Optimal Combining of Instantaneous and Statistical CSI in the SIMO Interference Channel

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Abstract—The uplink of the two-user multiple-antenna interference channel is considered and the optimal (in the ergodic rate sense) beamforming (BF) problem is posed and solved. Specifically, a feedback scenario is studied whereby a base station (BS) is allowed to estimate the instantaneous channel vector information from the users it serves, but not from out-of-cell interference users. That is to say, only statistical information can be obtained regarding the interference. In contrast with most previous works, the motivation behind the presumed feedback scenario is the compliance with current cellular network standards. For this scenario, we derive new expressions for the ergodic user rates. Exploiting the derived expressions, the optimal, with respect to ergodic rate maximization, receive BF vectors are found in closed-form. Finally, new user scheduling schemes are proposed, which exploit the derived BF solution and allow an efficient use of combined instantaneous and statistical information.

Index Terms—multicell system, covariance information, channel statistics, user scheduling, receive beamforming.

I. INTRODUCTION

The study of optimal transmission schemes over the interference channel has attracted significant attention in the recent years. While the capacity achieving scheme remains unknown for the general case, much progress has been reached when it comes to describing optimal strategies in specific settings, one example among others being the Pareto optimal linear transmit beamforming (BF) problem under single user decoding capability [1]. While rate-optimal BF investigations are typically carried out with little limitations regarding available channel state information (CSI), practical networks do pose constraints as to how much instantaneous CSI can be acquired, via feedback or pilot schemes. A classical approach for circumventing the practical CSI limitations consists in assuming a quantization scheme with a finite number of bits [2] or in deriving BF solutions from channel statistics alone. BF solutions based on covariance information were reported in [3]–[7] as well as in [8]–[13]. More recently, an optimal BF scheme suited to the downlink of a two-user interference channel based solely on covariance information was derived in [14]. There, the optimal BF vector based on the interference and the desired channel covariance matrix is shown to be in the form of a generalized eigenvector.

In this paper we argue that in the light of current standardization discussions, the assumptions of a CSI setting where only instantaneous (resp. only statistical) channel information is available to all interfering devices is overly optimistic (resp. pessimistic). In fact, forthcoming standard releases for 4G wireless systems stipulate that a given user is allowed to report instantaneous CSI to its home base station (BS), while it cannot report such information to interfering BSs (at least not directly). Such a constraint motivates the use of a hybrid CSI scenario where direct (useful) CSI is made available in instantaneous form while the interfering channels will be known only based on their second order statistics.

In such a setting, we propose to revisit the optimal BF problem for a two-BS interference network, in the uplink. More specifically, our contributions are as follows:

• We derive a closed form expression for the user rates under single user decoding capability which is a function of the instantaneous desired channel vectors and the covariance matrices of interfering channels.

• By focusing on interference dominated systems, i.e., assuming that interference is the dominant, as compared to noise, source of degradation, we derive a novel, simple approximation for the ergodic rate which leads to a simplified formulation of an optimal receive BF design.

• We derive the optimal receive BF vector in the sense of the ergodic rate for the aforementioned interference dominated regime.

• We make use of the optimal BF design in order to suggest new, low complexity multi-user scheduling schemes. The new schedulers allow to reduce the impact of interference substantially as the number of users increases.

Throughout the paper, the following notations are adopted: all boldface letters indicate vectors (lower case) or matrices (upper case). Superscript $(\cdot)^H$ stands for Hermitian transpose, $\mathbb{C}^{m \times n}$ is used to denote the set of $m \times n$ complex matrices and $\mathbb{E}\{\cdot\}$ stands for the expectation operator. The identity matrix of dimension $n \times n$ is denoted by $I_n$. Finally, $\|\cdot\|$ denotes the Euclidean norm, and $E_1(\cdot)$ stands for the exponential integral function, which is defined as: $E_1(x) = \int_x^{\infty} \frac{e^{-t}}{t} \, dt$ [15, 5.1.1].

