Optimized Data Symbol Allocation in Multicell MIMO Channels

Rajeev Gangula, Paul de Kerret, David Gesbert and Maha Al Odeh
Mobile Communications Department, Eurecom
2229 route des Crêtes, 06560 Sophia Antipolis, France
{gangula,dekerret,gesbert,Al-Odeh}@eurecom.fr

Abstract—In this work, we consider the joint precoding across K distant transmitters (TXs) towards K single-antenna receivers (RXs). In practical networks, cooperation between TXs is limited by the constraints on the backhaul network and the common approach to limit the backhaul overhead is to form small disjoint clusters of cooperating TXs. Yet, this limits the performance due to interference at the cluster edge. We overcome this problem by directly optimizing the allocation of the user’s data symbol without clustering but solely subject to a constraint on the total number of symbols allocated. Since the problem of optimal data symbol allocation is of combinatorial nature, we use a greedy approach and develop greedy algorithms having low complexity while incurring only small losses compared to the optimal data symbol allocation. Moreover, the algorithms are shown to be Multiplexing Gain (MG) optimal in many settings. Simulations results confirm that our approach outperforms dynamic clustering methods from the literature.

I. INTRODUCTION

In order to achieve high spectral efficiencies in future cellular systems, full frequency reuse is considered. However, such systems severely suffer from Inter-Cell Interference (ICI) thus degrading the throughput, especially for cell edge users. Promising approaches for mitigating the ICI are Multicell MIMO methods (or CoMP in the 3GPP terminology) and are considered for next generation wireless networks. In Multicell MIMO the set of user’s data symbols are shared across a group of cooperating TXs thereby jointly serving the set of users in a distributed MIMO fashion. With full data symbol sharing and CSI sharing, Multicell MIMO can be seen as virtual multiple-antenna broadcast channel (BC) channel [1].

Sharing of data symbols and Channel State Information (CSI) implies high backhaul and feedback overhead which increases as the number of cooperating terminals in the network increases. The common approach to reduce the overhead and make cooperation suitable for practical application is to form disjoint clusters of cooperating TXs [2]. Yet, the cluster-edge users still suffer from inter cluster interference. This method is improved in [3] where greedy algorithm which aims at optimizing the formation of disjoint cluster of TXs at each time slot is developed. Yet, dynamic clustering still suffers from inter-cluster interference and becomes complicated when the cluster size increases. In [4], a scheme is presented where each RX chooses the set of TXs serving it such that overlapping clusters are formed. However, the design of the clusters is not optimized and the set of TXs is selected based on simple heuristics. Finally, in [5]–[7], the impact of partially sharing the users data is analytically studied for one dimensional networks.

In this paper we consider an alternative to clustering and we directly optimize the data symbol allocation subject to a constraint on the total number of symbols being routed. Note that in directly optimizing the data symbol allocation, the knowledge of the CSI for the whole multiuser channel is necessary for all TXs to apply joint precoding. This represents a strong requirement for large cooperation areas and a method reducing the CSI sharing necessary for joint precoding is proposed in a companion paper [8].

The main contributions of the paper are as follows: First, we develop a precoding scheme adapted to this partial data sharing setting. We analyze the MG and then we develop a greedy algorithm to obtain a trade off solution with good performance and low complexity. Our approach is based on extending the approach used for TX antenna selection to Multicell MIMO system, which has been studied in the context of single user and Multiuser MIMO systems [9], [10].

II. SYSTEM MODEL

We first present the multicell MIMO network model and then we introduce the model for the backhaul data sharing.

