Transmission Techniques and Channel Estimation for Spatial Interweave TDD Cognitive Radio Systems

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Abstract— In this paper we propose a new method of designing the beamformer subspace in MIMO interference channel with a Time-Division Duplex (TDD) transmission scheme. In particular, this method is applied to a Spatial Interweave Cognitive Radio scenario. In our model we do not require a priori knowledge of the Channel State Information (CSI) at the transmitters. The primary and the opportunistic (cognitive) users are able to obtain information required for Tx beamforming through smart exploitation of received signal during a TDD time slot, exploiting channel reciprocity thus reducing overhead for channel estimation. The opportunistic user designs its beamformer in order to span the noise subspace at the primary receiver, thus intertwining its signal with the primary’s so that its signal lies within the spatial whitespaces of the primary, possibly licensed system, causing no interference to the latter.

I. INTRODUCTION

In the last few decades the number of wireless communication systems has grown exponentially and hence the electromagnetic spectrum has become more crowded. This is the reason for the popularity of the Cognitive Radio (CR) concept [1]. In the CR paradigm a secondary user is allowed to opportunistically communicate using the same spectrum as a licensed player, as a result increasing the spectrum efficiency. In the Interweave (IW) paradigm of CR, see [2] for more on CR terminology, the opportunistic user can transmit using the temporary space-time-frequency voids of the licensed communication without generating any kind of interference at the primary receiver. In this scenario the secondary transmitter can apply the concept of Interference Alignment (IA) [3], to design its transmitted (Tx) signal, hence the primary receiver (Rx) receives the opportunistic transmission into the dimension that is unused by the licensed user. As a result there is no degradation of the performance of the primary, possibly legacy, system.

For efficient beamformer design the knowledge of channel state information (CSI) is required at the transmitter. This makes Time-Division Duplex (TDD) systems desirable since they can in theory exploit the uplink (UL) downlink (DL) reciprocity in the radio propagation channel. Using this transmission strategy the transceiver can obtain DL (UL) channel knowledge using an estimate of the UL (DL) channel. However, in order to exploit channel reciprocity it is important to compensate for the mismatch between the analog Tx/Rx circuitry at both ends, this process is called calibration [4].

In this paper we show how CR users can achieve channel information of the primary link exploring opportunistic the TDD communication between licensed devices. In addition, we discuss the design of secondary transmitter signal so as to cause little interference to the primary communication. In particular, the secondary system is a spatial IW cognitive radio that exploits spatial holes resulting from unused spatial modes in the latter.

During the course of this work, the authors came across another independent work [5] that addresses a similar CR beamforming problem (called opportunistic interference alignment there) assuming perfect knowledge of all channels and same antenna configurations for the primary and secondary systems. Our work is in a more general setting and includes an inventory of quantities to be estimated for solving the beamforming problem. The main contribution compared to [5] is the demonstration that TDD is not just a possible option, but is crucial for spatial IW cognitive radio if unrealistic overheads and communications between the two systems are to be avoided. We also address calibration of Tx/Rx electronics that is a critical requirement in TDD systems and show that even though the opportunistic Tx needs to know the noise subspace at primary Rx, calibration between non cooperative Tx and Rx is not needed for beamformer design.

II. SYSTEM MODEL

We focus on the MIMO interference channel where two point-to-point bidirectional links transmit using a TDD transmission scheme. Even if our work can be applied to more general system to simplify the notation we will refer to a primary link composed of a licensee Base Station ($BS_1$), that communicates with the respective Mobile User ($MU_1$) ignoring completely the presence of a secondary transmission...
in its vicinity. At the same time a cognitive Base Station (BS2) tries to opportunistically communicate to a cognitive Mobile User (MU2) without degrading the licensee’s communication. BS1 and MU1 are equipped with the same number of antennas N1 and also BS2 and MU2 have N2 antennas. We focus on the case where the opportunistic users have a number of antennas greater than the primary users N2 ≥ N1. The matrices \( H_{ij} \) and \( H_{ij} \) ∈ \( \mathbb{C}^{N_i \times N_j} \) are, respectively, the DL and UL channel matrices from transmitter j to receiver i, where \( i, j \in \{1, 2\} \). The entries of these matrices are i.i.d. complex Gaussian random variable. In the following we will assume that all the channel matrices are fixed, this corresponds to assuming that the channel remains constant for a sufficient number of TDD slots.