II. SYSTEM MODEL

A system consisting of two BSs employing the same frequency resources, as shown in Fig.1, is considered. Both BSs, $BS_1$ and $BS_2$, are equipped with $M$ receive antennas, whereas for the two users operating in the same frequency slot, i.e., $U_1$, $U_2$, served by $BS_1$ and $BS_2$ respectively, the use of single antenna terminals is assumed. Focusing on the uplink, the
The interfering channel vector. From (2), \( y_i = w_i h_{ij_1} s_1 + w_i h_{ij_2} s_2 + w_i z_i \) (1), where \( h_{ij}, j = 1, 2 \) is the \( M \times 1 \) single-input, multiple-output (SIMO) Rayleigh fading channel between \( U_j \) and \( B_S \), undergoing correlation due to the finite multipath angle spread at the BS, \( w_i \in \mathbb{C}^{1 \times M} \), \( i = 1, 2 \) is the unit norm, i.e., \( \| w_i \| = 1 \), receive BF vector at \( B_S \), and \( z_i \) is the \( M \times 1 \) noise vector, the elements of which are, without loss of generality, assumed to be i.i.d. standard complex Gaussian RVs, i.e., \( z_i \sim \mathcal{CN}(0, I_M) \). Assuming that Gaussian codebooks are used for both users’ information signals, \( s_i \), it holds that \( s_i \sim \mathcal{CN}(0, 1) \), \( i = 1, 2 \).

The channel vector \( h_{ji}, i, j \in \{1, 2\} \) can be written as \( h_{ji} = R_{ji}^{1/2} h_{ji}^{(w)} \) (2), where \( R_{ji}^{1/2} \) is the symmetric square root of the covariance matrix \( R_{ji} \) of vector \( h_{ji} \), and \( h_{ji}^{(w)} \sim \mathcal{CN}(0, I_M) \). Moreover, the covariance matrix encompasses the transmit power \( P_i \), \( i = 1, 2 \) of both users and the deterministic path loss induced by the wireless channels.

Based on (1) it is easy to show that the achievable rate for communication of \( U_i \) with \( B_S \) is defined as \(^1\) \( R_i = \log \left( 1 + \frac{|s_i|^2}{1 + |\pi_i|^2} \right) \) (3), where \( s_i = w_i h_{ij} \) and \( \pi_i = w_i h_{ji}, j \neq i \) denoting the interfering channel vector. From (2), \( \pi_i \) is written as \( \pi_i = w_i R_{ji}^{1/2} h_{ji}^{(w)} \) (4).

By defining the quantity \( \alpha_i = 1 + |\pi_i|^2 \), (3) is readily expressed as \( R_i = \log(\alpha_i + |\pi_i|^2) - \log(1 + |\pi_i|^2) \) (5).

One of the most commonly adopted criteria for user scheduling and BF design in the presence of interfering users is the sum rate maximization criterion given instantaneous CSI for all, direct and interfering, channels \([16]–[20]\). In the system model that we investigate, the use of such scheduling and BF criteria would require acquiring and exchanging instantaneous CSI that, among others, preserves high rate communication links for the cooperation of BSs and increases the cost of the network. Therefore, in our analysis we investigate a realistic scheme that is based on the exchange of statistical CSI between the BSs and, specifically the covariance of channel \( h_{ji} \). Since such statistics are expected to vary slowly with time, the information exchange required in order to obtain \( \hat{R}_{ji} \) is significantly smaller as compared to the case of exchanging instantaneous CSI. Hence, we assume knowledge of \( \hat{R}_{ji}, j \neq i \) at \( B_S \), and we pose and solve the BF problem at \( B_S \) such as to maximize its expected rate, given the instantaneous direct channel \( h_{ii} \), denoted as \( \mathbb{E}_{h_{ii}} \{ R_i \} \). To do so, we calculate the statistical distribution of interference term \( |\pi_i|^2 \) and then \( \mathbb{E}_{h_{ii}} \{ R_i \} \). In the sequel, an approximation of \( \mathbb{E}_{h_{ii}} \{ R_i \} \) for an interference dominated system is derived. Following that, an analytical method is given for optimal receive BF design in terms of maximizing \( \mathbb{E}_{h_{ii}} \{ R_i \} \). These two steps are further described in the two following sections.