A. Multi-cell MIMO

We consider a network that consists of K TXs, with TX k equipped with N_k antennas and K single-antenna RX’s. The total number of antennas at the transmit side is denoted by \( N_T = \sum_{k=1}^{K} N_k \). The k-th RX receives

\[
y_k = h_k^H x + \eta_k
\]  

(1)

where \( h_k^H \in \mathbb{C}^{1 \times N_T} \) represents the channel vector corresponding to the k-th RX, \( x \in \mathbb{C}^{N_T \times 1} \) represents the combined transmit signals of all users sent by all the transmit antennas and \( \eta_k \sim \mathcal{C}\mathcal{N}(0, \sigma^2) \) represents the i.i.d. complex circular-symmetric additive Gaussian noise at the k-th RX. The whole multiuser channel matrix of the system is

\[
H = [h_1, h_2, \ldots, h_K]^H.
\]  

(2)
The channel is block fading and models a Rayleigh fading scenario with a long term pathloss corresponding to a cellular setting. Thus, the entries of the channel matrix $H$ read as $H_{ij} = \gamma_{ij} G_{ij}$ where $G_{ij}$ is i.i.d. $CN(0, 1)$ to model the Rayleigh fading and $\gamma_{ij}$ is a positive real number modeling the long term attenuation. It is assumed that all the TXs have the knowledge of the CSI of the whole multiuser channel. Multi-transmitter cooperative processing in the form of joint linear precoding is adopted. Thus, the transmitted signal $x$ is obtained from

$$ x = Ts = \begin{bmatrix} t_1 \\ \vdots \\ t_K \end{bmatrix} \begin{bmatrix} s_1 \\ \vdots \\ s_K \end{bmatrix} $$

where $s \in \mathbb{C}^{K \times 1}$ is the vector of transmit symbols (whose entries are independent $CN(0, 1)$), $T$ is the precoding matrix and $t_k \in \mathbb{C}_{NJ}^{N \times 1}$ is the beamforming vector of the symbol $k$.

Noting that in a statistically symmetric isotropic network, fulfilling a sum power constraint will lead to an equal average power used per TX, we consider for simplicity a sum power constraint $\|T\|_F^2 = K \times P$. We also assume that all data streams are allocated with an equal amount of power so that $\forall k, \|t_k\|^2 = P$.

### B. Zero-Forcing Precoders

Conventional Zero Forcing (ZF) results in complete removal of interference at the receivers. This is optimal at high SNR but not necessarily at intermediate SNR. To further improve the performance of the precoder, regularized ZF precoder, which achieves good performance even at intermediate SNR is used and is given by [11]

$$ T = \frac{\sqrt{KP}}{\|HH^H + \alpha I\|_F^{-1}} HH^H (HH^H + \alpha I)^{-1} $$

(4)

where $\alpha = K \times (\sigma^2/P)$ is the regularization constant.

### C. System Performance model

In this work we aim at maximizing the sum-rate of the system under the constrained backhaul overhead. The sum-rate of the system is equal to

$$ R = \sum_{k=1}^{K} R_k = \sum_{k=1}^{K} \log_2 (1 + \text{SINR}_k) $$

(5)

where the signal to interference plus noise ratio (SINR) of the $k$-th data stream is given by

$$ \text{SINR}_k = \frac{|h_k^H t_k|^2}{\sigma^2 + \sum_{j=1, j\neq k}^{K} |h_j^H t_j|^2} $$

(6)

Furthermore, the Multiplexing gain (MG) of the system is defined as

$$ M_G \triangleq \sum_{k=1}^{K} M_{G_k} \triangleq \sum_{k=1}^{K} \lim_{P \rightarrow \infty} R_k(P) / \log_2(P) $$

(7)

### D. Backhaul Data Symbol Routing

To represent the effect of the allocation of the user’s data symbols to the TXs, we introduce the concept of routing matrix which specifies to which TXs the symbol of a given user is being routed to, independently of any pre-determined clustering concept. In realistic settings, e.g. cellular networks, we are concerned with the sharing of the user’s data symbols to the TXs and not to each antenna individually, as the different antennas at a given TX are collocated and perfectly cooperate. To model the data symbol sharing we define the routing matrix $D \in \{0, 1\}^{K \times K}$ as the matrix whose element $D_{ij} \in \{0, 1\}$ is 1 if symbol $s_j$ is allocated to TX $i$ and 0 otherwise. The number of user’s data symbol shared in the backhaul network, that we also call routing links can be seen to be equal to $d = \|D\|_F^2$.