In a TDD transmission scheme assuming perfect Tx/Rx calibration the UL channel is the transpose of the relative downlink one [4] due to channel reciprocity.

\[
\hat{H}_{ij} = H^*_ji
\]

Thus an UL channel estimate can be used for designing the transmit beamformer. We assume that channel estimates are obtained through pilot symbols.

### III. Transmission Techniques and Channel Estimation

![Downlink Channel](image)

In the Interweave cognitive scenario, licensee (primary) systems are not aware of the presence of secondaries which should ideally cause no interference. The primary Tx is therefore assumed to be a Single User MIMO link (SU-MIMO). In this system the transmitter and receiver filters are designed in order to maximize the transmission rate and the capacity-achieving solution is SVD beamforming and Water-Filling (WF) [6]. Assuming low-rank Tx, the primary link can decomposes into a signal and a complementary (noise) subspace,

\[
H = U \Delta V^H = [U_s, U_n] \begin{bmatrix} \Delta_s & \Delta_n \end{bmatrix} \begin{bmatrix} V^H_s \ V^H_n \end{bmatrix}
\]

where subscripts s or n refer to signal subspace and noise subspace respectively. The matrices \( U \) and \( V \) are unitary matrices and \( \Delta \) is a diagonal matrix that contains the singular values of the channel matrix. In order to waterfill in UL and DL, both BS1 and MU1 must have complete knowledge of the primary channel and Rx noise variances. This information can be obtained partially through TDD reciprocity (pilots for channel estimation) and partially through unavoidable feedback.

In the interweave scenario unlicensed users must transmit without deterioring the licensed transmission. Because at low to medium signal-to-noise ratios (SNR) the primary transmitters are expected to exploit a limited number of channel modes, the opportunistic transmitter can beamform its signal in order to fall in the noise subspace of the licensed communication. This has been labelled an interference alignment technique in [5]. To adapt its communication the secondary Tx has to know what is signal subspace at the primary Rx. As discussed in the following this subspace can be learnt by an opportunistic exploitation of the primary’s signal.

All TDD frames in both UL and DL are composed of two time segments, one comprising possibly multiple data streams and the second pilots embedded for channel estimation in the relevant link. In the primary link only data part of the frame is beamformed but not pilots. This implies that they span the entire channel space. On the other hand in the cognitive transmission pilots are also beamformed, thus ensuring that they do not interfere with the primary transmission. We assume that the secondary TDD slots are aligned with the primary’s using classical spectrum sensing techniques.

#### A. First TDD Slot

In this first slot all devices in the system should start to get the knowledge that they need to transmit. In particular the licensed BS transmits without knowledge of the downlink channel and therefore cannot beamform transmitting over the entire channel. MU1 can estimate the channel matrix \( H_{11} \) using pilots. Cognitive users are assumed to be inactive at this time.

#### B. Second TDD Slot

MU1 now knows the downlink channel matrix and hence it can construct the beamforming subspace \( T_{MU1} \in \mathbb{C}^{N_1 \times d_1} \) using the reciprocity in equation (1), where \( d_1 \) is the number of transmitted streams and is equal to the signal subspace dimension. The received signal at BS1 has the following structure.

\[
\hat{y}_1 = H_{11}T_{MU1}\hat{s}_1 + \hat{n}_1
\]

\( \hat{y}_1 \in \mathbb{C}^{N_1 \times 1} \) is the received signal vector, \( \hat{s}_1 \in \mathbb{C}^{d_1 \times 1} \) is the transmitted signal vector and \( \hat{n}_1 \in \mathbb{C}^{N_1 \times 1} \) is the spatially white Gaussian noise with zero mean and variance \( \sigma^2_1 \).

MU1 proceeds with a SVD decomposition of the downlink dual channel, \( H^T_{11} = V^*_1\Delta_{11}U^*_1 \), uses as Tx beamformer \( T_{MU1} = U^*_1a \), taking the columns of \( U^*_1 \) according to the WF solution. The BS1 can design its Rx filter as \( R_{BS1} = V^T_{1\alpha} \in \mathbb{C}^{d_1 \times N_1} \) from the SVD of the UL channel. The signal at the Rx output, \( \tilde{r}_1 \in \mathbb{C}^{d_1 \times 1} \) at BS1 is written as

\[
\tilde{r}_1 = R_{BS1}H_{11}T_{MU1}\hat{s}_1 + R_{BS1}\hat{n}_1
\]