III. CALCULATION OF THE CONDITIONAL EXPECTED SUM RATE

In order to calculate \( \mathbb{E}_{h_{ii}} \{ R_i \} \), first the probability density function (PDF) of the interference term \( |\pi_i|^2 \) is derived as shown in the following analysis.

A. Distribution of the interference term

Using standard properties of complex Gaussian RVs, it is easy to prove that \( \pi_i \) for a given \( w_i \) is a complex Gaussian random variable (RV) with variance equal to \( \sigma_{\pi_i}^2 = \| w_i \|^2 R_{ji} \). As a result, the RV \( \beta_i = |\pi_i|^2 \) follows a chi-squared distribution of the form \( p_{\beta_i}(x) = \frac{1}{\sigma_{\pi_i}^2} \exp \left( -\frac{x}{\sigma_{\pi_i}^2} \right) \). (6)

In what follows, (6) is employed in order to calculate \( \mathbb{E}_{h_{ii}} \{ R_i \} \).

B. Calculation of \( \mathbb{E}_{h_{ii}} \{ R_i \} \)

Combining (5) and (6), \( \mathbb{E}_{h_{ii}} \{ R_i \} \) is written as

\[
\mathbb{E}_{h_{ii}} \{ R_i \} = \int_0^\infty (\log(\alpha_i + \beta_i) - \log(1 + \beta_i)) p_{\beta_i}(\beta_i) d\beta_i.
\] (7)

The following theorem states the result.

**Theorem 1.** The expected capacity \( \mathbb{E}_{h_{ii}} \{ R_i \} \) defined in (7) for the system model in Section II is expressed as

\[
\mathbb{E}_{h_{ii}} \{ R_i \} = \log(\alpha_i) + e^{\frac{\alpha_i}{\sigma_{\pi_i}^2}} E_1 \left( \frac{\alpha_i}{\sigma_{\pi_i}^2} \right) - e^{\frac{\alpha_i}{\sigma_{\pi_i}^2}} E_1 \left( \frac{1}{\sigma_{\pi_i}^2} \right).
\] (8)

**Proof:** Using integration by parts along with \([15], 5.1.28\) it is easy to prove that for any positive \( \alpha \) and \( \delta \)

\[
\int_0^\infty \frac{1}{\delta} e^{-\frac{x}{\delta}} \log(\alpha + \beta) d\beta = \log(\alpha) + e^{\frac{\alpha}{\delta}} E_1 \left( \frac{\alpha}{\delta} \right).
\] (9)

Therefore, by substituting (9) in (7), (8) is derived.
Focusing on the behavior of (8) in the case of interference dominated systems the following proposition can be proved.

**Proposition 1.** For an interference dominated system, the expected rate (8) can be closely approximated as

$$E_{|h_i|} \{ R_i \} = \log \frac{\alpha_i}{\sigma^2_{\alpha i}} + e^{\gamma} E_1 \left( \frac{\alpha_i}{\sigma^2_{\alpha i}} \right) + \gamma$$

(10)

where $\gamma \approx 0.5772$ is the Euler-Mascheroni constant.

**Proof:** Assuming that interference is the dominant degradation factor, or equivalently that interference power is sufficiently larger than noise power, it holds that $\sigma^2_{\alpha i} \gg 1$. Hence, the last term on the right hand side of (8) asymptotically converges to [15, 5.1.11]:

$$e^{\gamma} E_1 \left( \frac{1}{\sigma^2_{\alpha i}} \right) \to -\log \left( \frac{\alpha_i}{\sigma^2_{\alpha i}} \right).$$

(11)

Consequently, one can approximate (8) by:

$$E_{|h_i|} \{ R_i \} = \log \left( \frac{\alpha_i}{\sigma^2_{\alpha i}} \right) + e^{\gamma} E_1 \left( \frac{\alpha_i}{\sigma^2_{\alpha i}} \right) + \gamma.$$  

(12)

In the following section, a receive BF design is proposed based on maximizing the expected rates $E_{|h_i|} \{ R_i \}, i = \{1, 2\}$ as calculated by employing (10).