Yet, the antenna configuration has not been taken into account in the routing matrix. Therefore, we need to introduce more notations to represent it. Thus, we define the expansion matrix $E \in \mathbb{C}^{NJ \times K}$ as

$$ E \triangleq [A_1^T \cdots A_K^T]^T $$

(8)

where the matrix $A_k \in \mathbb{C}^{NJ \times K}$ is defined as $A_k \triangleq 1_{[NJ \times 1]} e_k$. The vector $e_k \in \mathbb{C}^{NJ \times 1}$ is the $k$-th vector from the canonical basis. Now the matrix $D \triangleq ED$ can be defined and represents well the data sharing constraints, as will be shown in Section III.

### E. Optimization Problem

In order to optimize the backhaul routing directly, we formulate the following sum-rate maximization problem:

$$ \begin{align*}
\text{maximize} & \quad \sum_{k=1}^{K} R_k \\
\text{subject to} & \quad d = \|D\|_F^2 \leq d^*.
\end{align*} $$

(9)

where $d^*$ is the constraint on the backhaul routing overhead. Above problem is a discrete combinatorial optimization problem and generally exhaustive search is required to find the optimal data symbol allocation. For exhaustive search, a total of $\binom{K}{d^*}$ data allocation combinations need to be searched over, i.e the complexity grows as $\binom{K}{d^*}$. This is prohibitive even for small number of cooperating nodes, for example $K = 7$ considered in this paper. Therefore, we propose two low complexity greedy algorithms. At each iteration of the algorithm a data symbol is rooted (resp. removed) to (from) TX so as to achieve the largest sum-rate. This is done until the constraint on the number of routing links is reached.

### III. PRECODER OPTIMIZATION

In this section we consider the design of the precoder $T$ given the routing matrix $D$. Note that each beamforming vector can be derived independently due to the ZF constraints. If one TX does not receive one symbol, it cannot participate into the transmission of that symbol and the coefficient used for that beamformer at that TX is then 0. Therefore the precoder with constrained backhaul overhead will be of the
form $\mathbf{T} \odot \mathbf{D}$, where $\odot$ is the element wise product. The beamforming vector $t_k$ transmitting the symbol $k$ is obtained from the following optimization:

$$\text{minimize} \left\| \mathbf{H} \left( t_k \odot \mathbf{D}_{:,k} \right) - \phi \right\|_2^2$$

(10)

where $\phi = \mathbf{HT}$ and the precoder $\mathbf{T}$ is obtained using (4) with full routing.

In optimization (9) we consider no predefined constraint on the routing pattern's structure. Therefore, there may be some columns of the routing matrix containing only zero i.e. a user is not served. This is some kind of user selection and accounts for a positive MG with partial data sharing. Thus, we define a non-active user as a user whose data symbol is not routed to any TX, i.e., $\mathbf{D}_{:,k} = 0_{K \times 1}$, or the power allocated to the user $p_k = 0$. If there are non-active users in the system, the precoding scheme needs to be modified so that interference is removed only at active users. We start by introducing some notations and we denote the set of indices such that $t_k \odot \mathbf{D}_{:,k} \neq 0$ by $\mathcal{J}$ and the reduced vector $\tilde{t}_k(\mathcal{J}) \in \mathbb{C}^{n_2 \times 1}$ with $n_2 = |\mathcal{J}|$ made of the elements of $\mathcal{J}$. Further more, the set of indices corresponding to the active-users is denoted by $\mathcal{A}$, with $n_1 = |\mathcal{A}|$. The matrix $\mathbf{H}(\mathcal{A},\mathcal{J}) \in \mathbb{C}^{n_1 \times n_2}$ is used to represent the channel containing only the rows and columns consisting in $\mathcal{A}$ and $\mathcal{J}$ respectively. Finally, $\tilde{\phi} = \phi(\mathcal{A},\mathcal{J})$ represents a sub matrix of $\phi$ formed by keeping the rows and columns of the active-users. The beamforming vector $\tilde{t}_k$ can now be obtained from the following optimization:

$$\text{minimize} \left\| \mathbf{H}(\mathcal{A},\mathcal{J}) \tilde{t}_k(\mathcal{J}) - \tilde{\phi}_k \right\|_2^2$$

(11)

which can be solved as a conventional Least Square problem. The beamforming vector of user $k$ is then obtained by reinserting the coefficients obtained in $\tilde{t}_k$ at the positions corresponding to $\mathcal{J}$ in $t_k$.

