RECEIVER DIVERSITY WITH BLIND FIR SIMO CHANNEL ESTIMATES

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ABSTRACT

Traditionally, the performance of blind SIMO channel estimates has been characterized in a deterministic fashion, by identifying those channel realizations that are not blindly identifiable. In this paper, we focus instead on the performance of Linear Equalizers for fading channels when they are based on blind channel estimates. Our analysis shows that with Zero Forcing Linear Equalizer (ZF-LE) at least one order of the diversity is lost depending on the way by which the scalar ambiguity that results from the blind channel estimation is resolved. However, in some Tx scenarios we are able to recover the diversity with MMSE-LE. Various Tx scenarios are considered in detail.

Index Terms— channel estimation, blind, receiver diversity.

1. INTRODUCTION

Consider a linear modulation scheme and single-carrier transmission over a Single Input Multiple Output (SIMO) linear channel with additive white noise. The multiple (subchannel) outputs will be mainly thought of as corresponding to multiple antennas. After a receive (Rx[1]) filter (possibly noise whitening), we sample the Rx signal to obtain a discrete-time system at symbol rate[2]. When stacking the samples corresponding to multiple Rx antennas in column vectors, the discrete-time communication system is described by:

$$\underbrace{\boldsymbol{y}_k}_{n_r\times 1} = \underbrace{\mathbf{h}[q]}_{n_r\times 1}\,\underbrace{\boldsymbol{a}_k}_{1\times 1} + \underbrace{\boldsymbol{v}_k}_{n_r\times 1} \qquad (1)$$

where k is the symbol (sample) period index, n_r is the number of Rx antennas. The noise power spectral density matrix is $S_{\boldsymbol{vv}}(z) = \sigma_v^2\, I_{n_r}$, q^{-1} is the unit sample delay operator: $q^{-1}\, a_k = a_{k-1}$, and $\mathbf{h}[z] = \sum_{i=0}^{L} \mathbf{h}_i\, z^{-i}$ is the SIMO channel transfer function in the z domain. The channel delay spread is L symbol periods. In the Fourier domain we get the vector transfer function $\mathbf{h}(f) = \mathbf{h}[e^{j2\pi f}]$.

We introduce the vector containing the SIMO impulse response coefficients[3] $\mathbf{h} = [\mathbf{h}_0^T \cdots \mathbf{h}_L^T]^T$. Assume the energy normalization $\text{tr}\{R_{\mathbf{hh}}\} = n_r$ with $R_{\mathbf{hh}} = \text{E}\,\{\mathbf{hh}^H\}$. By default we shall assume the i.i.d. complex Gaussian channel model: $\mathbf{h} \sim \mathcal{CN}(0, \frac{1}{L+1} I_{n_r(L+1)})$ so that spatio-temporal diversity of order $n_r\,(L+1)$ is available (which is the case from the moment $R_{\mathbf{hh}}$ is nonsingular). The average per Rx antenna SNR is $\rho = \frac{\sigma_a^2}{\sigma_v^2}$.

In practice the Linear Equalizer (LE) is often used since its settings are easier to compute and there is no error propagation. Also in practice, for both LE and Decision Feedback Equalizer(DFE), only a limited degree of non-causality (delay) can be used and the filters are usually of finite length (FIR). Analytical investigations into the diversity for SISO with LEs are much more recent, see [1],[8] for linearly precoded OFDM and [9] for Single-Carrier with Cyclic Prefix (SC-CP). In [1], it was shown that the introduction of redundant linear precoding in OFDM allows a MMSE-ZF linear block receiver to regain full diversity in the SISO (or SIMO) case. The ZP introduces redundancy in the time (delay) dimension which allows a LE of inter-symbol interference (ISI) to maintain full diversity: every input symbol can be recovered linearly unless the whole channel impulse response becomes zero. Moreover analysis in [2] reveal a fundamental link between a channel parameter orthogonality deficiency (od) of the channel matrix and the diversity and capacity of LEs. In all the above mentioned references the channel was assumed to be perfectly known at the Rx and in some cases at the Tx too. However, practical receivers must estimate the channel, thereby incurring estimation error that needs to be accounted for in the performance analysis. In [3] the effect of channel estimation error on the performance of the viterbi equalizer is studied in the SIMO framework. In [4] the bit-error rate (BER) performance of multilevel quadrature amplitude modulation with pilot-symbol-assisted modulation channel estimation in static and Rayleigh fading channels is derived, both for single branch reception and maximal ratio combining diversity receiver systems. However, in [5] they show that the practical ML channel estimator preserves the diversity order of MRC (Maximum Ratio Combining), see also [6] for more profound analysis. In this paper we consider the channel to be estimated at the Rx using blind deterministic algorithms then we investigate the effect of the resulting channel estimation error on the diversity achieved by LE.