**IV. OPTIMAL RECEIVE BEAMFORMING**

Capitalizing on the simple closed form expression (10), one can explicitly derive the optimal BF vector for interference dominated systems, i.e., the BF vector that maximizes (10). This is illustrated in the following proposition:

**Proposition 2.** The optimal receive BF vector at BS$_i$ is the unit-norm dominant generalized eigenvector (DGE) $^2$ of the pair $((I_M + h_{ij}h_{ij}^H), R_{ji}), i \neq j$.

**Proof:** The optimal solution is reached by solving the following optimization problem at BS$_i$:

$$w_i^{opt} = \arg \max_{w_i} E_{|h_i|} \{ R_i \}, i = \{1, 2\}.$$  

(13)

To start, one can rewrite $E_{|h_i|} \{ R_i \}$ as

$$E_{|h_i|} \{ R_i \} = U(\lambda),$$

where

$$\lambda_{w_i} = \frac{1 + w_i h_i h_i^H w_i^H}{w_i R_{ji} w_i^H}, i = \{1, 2\}. $$

(14)

The function $U(\cdot)$ is defined as

$$U(\lambda) = \log(\lambda) + e^\lambda E_1(\lambda) + \gamma$$

(15)

where $\lambda$ is a dummy variable. Since $\|w_i\| = 1$, $\lambda_{w_i}$ can be expressed as

$$\lambda_{w_i} = \frac{w_i (I_M + h_i h_i^H) w_i^H}{w_i R_{ji} w_i^H}, i \in \{1, 2\}.$$  

(16)

By differentiating $U(\lambda)$ and using [15, eq. 5.1.26] one can prove that $U(\lambda)$ is an increasing function of $\lambda$. Thus, the optimization problem (13) is equivalent to the Rayleigh - Ritz ratio optimization problem

$$w_i^{opt} = \arg \max_{\lambda_i} \lambda_{w_i}, i \in \{1, 2\}.$$  

(17)

By setting the derivative of (16) equal to zero, it can be seen that the optimal BF vector is the one that satisfies the equation

$$(I_M + h_i h_i^H) w_i^H = \lambda w_i R_{ji} w_i^H.$$  

(18)

As a result, by inspecting (18) it can be seen that the optimal BF vector for BS$_i$ is the eigenvector corresponding to the maximum eigenvalue of $R_{ji}^{-1}(I_M + h_i h_i^H)$, that is, the DGE of the matrix pair $((I_M + h_i h_i^H), R_{ji})$.

The above solution is reminiscent of the one in [14], with the key difference lying in the fact that the left hand side of (18) uses an instantaneous ‘estimate’ of the covariance $R_{ji}$ exploited in [14]. Moreover, unlike [14], the obtained solution is independent of the number of receive antennas. In the following section, new, low complexity user scheduling algorithms are introduced that exploit the derived BF.

**V. COORDINATED USER SCHEDULING ALGORITHM**

Given $N_i, i = 1, 2$, users within the coverage area (CA) of BS$_i$, optimal user selection with respect to the criterion adopted for BF design, would involve searching over the $N_i \times N_j$ possible user pairs in order to find the pair $\{\kappa_1, \kappa_2\}, \kappa_1 \in [1, N_i], \kappa_2 \in [1, N_j]$ that maximizes the sum of conditional expected rates, i.e. the pair

$$\{\kappa_1, \kappa_2\} = \arg \max_{k_1, k_2} \left( E_{|h_{k_1}|} \{ R_{k_1}(k_1, k_2) \} + E_{|h_{k_2}|} \{ R_{k_2}(k_1, k_2) \} \right)$$