IV. MULTIPLEXING GAIN ANALYSIS

In this section we consider the fundamental limit behind the optimization problem (9) and look for the maximum MG achievable when there is a constraint on the number of data symbols allocated. In the following analysis we assume that there will always be enough users to serve in the system so that the MG is not restricted by the total degrees of freedom available at the RXs. Mathematically, if $\gamma$ is the maximum MG, then $K \geq \gamma$.

**Proposition 1.** In order to achieve the maximum MG $\gamma$ under the constraint $\|\mathbf{D}\|_F^2 \leq d^*$, the following conditions needs to be satisfied

$$\sum_{k=1}^{K_{TX}} N_k \geq \gamma \text{ (ZF Feasibility )},$$

$$\gamma \times K_{TX} \leq d^* \text{ (Sharing Constraint )}.$$  

(12)

**Proof:** W.l.o.g, we consider the TXs to be ordered in decreasing number of antennas $N_1 \geq N_2 \geq \ldots \geq N_K$. To achieve maximum MG of $\gamma$ with ZF precoding, we need to satisfy the following conditions.

- **ZF Feasibility:** To cancel the interference at $\gamma - 1$ TXs, each one of the $\gamma$ data symbols should be transmitted from at least $\gamma$ antennas.
- **Sharing Constraint:** To achieve ZF Feasibility, let all the $\gamma$ data symbols are routed to $K_{TX}$ TXs. Then the total number of routing links can be seen to be equal to $\gamma \times K_{TX}$ and it should be less than or equal to the given constraint on the number of routing links $d^*$.

**Corollary 1.** The maximum MG $\gamma$ achieved with all TXs having $N$ antennas under the constraint $\|\mathbf{D}\|_F \leq d^*$ is given by

$$\gamma = \max \left\{ N \left\lfloor \sqrt{d^*/N} \right\rfloor, \left\lfloor \frac{d^*}{\sqrt{d^*/N}} \right\rfloor \right\}.$$  

(13)

**Proof:** Since all the TXs have $N$ antennas, $N_k = N, \forall k$. Rewriting the constraints of the problem (12), we get

$$\gamma \leq \frac{d^*}{K_{TX}} \text{ and } \gamma \leq NK_{TX}.$$  

(14)

Therefore, the optimization problem can be reformulated as

$$\gamma = \max \min_{K_{TX}} \left\{ \frac{d^*}{K_{TX}}, NK_{TX} \right\}.$$  

(15)

Since the first argument in the minimization is increasing in $K_{TX}$ and the second one is decreasing in $K_{TX}$, the maximum occurs when both terms are equal such that $K_{TX} = \sqrt{d^*/N}$. Yet, $K_{TX}$ has to be an integer, and from the analysis of (15), we conclude that $K_{TX} = \left\lfloor \sqrt{d^*/N} \right\rfloor$ or $K_{TX} = \left\lceil \sqrt{d^*/N} \right\rceil$.

If $K_{TX} = \sqrt{d^*/N} \notin \mathbb{N}$ we look for the following cases.

- If $K_{TX} = \left\lfloor \sqrt{d^*/N} \right\rfloor$ then $\frac{d^*}{K_{TX}} \geq NK_{TX}$ and

$$\gamma = N \left\lfloor \sqrt{d^*/N} \right\rfloor.$$  

(16)

- If $K_{TX} = \left\lceil \sqrt{d^*/N} \right\rceil$ then $\frac{d^*}{K_{TX}} < NK_{TX}$ and

$$\gamma = \left\lfloor \frac{d^*}{\sqrt{d^*/N}} \right\rfloor.$$  

(17)

Inserting (16), (17) in (15) concludes the proof.

From the MG analysis, we now know how many users should be served to maximize the MG, which is in itself interesting and will be useful in evaluating the MG optimality of our algorithm.