2. OUTAGE ANALYSIS OF SUBOPTIMAL RECEIVER SINRS

A perfect outage occurs when $\text{SINR} = 0$. For the MFB this can only occur if $\mathbf{h} = 0$. For a suboptimal Rx however, the SINR can vanish for any $\mathbf{h}$ on the *Outage Manifold* $\mathcal{M} = \{\mathbf{h} : \text{SINR}(\mathbf{h}) = 0\}$. At fixed rate R, the diversity order is the codimension of (the tangent

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[1] In this paper, "Rx" stands for "receive" or "receiver" or "reception" etc., and similarly for "Tx" and "transmit", ...

[2] In the case of additional oversampling with integer factor w.r.t. the symbol rate, the Rx dimension would get multiplied by the oversampling factor.

[3] In this paper, $.^*$, $.^T$, and $.^H$ denote complex conjugate, transpose and Hermitian (complex conjugate) transpose respectively, and $\mathbf{h}^\dagger[z] = \mathbf{h}^H[1/z^*]$ denotes the paraconjugate (matched filter). Note that $\mathbf{h}^\dagger[e^{j2\pi f}] = \mathbf{h}^H(f)$.

subspace of) the outage manifold, assuming this codimension is constant almost everywhere and assuming a channel distribution with finite positive density everywhere (e.g. Gaussian with non-singular covariance matrix). For example, for the MFB (which only depends on $\mathbf{h}$) the outage manifold is the origin, the codimension of which is the total size of $\mathbf{h}$. The codimension is the (minimum) number of complex constraints imposed on the complex elements of $\mathbf{h}$ by putting SINR($\mathbf{h}$) $= 0$. Some care has to be exercised with complex numbers. Valid complex constraints (which imply two real constraints) are such that their number becomes an equal number of real constraints if the channel coefficients were to be real. A constraint on a coefficient magnitude however, which is in principle only one real constraint, counts as a valid complex constraint (at least if the channel coefficient distributions are insensitive to phase changes). An actual outage occurs whenever $\mathbf{h}$ lies in the *Outage Shell*, a (thin) shell containing the outage manifold. The thickness of this shell shrinks as the rate increases.

3. LINEAR EQUALIZATION (LE) IN SINGLE CARRIER CYCLIC PREFIX (SC-CP) SYSTEMS

The diversity of LE for SC-CP systems has been studied in [9] for the SISO case with i.i.d. Gaussian channel elements, fixed rate R and block size $N = L{+}1$. The LE DMT for SIMO SC-CP systems appears in [10]. Consider a block of N symbol periods preceded by a cyclic prefix (CP) of length L (as a result of the CP insertion, actual rates are reduced by a factor $\frac{N}{N+L}$, which is ignored here in what follows). The channel input-output relation over one block can be written as

$$\boldsymbol{Y} = \mathbf{H}\,\boldsymbol{A} + \boldsymbol{V} \tag{2}$$

where $\boldsymbol{Y} = \boldsymbol{Y}_k = [\boldsymbol{y}_k^T\;\boldsymbol{y}_{k+1}^T \cdots \boldsymbol{y}_{k+N-1}^T]^T$ etc. and $\mathbf{H}$ is a banded block-circulant matrix (see (13) in [10]). Now apply an N-point DFT (with matrix F_N) to each subchannel received signal, then we get

$$\underbrace{F_{N,n_r}\boldsymbol{Y}}_{\boldsymbol{U}} = \underbrace{F_{N,n_r}\mathbf{H}\,F_N^{-1}}_{\mathcal{H}}\;\underbrace{F_N\;\boldsymbol{A}}_{\boldsymbol{X}} + \underbrace{F_{N,n_r}\boldsymbol{V}}_{\boldsymbol{W}} \tag{3}$$