(19)

where $R_{k_i}(k_1, k_2)$ denotes the instantaneous information rate at BS$_i, i = 1, 2$, when users indexed by $k_1$ and $k_2$ are scheduled by BS$_1$ and BS$_2$ respectively. To overcome the increased complexity of this exhaustive coordinated approach, motivated by [21], we propose a novel greedy user selection algorithm. The algorithm is based on an iterative optimization procedure where, on each iteration, BS$_i$ decides in favor of user $\kappa_i$ that maximizes the sum of conditional expected rates $E_{|h_{k_1}|} \{ R_{k_1}(\cdot, \cdot) \} + E_{|h_{k_2}|} \{ R_{k_2}(\cdot, \cdot) \}$, given that BS$_j, j \neq i$ has decided upon user $\kappa_j$. In what follows this algorithm, henceforth characterized as the centralized coordinated algorithm, is summarized:

1) **Initialization:**

- Randomly select a user $\kappa_2$ in BS area 2
- Set the iteration number, $\nu = 0$

2) **Step 1:**

- Set $\nu = \nu + 1$
- Select the user with index $\kappa_1$ in BS area 1 that satisfies

$$\kappa_1 = \arg \max_{k_1} \left( E_{|h_{k_1}|} \{ R_{k_1}(k_1, \kappa_2) \} + E_{|h_{k_2}|} \{ R_{k_2}(k_1, \kappa_2) \} \right).$$

(20)

3) **Step 2:**

- Select the user with index $\kappa_2$ in BS area 2 that satisfies

$$\kappa_2 = \arg \max_{k_2} \left( E_{|h_{k_1}|} \{ R_{k_1}(\kappa_1, k_2) \} + E_{|h_{k_2}|} \{ R_{k_2}(\kappa_1, k_2) \} \right).$$

(21)
4) Step 3:

- If the number of iterations \( \nu \) is smaller than a predefined threshold, go back to Step 1 and iterate.

With the proposed algorithm, at each iteration the decision of \( BS_j \), given the previous decision of \( BS_i \), leads to the selection of a user that increases, or, in the worst case, preserves the expected sum rate obtained from the previous iteration. Thus, the algorithm always converges to a solution. However, a drawback of this algorithm is the fact that apart from slow-varying covariance information, also instantaneous CSI needs to be exchanged between the BSs. As a result, in addition to the centralized coordinated algorithm, we propose an iterative distributed coordinated scheduling approach, in which at each iteration, \( BS_i \) selects user \( \kappa_i \) that maximizes the expected rate of its own CA rather than the expected sum rate, that is

\[
\kappa_i = \arg \max_{k_i} \mathbb{E}_{\mathbf{h}_i}[R_i(k_i, \kappa_j)], i \in \{1, 2\}, j \neq i \quad (22)
\]

where \( \kappa_j \) is the user already selected by \( BS_j \).

VI. NUMERICAL RESULTS

In order to evaluate the performance of the proposed receive BF and user scheduling schemes, extensive Monte Carlo (MC) simulations of a two-BS system have been performed. In the simulations setup, a spectrum sharing scenario is considered, in which two BSs are located in such a way that the two CAs overlap with each other (by a factor of 50% in our scenario). In this scenario it has been assumed that the users in each CA were uniformly distributed and that both BS CAs have common signal to noise ratio (SNR) characteristics, i.e., the same transmit power and BS CA radius. Some further simulation parameters are given in Table I.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>BASIC SIMULATION PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>BS CA radius</td>
<td>1 km</td>
</tr>
<tr>
<td>Number of BS antennas</td>
<td>2</td>
</tr>
<tr>
<td>Path loss exponent</td>
<td>3</td>
</tr>
<tr>
<td>Carrier frequency</td>
<td>2 GHz</td>
</tr>
<tr>
<td>Antenna spacing</td>
<td>( \lambda/2 )</td>
</tr>
<tr>
<td>AOA distribution</td>
<td>Gaussian</td>
</tr>
<tr>
<td>Multipath angle spread</td>
<td>30 degrees</td>
</tr>
<tr>
<td>Number of spatially independent paths</td>
<td>30</td>
</tr>
</tbody>
</table>