\( \hat{y}_2 = H^T_{12}T_{MU1}\hat{s}_1 + \hat{n}_1 = H^T_{12}U^*_1a\hat{s}_1 + \hat{n}_1 \).
Assuming sufficient data samples, we can obtain at BS$_2$ a consistent estimate (in the SNR sense) of the primary Rx signal subspace as $y_2 y_2^T = E[y_2 y_2^T]$. Knowing $U_{1,s}^*$, the BS$_2$ Tx beamformer $T_{BS2} \in \mathbb{C}^{N_2 \times d_2}$ can send at most $d_2$ streams while ensuring its signal lies in the noise subspace at the primary Rx. This implies that

$$R_{MU_1} H_{12} T_{BS2} = 0 \implies T_{BS2} = (R_{MU_1} H_{12})^\perp$$  \hspace{1cm} (6)

where $A^\perp$ represent the orthogonal complement of the row space of the matrix $A$.

Taking the $MU_1$ Rx in the definition of $T_{BS2}$ has the advantage that in the low to medium SNR of the primary link, where the primary Tx sends only $d_1 < N_1$ of the total available signaling dimension $N_1$, the secondary Tx can (opportunistically) transmit at most $d_2 = N_2 - d_1$ streams. On the other hand in the high SNR region, when the primary link use up its entire degrees of freedom (DoF) for spatial multiplexing, the secondary can always transmit $d_2 = N_2 - N_1$ streams.

**C. Third TDD Slot**

From this TDD time slot onwards starts the steady state of the system. This means that also the cognitive BS starts to transmit to the $MU_2$. As for the reverse link, in the primary forward link BS$_1$ constructs its beamforming subspace using SVD of the channel matrix $H_{11}$, $T_{BS1} = V_{1,s}$, and $MU_1$ uses as Rx, $R_{MU_1} = U_{1,s}^*$. The opportunistic BS starts to transmit its data hence the received signal at primary MU is

$$y_1 = H_{11} T_{BS1} s_1 + H_{12} T_{BS2} s_2 + n_1$$  \hspace{1cm} (7)

In order to extract the useful data $MU_1$ applies the Rx filter to the received signal: $r_1 = R_{MU_1} y_1$. The $BS_2$ beamformed signal lies in the noise subspace, $MU_1$ sees no interference. On the other hand $MU_2$ receives signal from both $BS_1$ and $BS_2$:

$$y_2 = H_{22} T_{BS2} s_2 + H_{21} T_{BS2} s_1 + n_2$$  \hspace{1cm} (8)

$MU_2$, using the beamformed pilots incorporated into the secondary data frame, can estimate the secondary link beamformed channel $H_{22} T_{BS2}$. Using this information it determines the transmitter subspace of the primary downlink using second-order statistics (SOS) of the received signal $y_2$. Similarly to $BS_2$ the beamformer subspace at $MU_2$ is: $T_{MU_2} = (R_{BS2} H_{22}^H)^\perp$

**D. Fourth TDD slot**

In this slot all nodes have the knowledge they need to transmit to corresponding receivers. The received signal of the primary UL transmission is

$$y_1 = H_{11}^H T_{MU_1} s_1 + H_{12}^T T_{MU_2} s_2 + n_1$$  \hspace{1cm} (9)

The Rx filter at BS$_1$ suppresses the opportunistic Tx from $MU_2$. The received signal at BS$_2$ nevertheless contains interference due to $MU_1$.

$$\tilde{y}_2 = H_{22}^T T_{MU_2} s_2 + H_{12}^T T_{MU_1} s_1 + \tilde{n}_2$$  \hspace{1cm} (10)

**IV. SECONDARY LINK OPTIMIZATION**

Once the secondary link beamformer subspace is defined in order to cause zero interference at the primary receivers, we can optimize for the secondary link by designing a $d_2 \times d_2$ square beamforming matrix $Q_{BS2}$ such that $T_{BS2} Q_{BS2} \in \text{span}(T_{BS2})$.

The received signal at $MU_2$ is given in (8). To find the matrix $Q_{BS2}$ we need to solve the following optimization problem:

$$\max_{Q_{BS2}} \log_2 \det \left( I + Q_{BS2}^H T_{BS2}^H H_{22}^{-1} T_{BS2} Q_{BS2} \right) \quad \text{s.t.} \quad \text{trace}(T_{BS2} Q_{BS2} Q_{BS2}^H T_{BS2}^H) = P_2$$  \hspace{1cm} (11)

$P_2$ represents the transmit power constraint at the secondary link and $R_{int} = H_{22}^T T_{BS2} Q_{BS2} H_{22}^H + \sigma_n^2 I$ is the interference plus noise covariance matrix. The problem is the traditional waterfilling in colored noise.