where $F_{N,n} = F_N \otimes I_n$ (Kronecker product: $A \otimes B = [a_{ij}B]$), $\mathcal{H} =$ blockdiag$\{\boldsymbol{h}_0, \ldots, \boldsymbol{h}_{N-1}\}$ with $\boldsymbol{h}_n = \mathbf{h}(f_n)$, the $n_r \times 1$ channel transfer function at tone n: $f_n = \frac{n}{N}$, at which we have $\mathbf{u}_n = \boldsymbol{h}_n\,\boldsymbol{x}_n + \mathbf{w}_n$. The $\boldsymbol{x}_n$ components are i.i.d. and independent of the i.i.d. $\mathbf{w}_n$ components with $\sigma_x^2 = N\,\sigma_a^2,\ \sigma_w^2 = N\,\sigma_v^2$. Now we proceed to introduce the channel estimation error by splitting the true channel into two parts, the estimated part $\widehat{\boldsymbol{h}}_n$ and the corresponding error part $\widetilde{\boldsymbol{h}}_n$ that results from the estimation process. However, one of the drawbacks of the blind channel estimation is that it yields $\widehat{\boldsymbol{h}}_n$ with a scalar ambiguity. Thus, after the blind channel estimation process we have $\alpha\widehat{\boldsymbol{h}}_n$ and not $\widehat{\boldsymbol{h}}_n$ where α is the same across all tones. There are many methods to resolve this scalar ambiguity available in the literature either by using a differential transmission or by forcing a constraint. Hence, after resolving the scalar ambiguity $\widehat{\widehat{\boldsymbol{h}}}_n$ and $\widetilde{\widetilde{\boldsymbol{h}}}_n$ denote respectively the resolved estimated channel and the corresponding resolved error.

$$\begin{aligned} \mathbf{u}_n &= \widehat{\widehat{\boldsymbol{h}}}_n\,\boldsymbol{x}_n + \widetilde{\widetilde{\boldsymbol{h}}}_n\,\boldsymbol{x}_n + \mathbf{w}_n \\ &= \widehat{\widehat{\boldsymbol{h}}}_n\,\boldsymbol{x}_n + \mathbf{z}_n \end{aligned} \tag{4}$$

Where $S_{\mathbf{z}_n\mathbf{z}_n} = \mathrm{E}\,\{\mathbf{z}_n\mathbf{z}_n^H\} = \sigma_x^2\,S_{\widetilde{\widetilde{h}}_n\widetilde{\widetilde{h}}_n} + \sigma_w^2\,I$. It is worthy to note that if we treat $\mathbf{z}_n$ as independent of $\widehat{\widehat{\boldsymbol{h}}}_n\;\boldsymbol{x}_n$ then we get a capacity (or mutual info) lower bound (that is fairly tight). We can also notice that $S_{\mathbf{z}_n\mathbf{z}_n}$ is spatially colored due to the color of the estimation error that results from the blind channel estimation process. Moreover, we assume here that the blind channel estimation error is independent of the Tx symbols. This is true if we estimate the channel from one Rx block and use that channel to detect the symbols in the following Rx block. Thus, a ZF ($\delta = 0$) or MMSE ($\delta = 1$) LE is defined per tone as:

$$\mathbf{f}_n = \left(\widehat{\widehat{\boldsymbol{h}}}_n^H S_{\mathbf{z}_n\mathbf{z}_n}^{-1}\,\widehat{\widehat{\boldsymbol{h}}}_n + \delta\sigma_x^{-2}\right)^{-1}\widehat{\widehat{\boldsymbol{h}}}_n^H S_{\mathbf{z}_n\mathbf{z}_n}^{-1} \tag{5}$$

Applying this LE to the received signal we get $\widehat{\boldsymbol{x}}_n = \mathbf{f}_n\mathbf{u}_n$ from which $\widehat{\boldsymbol{a}}_n$ is obtained after IDFT with

$$\mathrm{SINR}_{CP-LE}^{\delta} = \frac{\sigma_x^2}{\frac{1}{N}\sum_{n=0}^{N-1}\left(\widehat{\widehat{\boldsymbol{h}}}_n^H S_{\mathbf{z}_n\mathbf{z}_n}^{-1}\,\widehat{\widehat{\boldsymbol{h}}}_n + \delta\sigma_x^{-2}\right)^{-1}} - \delta. \tag{6}$$