The coefficients of the covariance matrices \( \mathbf{R}_{ji}, i, j \in \{1, 2\} \) are computed as in [22]:

\[
\mathbf{R}_{ji}(m, n) = \frac{\beta_{ji}}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} e^{i2\pi \frac{D}{\lambda}(n-m)\cos(\theta+\bar{\theta}_{ji})- \frac{\sigma^2}{2\sigma^2}\theta} \, d\theta
\]

where \( \beta_{ji} \) is the distance-based path loss between the user scheduled by \( BS_j \) and \( BS_i \), \( \sigma \) is the multipath angle spread, considered to be the same for both desired and interference channels, \( D \) is the antenna spacing at BS side, \( \lambda \) is the wavelength of the signal and \( \bar{\theta}_{ji} \) is the mean Angle of Arrival (AOA) between the user scheduled by \( BS_j \) and \( BS_i \). The distance-based path loss is defined as:

\[
\beta_{ji} = \frac{\alpha}{d_{ji}^{\gamma_{path}}}
\]

where \( \alpha \) stands for a constant which depends on the prescribed average SNR at the BS CA edge, \( d_{ji} \) is the geographical distance between the user scheduled by \( BS_j \) and \( BS_i \) and \( \gamma_{path} \) denotes the path loss exponent.

Fig. 2. Expected sum rate vs. SNR for the two examined BF's for \( N_1 = N_2 = 1 \) user/BS CA.

Fig. 3. Expected sum rate vs. SNR for \( N_1 = N_2 = 10 \) users/BS CA.

Fig. 4. Expected sum rate vs. \( N \), SNR=15dB.

In Fig. 2 the achievable expected sum rate of the proposed BF scheme is depicted as a function of the average SNR at the BS CA edge and compared with the expected sum rate of the Maximal Ratio Combining (MRC) BF. The results correspond to the case that \( N_1 = N_2 = 1 \). MC averaging is performed with respect to the two users’ positions/covariance structure and instantaneous channel realizations. The motivation behind
this comparison is the fact that MRC is perhaps the closest, in terms of required CSI, to the proposed BF scheme. The sum rate curves demonstrate the superiority of the proposed BF scheme for almost all the examined SNR levels.

In Fig. 3 the expected sum rate for systems employing the proposed BF and scheduling schemes is depicted as a function of the average SNR at the CA edge for the case that $N_1 = N_2 = 10$. Specifically, four different user scheduling scenarios are compared: the uncoordinated one (random user selection), the exhaustive coordinated one, the centralized coordinated one as well as the distributed coordinated one (one iteration). It is evident that both the centralized and distributed greedy approaches achieve a performance which is marginally lower than the exhaustive one. Additionally, the coordination gain is evident for the whole SNR range.

In Fig. 4 the expected system’s capacity is depicted with respect to the number of users per BS CA, assuming that $N_1 = N_2 = N$. The average SNR level at the BS CA edge for our simulation setup was set equal to 15dB. We observe that both the centralized and the distributed (with only one iteration) low complexity approaches achieve a slightly inferior performance with respect to the coordinated exhaustive approach. Moreover, both the proposed and the coordinated exhaustive algorithms, offer a clear multi-user diversity gain that increases with respect to $N$.

VII. CONCLUSIONS

The uplink of the two-user multiple antenna interference channel has been investigated. Closed form expressions for the expected rate conditioned on instantaneous CSI for the direct channels have been derived. By examining the asymptotic behavior of the derived expression with respect to interference dominated systems, an optimal receive BF scheme has been developed that exploits both instantaneous (for direct channels) and statistical (for interfering channels) CSI. Finally, novel low complexity user scheduling algorithms have been proposed, that achieve performance similar to the optimum, with respect to the available CSI, scheduling policy. Interesting extensions can be made concerning the case of multi-interference BSs.

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