**A. Feedback Requirements and Differential Feedback**

To find the solution of the optimization problem above $BS_2$ should know the covariance matrix $K$. It must be remarked that even using TDD transmission scheme there is no way for $BS_2$ to know the interference plus noise covariance matrix, $R_{int}$ at $MU_2$. A feedback of $K$ to $BS_2$ is therefore inevitable. In order to reduce the rate penalty due to feedback the entire matrix, we propose differential feedback [7].

In this technique the Rx and Tx both generate a common random codebook of Hermitian matrices from which they choose the appropriate matrix. In particular the receiver, according to the received signal, chooses the Hermitian matrix that is closer to the real covariance matrix. The information that is feedback is the index corresponding to the chosen matrix in the codebook. Using the index and the corresponding random matrix the transmitter finds the Tx filter through WF. This process continues until convergence or a certain number of iteration is reached, refer to [7] for more details.

The main advantage of differential method is that the amount of feedback is not related to the matrix dimensions. The number of bits required is $b = \log_2(Q)$, where $Q$ is the cardinality of the codebook. The disadvantage of this method is that it is sensible to transmission error, in particular if the transmitter chooses the wrong matrix, due to feedback errors, the beamformer matrix is no longer optimal. Fortunately, it turns out that differential feedback is robust against transmission errors. At every iteration before finding the new covariance matrix, the receiver should verify if the
transmitter has used the right matrix to design the beamformer. In particular it computes an appropriate cost function using the received covariance matrix. In addition it computes the same cost function using the covariance matrix that it would have received if the transmitter would have used the covariance matrix corresponding to the right feedback index. It compares the results and if they are different it tries to find out the covariance matrix that the transmitter chose, and uses this matrix for the next iteration.

V. UPLINK DOWNLINK CALIBRATION

\[
\begin{align*}
D_{ii} & = R_B H_{ii}^T T_M \\
D_{i} & = R_M H_{i}^T T_B
\end{align*}
\]

where the matrices \(T_B, R_B\) and \(T_M, R_M\) represent transmit and receive circuitry at BS and MU respectively and with dimensions \(N_i \times N_i\). It is possible to express the DL channel in function of the UL channel, and vice versa:

\[
D_{ii} = R_M T_M^T U_{ii}^T R_B^T T_B
\]

The overall UL and DL channels, Fig. 3, can be written as:

\[
\begin{align*}
U_{ii} & = R_B H_{ii}^T T_M \\
D_{i} & = R_M H_{i}^T T_B
\end{align*}
\]

as function of the UL channel SVD decomposition using the calibration filters as:

\[
D_{11} = P_{MU_i} U_{11}^T P_{BS_i} = P_{MU_i} W^T D Z^T P_{BS_i}
\]

in order to diagonalize the DL channel \(B_S\) designs its beamformer subspace as \(T_{BS_i} = P_{BS_i}^{-1} Z\), and hence the receiver filter at \(MU_i\) is given by: \(R_{MU_i} = W^T P_{MU_i}^{-1}\).

During UL transmission it is possible to design the transmitter and receiver filters using the UL channel as reference. In doing so, calibration filters do not appear in the expression and thus the transmitter matrix at \(MU_i\) is \(T_{MU_i} = W\) and the receiver filter at \(BS_i\) is: \(R_{BS_i} = Z^H\).

B. Secondary Beamformer Design without Crosslink Calibration

The signal at secondary BS due to primary and secondary Tx is expressed as

\[
\hat{y}_2 = U_{21} T_{MU_i} \hat{s}_1 + U_{22} T_{MU_2} \hat{s}_2 + n_2
\]

Knowing \(U_{22} T_{MU_2}\) estimated through \(MU_2\) beamformed pilots, \(BS_2\) can determine the \(MU_1\) Tx subspace \(U_{21} W\) using second order statistics.

Now let us consider the signal at \(MU_1\), after the Rx filter, which is given by

\[
r_1 = R_{MU_1} D_{11} T_{BS_i} \hat{s}_1 + R_{MU_1} D_{12} T_{BS_2} \hat{s}_2 + n_1
\]

where \(r_{1,s}\) represent the useful signal part and \(r_{1,int}\) contains the interference term.