Practically, $S_{\mathbf{z}_n\mathbf{z}_n}$ is not known in advance. Moreover, concerning the other cases where the channel and the Rx symbols are detected from the same Rx block, the previously mentioned formula for $S_{\mathbf{z}_n\mathbf{z}_n}$ is not applicable. Hence, we use an approximate LE $\mathbf{f}_n = \left(\widehat{\widehat{\boldsymbol{h}}}_n^H\widehat{\widehat{\boldsymbol{h}}}_n + \delta\sigma_x^{-2}\right)^{-1}\widehat{\widehat{\boldsymbol{h}}}_n^H$. Our simulations show that all the conclusions drawn in this paper are still valid even in such cases. On the other hand, as we will see in the following sections, the way by which we resolve the scalar ambiguity has a major effect on the diversity achieved by the receiver. In this paper we deal with three different constraints namely, Linear constraint, Least square constraint and Fixing one-tap constraint. Moreover, we simulate a differential transmission scenario within the context of OFDM where it is possible to define a ZF-LE as in the non-differential case.

3.1. Linear Constraint

Generally, the cost function of any blind deterministic channel estimation can be represented by $\widehat{\widehat{\mathbf{h}}}^H\;\mathbf{Q}\;\widehat{\widehat{\mathbf{h}}}$. To resolve the scalar ambiguity we minimize this cost function subject to the linear constraint as follows: $\min_{\mathbf{h}^H\widehat{\widehat{\mathbf{h}}}=\mathbf{h}^H\mathbf{h}} ||\widehat{\widehat{\mathbf{h}}}^H\;\mathbf{Q}\;\widehat{\widehat{\mathbf{h}}}||^2$. Applying the Lagrange multiplier we get:

$$\widehat{\widehat{\mathbf{h}}} = \frac{\mathbf{h}^H\mathbf{h}}{\mathbf{h}^H\mathbf{Q}^{-1}\mathbf{h}}\mathbf{Q}^{-1}\mathbf{h} \tag{7}$$

This constraint yields $\widetilde{\widetilde{\mathbf{h}}} \perp \mathbf{h}$ and leads to the minimal Cramer Rao lower Bound $\mathbf{CRB}$. Normally, $\mathbf{CRB}$ is defined as the inverse of the Fisher Information Matrix $\mathbf{FIM}$ while for singular $\mathbf{FIM}$ with the above mentioned linear constraint the formula of the $\mathbf{CRB}$ is given by the pseudo inverse of the $\mathbf{FIM}$.

$$\mathbf{CRB} = \left(\sum_{n=0}^{N-1} G_n^H\;\mathbf{FIM}_n\;G_n\right)^{\dagger} \tag{8}$$

$\mathbf{FIM}_n$ is the FIM over tone $\mathbf{n}$ and is given by:

$$\mathbf{FIM}_n = \sigma_w^{-2}\left(I_{n_r} - \frac{\boldsymbol{h}_n\boldsymbol{h}_n^H}{||\boldsymbol{h}_n||^2}\right) \tag{9}$$

and $\dagger$ denotes a Moore-Penrose pseudo inverse while G_n is a transformation matrix (containing DFT portions) such that $\boldsymbol{h}_n = G_n\;\mathbf{h}$. To be more accurate, G_n is of size $n_r \times n_r\,(L+1)$ such that it contains the first $n_r\,(L+1)$ elements of the nth block row of F_{N,n_r}. Generally speaking, we know that at high SNR the error covariance matrix of all well behaved deterministic blind channel estimation methods attain the CRB. With the linear constraint discussed above, $R_{\tilde{\mathbf{h}}\tilde{\mathbf{h}}}$ attains CRB in (8) and consequently $S_{\tilde{\tilde{h}}_n\tilde{\tilde{h}}_n}$ attains the per-tone $\mathbf{CRB_n}$ which can be obtained from (8) after transforming it back to frequency domain. Hence we can write: $S_{\mathbf{Z}_n\mathbf{Z}_n} = \mathrm{E}\left\{\mathbf{z}_n\mathbf{z}_n^H\right\} = \sigma_x^2\,\mathbf{CRB_n} + \sigma_w^2\,I$. We start with the ZF ($\delta = 0$), SINR in (6) vanishes when either $\hat{\hat{h}}_n$ vanishes or $S_{\mathbf{Z}_n\mathbf{Z}_n}$ blows up. This is valid for any tone $\mathbf{n}$. Since $\mathbf{Q}$ in (7) is singular then $\hat{\hat{h}}_n$ needs only $n_r - 1$ constraints to vanish.

$$d_{CP-ZF}^{Lin} = (n_r - 1) \tag{10}$$

Thus, ZF equalizer doesn't even attain the full spatial diversity. Moreover, any frequency diversity is lost.