V. GREEDY DATA SYMBOL ALLOCATION

A. Greedy Algorithms

We consider two possible versions of the greedy algorithms. In Decreasing greedy algorithm (DEC), we initialize the routing matrix $\mathbf{D} = 1_{N \times K}$ and at each iteration the element of $\mathbf{D}$ which causes the least degradation in the sum-rate is set to 0 (i.e. removing the routing link of one symbol to one of
the TX). This process is continued till the constraint on the number of backhaul links is reached. Similarly the Increasing greedy algorithm (INC) starts with $D = 0_{N \times K}$ and at each iteration the element of $D$ which leads to the largest increase in the sum-rate is set to 1 until the constraint on the number of backhaul links is reached. Due to the space limitation we omit the details of INC and we refer the reader to [12].

### Algorithm 1 Decreasing greedy algorithm

**Input:** $H$, $d^*$, **Output:** $D$, $p$

1. Initialize: $D = 1_{N \times K}$, $p = P \times 1_{K \times 1}$
2. $C_{\text{init}} = 0$, $D_{\text{temp}} = D$, $p = p^*$
3. for RX $k = 1$ to $K$ do
4. for TX $l = 1$ to $K$ do
5. if $\{D_{\text{temp}}\}_{lk} \neq 0$ then
6. $D_{\text{temp}}\_{lk} = 0$
7. $T = \text{precoding } (D_{\text{temp}}, H, p) \% (\text{From Sec.III})$
8. $C_{\text{sum}} = \text{sumrate}(T, H) \% (\text{using (5)})$
9. if $C_{\text{sum}} \geq C_{\text{init}}$ then
10. $m = l$, $n = k$, $C_{\text{init}} = C_{\text{sum}}$
11. end if
12. $D_{\text{temp}} = D$
13. end if
14. end for
15. $\{D\}_{mn} = 0$, $T = \text{precoding } (D, H, p)$
16. $C_{\text{sum}} = \text{sumrate}(T, H) \% (\text{using (5)})$
17. $p = \text{Power allocation}(p, T, C_{\text{sum}}) \% (\text{C.f Sec.V-B1})$
18. end for

### B. Analysis of Greedy Algorithms

Using the MG analysis in Section IV, we can make some improvements to the constrained greedy algorithms. The greedy algorithms at any given step try to maximize the performance at each step and they do not necessarily achieve the maximum MG. For example, in a system consisting of $K = 7$ TX/RX pairs, in the DEC we start with full cooperation and therefore MG is 7. Removing one symbol form an antenna results in creating interference to all the other streams such that MG is 1, whereas the optimal strategy (at high SNR) with 48 routing links is to serve 6 users so that MG of 6 is achieved. By not being MG optimal does not hurt the performance at low SNR but in intermediate and high SNR but it has a considerable effect on the performance of the algorithm. Therefore, we propose some improvements to the greedy algorithms.

#### 1) Binary Power Control: In Binary Power Control (BPC) power allocated to $k$-th user $p_k$ takes only two values 0 or $P$ [13]. In this we use the idea of BPC in Algorithm 1 in step 18 to make it MG optimal. After each step of removing a data symbol from $D$ we check whether by turning off a user completely, results in an increase in the sum-rate. If the sum-rate increases by turning off $k$-th user completely, then the power allocated to that user $p_k = 0$. We will show in the following proposition that MG can be increased by turning off the correct number of users at high SNR.

#### Proposition 2. With single antenna TXs, the DEC with BPC as described in Algorithm 1 achieves the optimal MG.

**Proof:** By using the definition of MG and ZF, proving the MG optimality is equivalent to show that the algorithm makes sure that $|A| = \lfloor \sqrt{d^*} \rfloor$ and $\forall k \in A$ the beamforming vectors $b_k(J) \in C_{n \times 1}$, $n_2 = |J| = \lfloor \sqrt{d^*} \rfloor$, where $A$ is the active user index set and $\lfloor \sqrt{d^*} \rfloor$ is the maximum MG that can be achieved with $d^*$ routing links [C.f. Section IV].

