groups for training and evaluation. The training data is dedicated to the nonparametric test which is represented in this work by the SVM classifier with a Gaussian kernel.

To make a noisy signal, white and babble noises from NOISEX-92 database are added to short-time frames (16ms) of clean speech, with 50% overlap at Signal-to-Noise Ratio (SNR) 5 dB. From each frame, we extract 6 PEM coefficients which are used in the parametric test. The covariance matrices $\mathbf{\Sigma}_0$ and $\mathbf{\Sigma}_1$ are initially estimated from a small set of frames. In order to evaluate the performance of the algorithms, we investigate the speech detection and false alarm probabilities for each VAD approach. Hence, the evaluation is based on the ROC curve. Fig. 2 shows the results. The parametric curve corresponds to the test $\delta_p(\boldsymbol{x}_i)$ based only on the decision function given in (13). The threshold of $\delta_p(\boldsymbol{x}_i)$ is computed such that this test satisfies a prescribed false alarm probability. The semiparametric ROC curve is computed for the SVM detector based on two features extracted from $\boldsymbol{x}_i^q$, namely its mean and its standard deviation. Regarding the performance of the SVM detector, it is clear that some discriminating information between $\mathcal{H}_0$ and $\mathcal{H}_1$ is contained in the subvector $\boldsymbol{x}_i^q$. It is also clear that the SVM detector has poor performances for a small false alarm rate. The semiparametric test (6) outperforms the two other detectors, especially for a small false alarm rate.

5. CONCLUSION

This paper proposed a novel VAD algorithm based on semiparametric test derived from the likelihood ratio test between a nonparametric binary decision value and a parametric model. Numerical simulations by means of ROC curves show that this new approach outperforms both a pure parametric test and a pure nonparametric test, especially for a small false alarm probability.

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