3.2. Least Square Constraint

In this case the minimization process is done in two steps. First we minimize as follows: $\min_{||\hat{\mathbf{h}}||=1} ||\hat{\mathbf{h}}^H\mathbf{Q}\,\hat{\mathbf{h}}||^2$ to get $\hat{\mathbf{h}} = V_{min}(\mathbf{Q})$ then the scalar ambiguity is resolved by forcing a least square constraint as follows: $\min_{\alpha} ||\mathbf{h} - \alpha\hat{\mathbf{h}}||^2$. After a little manipulation we get the following solution:

$$\hat{\hat{\mathbf{h}}} = \frac{\hat{\mathbf{h}}^H\mathbf{h}}{||\hat{\mathbf{h}}||^2}\hat{\mathbf{h}} \tag{11}$$

so that $\tilde{\tilde{\mathbf{h}}} \perp \hat{\hat{\mathbf{h}}}$ which is a well known feature of the least square estimation. Now following the same intuition presented in the previous section (linear constraint case) we conlcude that the number of constraints required for the SINR ($\delta = 0$) in (6) to vanish is $(n_r - 1)$

$$d_{CP-ZF}^{LeastSq} = (n_r - 1) \tag{12}$$

3.3. Fixing One Tap (FOT) Constraint

Again to resolve the scalar ambiguity we minimize the cost function by considering without loss of generality that the first tap of the channel that corresponds to the first Rx antenna is known. $\min_{\hat{\hat{\mathbf{h}}}[1]=\mathbf{h}[1]} ||\hat{\hat{\mathbf{h}}}^H\mathbf{Q}\,\hat{\hat{\mathbf{h}}}||^2$. Applying the Lagrange multiplier we get:

$$\hat{\hat{\mathbf{h}}} = \frac{\mathbf{h}\,[1]}{V\,[1]}V \tag{13}$$

Where V is the first column vector of $\mathbf{Q}^{-1}$ and $V\,[1]$ is the first element of that vector. It is obvious from (13) that for $\hat{\hat{\mathbf{h}}}$ to vanish it is sufficient that $\mathbf{h}\,[1]$ gets very small. Hence the diversity achieved is one regardless of the LE used.

$$d_{CP-LE}^{FOT} = 1 \tag{14}$$

4. LE IN OFDM SYSTEMS

For SC-CP as defined in (3) the Tx symbols are represented by $\mathbf{A}$ hence they are in time domain while in OFDM the same formula (3) is still applicable but now the Tx symbols are represented by $\mathbf{X}$ so they are in frequency domain. Now each tone is carrying a separate symbol hence,

$$\mathrm{SINR}_{OFDM-LE}^{\delta} = \frac{\sigma_x^2}{\left(\hat{\hat{h}}_n^H S_{\mathbf{Z}_n\mathbf{Z}_n}^{-1}\hat{\hat{h}}_n + \delta\sigma_x^{-2}\right)^{-1}} - \delta. \tag{15}$$

Following the same intuition used in the SC-CP case and since the matrix $\mathbf{Q}$ is still singular this means that the ZF-LE yields a diversity order $n_r - 1$ for both linear and least square constraints while it yields one for FOT constraint for the same reason discussed before. However, for MMSE the SINR and due to the regularization term σ_x^{-2} introduced in the denominator of (15) doesn't vanish when the previous $n_r - 1$ constraints are available. This means more constraints are required for outage to occur but it is well known that in case of OFDM the best diversity achieved is the full spatial diversity. This leads us to conclude that the diversity order achieved by MMSE-LE is n_r.

$$d_{OFDM-MMSE} = n_r \tag{16}$$

On the other hand, we are able here to introduce within the famework of OFDM a differential transmission system that uses a linear M-PSK modulation where the Tx information (symbols) are represented by the difference of phases between any two consecutive tones of the same OFDM symbol. At the Rx we can utilize the LE introduced in (5) to compute the estimated symbols at different tones as follows $\hat{\boldsymbol{x}}_n = \mathbf{f}_n\mathbf{u}_n$, $\hat{\boldsymbol{x}}_{n+1} = \mathbf{f}_{n+1}\mathbf{u}_{n+1}$ and so on then we can extract the information buried between any two consecutive tones by computing the angle of $\hat{\boldsymbol{x}}_{n+1}^*\hat{\boldsymbol{x}}_n$. The differential transmission usually leads only to a loss in the coding gain while it preserves the diversity order $(n_r - 1)$ and this what our simulations show as we will see later.