The objective of secondary user is to transmit without causing any interference to the primary system. So \(BS_2\) must design its beamformer subspace such that \(r_{1,int} = 0\). Expressing the DL channel \(D_{12}\) as function of the UL channel and the calibration filters we can write

\[
r_{1,int} = R_{MU_1} D_{12} T_{BS_i} \hat{s}_2 + W^T U_{21}^T P_{BS_i} T_{BS_2} \hat{s}_2
\]

because \(BS_2\) knows the calibration filter \(P_{BS_2}\) it is possible to parameterize \(T_{BS_2} = P_{BS_2}^{-1} T_{BS_2}\), so it is possible to design the beamformer subspace, in order to cause zero interference at \(MU_1\) after its receiver filter, as

\[
T_{BS_2} = (W^T U_{21}^T)^{-1}
\]

Similar treatment applies to the design of \(MU_2\) beamformer which are not discussed for lack of space.

It is important to remark that secondary transmitter can design the beamformer subspace using its own calibration factor, obtained during the calibration phase only with its intended receiver, the UL channel and the receiver subspace at \(MU_1\) that are estimated using second order statistics of the received signal. Calibration with non cooperative users is not required.

VI. PRACTICAL CONSIDERATIONS IN SPATIAL IW CR

Despite a pragmatic approach taken in this work to spatial interweave CR design, we nevertheless make one strong assumption, namely the Tx/Rx subspace is the same in the primary system. In practical system this condition may not
be satisfied for a multitude of reasons, for example different ratio of power constraint and noise variance between the $BS_1$ and $MU_1$ may lead to different number of streams in UL and DL. One subspace will be the subset of the other. A more drastic difference could be the presence at one end of colored noise instead of white noise. Interference covariance matrices is needed for channel whitening and primary beamformer design. In some way, the CR problem starts resembling a classical MIMO interference channel.

If the primary link is affected by colored noise due to secondary link leakage, one may observe that the CR is no longer strictly spatial interweave and fits the underlay paradigm [2]. When this happens, TDD is not enough to design Tx/Rx filters and feedback is also required between $BS_1$ and $MU_1$. Furthermore, estimation of interference plus noise covariance matrices is needed for channel whitening and primary beamformer design. In some way, the CR problem starts resembling a classical MIMO interference channel.

**VII. NUMERICAL RESULTS**

![Fig. 4: Rate for $N_1 = 6$ and $N_2 = 6$](image)

Fig. 4 depicts the rate curve for the primary and secondary links where the licensed users and the opportunistic ones have the same number of transmitting and receiving antennas $N_1 = N_2 = 6$. As we can see primary communication is not affected by the opportunistic transmission. The plot shows also that secondary transmission takes place only in the low SNR region because the opportunistic users can only communicate using unused modes of primary communication. When licensed users use all the possible modes there is no room for secondary transmission and hence the rate curve converge to zero.

Fig. 5 shows the rate curve for a licensed users with $N_1 = 6$ and the opportunistic ones have more antennas $N_2 = 8$. The main difference with the previous case is that in high SNR region the opportunistic users can still continue to transmit due to the fact that they have more antennas than the primary users. In this case the opportunistic user is able to sustain a significant rate. In both plots we show two curves for secondary transmission. The first assumes secondary link optimization using full CSIT while the second exploits estimates obtained through differential feedback with 100 iteration and $b = 4$ feedback bits. As can be seen there is little difference between the two techniques.

**VIII. CONCLUDING REMARKS**

We addressed beamformer design for secondary systems in an interweave CR system that acquire channel state information in an opportunistic fashion by exploiting primary signal statistics and the reciprocity of the underlying TDD channel. The beamformer for secondary Tx is designed so that the secondary signal lies in the noise subspace of the primary signal. It must be noted that the key assumption to guarantee success of such a scheme is the reciprocity of the TDD channel. Tx/Rx calibration is therefore mandatory. The main contribution of this paper is the discovery that despite the requirement for channel reciprocity between noncooperative users, calibration between crosslinks is not required. To optimize secondary-link communication, the beamformer is a cascade of two beamformers, the first ensuring zero interference to the primary Rx and the second diagonalizing the whitened channel of the secondary. To enable waterfilling in the secondary link, we make use of differential feedback in this link and propose a modification of the feedback algorithm in order to make it robust to transmission errors.

**REFERENCES**