We consider asymptotically high SNR such that the behavior of the algorithm can be predicted as it selects at each step the routing with the largest MG. Let start from the first step with $|A| = K$ and $\forall k \in A$, $b_k(J) \in C_{n \times 1}$, $n_2 = |J| = K$. Removing the $k$-th symbol from $l$-th TX i.e. $\{D\}_{lk} = 0$ results in $b_k(J) \in C_{n \times 1}$ with $n_2 = |J| = |A| - 1$. The reduced channel matrix corresponding to $b_k(J)$ is $H(A,J) \in C_{|A| \times |J|}$. Thus, rank $(H(A,J)) = |J|$ and the solution of (11) will not be able to ZF the interference, resulting in a MG of 1. At this stage turning off a data stream results in an increase in MG thus BPC will turnoff a user. Suppose $p_k = 0$, then the $i$-th user is non-active which results in $|A| = |J|$ and the system is in a square setting such that ZF is feasible and MG is $|A|$. Since $p_k = 0$, removing the $i$-th user’s data symbol from the other TXs does not diminish the MG or sum-rate. Therefore, in the following iterations the DEC algorithm will remove the $i$-th symbol from all the other TXs resulting in $T_{,i} = 0_{K \times 1}$. A square setting is then obtained and the same method can be repeated until the constraint $d^* = ||D||_2^2$ is reached.

#### VI. Simulations

We simulate a multicell network consisting of $K = 7$ TXs and RXs. The pathloss between the $l$-th TX and the $k$-th RX which are separated by distance $r_{lk}^{km}$ is $128 + 37.6 \log_{10}(r_{lk}^{km})$. The noise power at the receiver is $P_{\text{noise}} = -104$ dBm. In the simulations, an average cell edge SNR of 20 dB is maintained by selecting the transmit power $p_T = 50$ dBm and the cell radius equal to 1.5 km. The simulation results are averaged over 1000 uniformly randomly generated user positions (such that each cell has exactly one user) and Rayleigh fading realizations.

Fig. 1 shows the sum-rate achieved by Algorithm 1 with single antenna TXs for different data sharing in terms of the average SNR for cell edge user when the power per TX changes. The data sharing is represented by the percentage of data symbols shared compared to full cooperation i.e., $d^*/K^2$. For the dynamic clustering approach, we consider the algorithm from [3], with clusters of size 4 and 3 (Sharing % is $(4^2 + 3^2)/7^2 \approx 50\%$) and 6 and 1 $(6^2 + 1^2)/7^2 \approx 75\%$). As expected, the optimized data symbol allocation results in better performance than the clustering methods. It can be seen that the Algorithm 1 with 50% sharing outperforms the dynamic clustering solution with 75% sharing.

In Fig. 2 the performance of the two algorithms with multiple antennas at the TXs with $[N_1, N_2, ..., N_7] = [2 \ 1 \ 1 \ 2 \ 1 \ 1 \ 2]$.
3 1 2 1] is considered. The DEC is close to optimal at high percentage of data sharing and the INC is close to optimal for low percentage of data sharing. The greedy algorithms outperforms dynamic clustering at all percentages of data sharing. From the performance curves of both the INC and the DEC we get the nice intuition of the performance of the optimal data symbol allocation. Note that the sum-rate curve of the INC is not smooth. This is not a consequence of the averaging but caused due the variations in MG as the INC is not MG optimal(C.f V-B). Finally, the greedy algorithms outperforms dynamic clustering even when the the routing solution is obtained based on only the longterm CSI. In long term greedy, the routing is computed only once at the beginning when a user group selected randomly and kept same for remaining 100 Rayleigh fading realizations while keeping the selected user positions are fixed.

VII. CONCLUSION

In this work, an alternative to the clustering has been provided where direct optimization on data symbol allocation subject to the constraint on the backhaul overhead is done. By the simulation results we have shown that the proposed routing algorithm out performs the the dynamic clustering algorithms from the literature. This was an expected result as the data symbol allocation is optimized without the constraint of forming disjoint clusters. Furthermore, the routing solution based only on the long term information outperforms the dynamic clustering, while still having less cost on the network architecture and therefore appears as a practical alternative to clustering in Muticell MIMO systems. By exploiting the inherent sparsity in the channel in a large network, the approach of directly optimizing the data symbol allocation is extended for large cooperation domains [14]. Finally, the future work consists of studying the partial data sharing setting with multiple antenna RXs [15].

REFERENCES