5. FIR LINEAR EQUALIZATION

Consider now the use of an FIR LE of length N. For SIMO channels, there exist indeed FIR equalizers for FIR channels, due to the Bezout identity, as long as $N \geq \frac{L}{n_r-1}$. The LE design is based on a banded block Toeplitz input-output matrix $\overline{\mathbf{H}}$ which can be obtained by starting from a block circulant $\mathbf{H}$ (as in the CP case) of size $N+L$ and removing the top L block rows. Now, we play the same game as in the case of CP to introduce the channel estimation error.

$$\begin{aligned} \boldsymbol{Y} &= \hat{\hat{\overline{\mathbf{H}}}}\,\boldsymbol{A} + \tilde{\tilde{\overline{\mathbf{H}}}}\,\boldsymbol{A} + \boldsymbol{V} \\ &= \hat{\hat{\overline{\mathbf{H}}}}\,\boldsymbol{A} + \mathcal{A}\tilde{\mathbf{h}} + \boldsymbol{V} \\ &= \hat{\hat{\overline{\mathbf{H}}}}\,\boldsymbol{A} + \boldsymbol{V}' \end{aligned} \tag{17}$$

Where $R_{V'V'} = \sigma_v^2\,I + \mathbf{E}_a\left\{\mathcal{A}R_{\tilde{\mathbf{h}}\tilde{\mathbf{h}}}\mathcal{A}^H\right\}$, $\mathcal{A} = \mathcal{A}' \otimes \mathbf{I_{n_r}}$ and $\mathcal{A}'$ is a Hankel matrix filled with the elements of $\boldsymbol{A}$. As in the case of CP, the estimated channel has a scalar ambiguity that can be resolved using the same techniques utlizied over there. However, we omit the double hat and the double tilde notation here seeking for simplicity. Hence with no structure on the unknown $\boldsymbol{A}$, we can only do MMSE-ZF with $\hat{\overline{\mathbf{H}}}$. Hence, $\hat{\boldsymbol{A}} = (\hat{\overline{\mathbf{H}}}^H\hat{\overline{\mathbf{H}}})^{-1}\hat{\overline{\mathbf{H}}}^H\boldsymbol{Y}$ with error covariance matrix given by: $R_{\tilde{A}\tilde{A}} = (\hat{\overline{\mathbf{H}}}^H\hat{\overline{\mathbf{H}}})^{-1}\hat{\overline{\mathbf{H}}}^H R_{V'V'}\,\hat{\overline{\mathbf{H}}}(\hat{\overline{\mathbf{H}}}^H\hat{\overline{\mathbf{H}}})^{-1}$. Then for symbol $\boldsymbol{a}_k$ we get C = log $(1+\sigma_{\boldsymbol{a}}^2/\sigma_{\tilde{\boldsymbol{a}}_k}^2)$. This is the capacity for given $\hat{\overline{\mathbf{H}}}$ and given $\overline{\mathbf{H}}$ where $\sigma_{\tilde{\boldsymbol{a}}_k}^2$ is a kth diagonal element of $R_{\tilde{A}\tilde{A}}$. So we get outage whenever $\hat{\overline{\mathbf{H}}}$ loses full column rank or whenever $R_{\tilde{\mathbf{h}}\tilde{\mathbf{h}}}$ explodes, which happens whenever $\overline{\mathbf{H}}$ loses full column rank. This in turn occurs whenever $\mathbf{h}\,[z_0] = 0$ for some z_0, in other words, the subchannel transfer functions have a zero in common. This imposes on the $n_r - 1$ other subchannels to have a zero equal to a zero in the first subchannel. Hence the codimension of the outage manifold is $n_r - 1$. This is true when we resolve the scalar ambiguity by either linear or least square constraint or even using differential Tx.

$$d_{FIR-ZF}^{Lin,LeastSq,diff} = (n_r - 1) \tag{18}$$

However, if FOT is used then again, as in the SC-CP case, the diversity order is one, $d_{FIR-ZF}^{FOT} = 1$.

6. SIMULATIONS

We have used SRM (Subchannel Response Matching) as a blind channel estimation algorithm for ZF-LE. While we have used WCM (Weighted Covariance Matrix) with the MMSE-LE. The WCM has been initialized by PQML (Pseudo Quadratic Maximum Likelihood) and the latter initialized by SRM. In all the subsequent simulations we have used 8PSK and 3 Rx antennas ($n_r = 3$). In Fig. 1 we have simulated the ZF-LE with SRM within the SC-CP framework with the three different constraints explained in our article. We can observe that both linear and least square constraints attain a diversity of two that is $n_r - 1$ while the FOT attains only a diversity of one (i.e. no diversity). In Fig. 2 we show that the differential transmission scheme in the context of OFDM along with ZF-LE also leads to a loss of one order in the diversity so two out of three is only achieved. However, when MMSE-LE is used the diversity is recovered so full spatial diversity is achieved. Finally, in Fig. 3 we prove the results stated in the FIR-LE section namely, with FOT no diversity exists at all (div = 1) while with the linear constraint diversity loses one order, hence again here two out of three is attained.

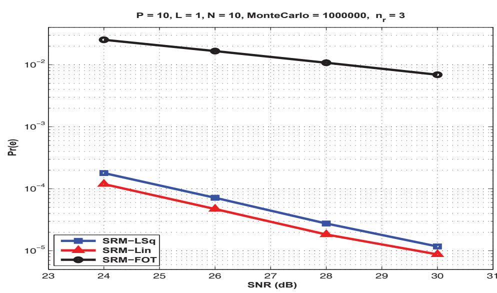

Fig. 1. Probability of error vs. SNR for SC-CP ZF-LE using SRM with different constraints.

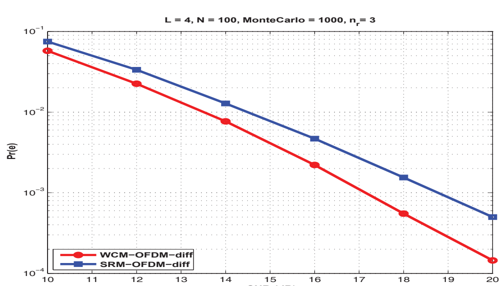

Fig. 2. Probability of error vs. SNR for OFDM with ZF-LE and MMSE-LE using SRM and WCM respectively with differential Tx.

7. CONCLUSION

We have investigated in this paper the diversity of the LE in the context of blind SIMO channel estimates. We have shown that the diversity achieved by the LE is highly affected by the method used to resolve the scalar ambiguity that results from the blind channel estimation process. We have treated both CP and non-CP cases showing that the ZF-LE for CP contrary to the case of perfect channel knowledge at the Rx is not capable of achieving the full spatial diversity n_r. Consequently, the diversity order loses one rank when either

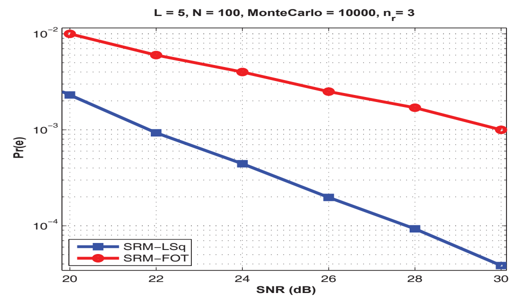

Fig. 3. Probability of error vs. SNR for FIR ZF-LE using SRM with different constraints.

the linear constraint or the least square constraint is used and the same result holds for non-CP with ZF-LE. The worst scenario occurs when the scalar ambiguity is resolved by considering one channel tap is known. In this case the diversity order is just one for both CP and non-CP. Moreover, as an alternative to resolving the scalar ambiguity by forcing a constraint, we have investigated a differential transmission scenario in the context of OFDM where the diversity of the ZF-LE is shown to be $n_r - 1$. However, using MMSE-LE then a full spatial diversity is recovered. Now we are working on the semiblind case which we think that it is promising in terms of the diversity order that it can attain.

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