ON THE GAINS OF FIXED RELAYS IN CELLULAR NETWORKS WITH INTERCELL INTERFERENCE

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ABSTRACT
In this paper, we consider a cellular network assisted by fixed relay stations (RS), which are used by mobile stations (MS) to access the base station (BS) via a relaying strategy, namely Amplify-and-forward (AF) and Compress-and-forward (CF). We analyze the achievable sum-of-rates for uplink communications. It is assumed that mobile signals and relay signals are emitted on orthogonal bands, with the possibility of having a larger bandwidth (BW) on the relay-to-base links. Our key result is that with a relay bandwidth just twice that of the mobile’s bandwidth, the system capacity approaches that of an ideal distributed antenna system (DAS), while the ideal DAS requires new backhaul links with very high capacity. Moreover, with the successive interference cancellation (SIC) decoder at the BS, it is shown that under certain conditions the fairness performance in terms of minimum user rates achieved by relay-assisted cellular systems is the same as that of an ideal DAS.

1. INTRODUCTION
Lately, deployment of relay stations in next generation cellular systems has been envisaged as it provides better link quality, coverage and/or higher network throughput, and hence improves the overall system performance [1, 2, 3]. Up to now different relaying strategies have been widely studied to improve the spectral efficiency and system performance where AF, CF and decode-forward (DF) are the main relaying strategies. In this paper, we consider the AF and CF relaying strategies in order to benefit from joint processing at the BS which is possible due to a strong relay-to-base link.

Recently, there has been a great deal of research on relay-assisted infrastructure based networks due to the potential improvements in system performance provided by the relaying mechanism. The performance improvements take the form of reducing deployment cost, reducing terminal transmission power, enhancing network capacity, extending radio range, mitigating shadowing effect and providing spatial diversity [4], [5], [6]. In addition to relaying, there have been some other proposals for next generation wireless networks to increase system capacity and coverage fairness such as DAS and multi-cell coordination. The impact of limited-capacity backhaul on both multi-cell processing and MS cooperation for the uplink (UL) and the downlink for non-fading Gaussian scenarios have been studied in [7, 8]. Even though the above proposals provide huge system performance gains, the deployment of RSs in cellular networks is preferable since it is easier (due to relay terminal size) and more flexible (due to having wireless links between RSs and BSs).

In this paper, we analyze the achievable UL sum-of-rates of relay-aided cellular systems. We compare the AF and CF relaying strategies, with two well-known cellular systems where in the first case the BS antennas are assumed to be co-located (conventional cellular systems) and in the second case they are assumed to be distributed in the cell and be connected to the BS via very high capacity wired links (which complicates backhaul network deployment). The contributions of this paper are: firstly, we propose a theoretical analysis of the gains brought by fixed RSs in a cellular scenario. The key point is that we exploit the ability of the system designer to engineer near line-of-sight links between the RSs and the BS at deployment time. Secondly, a major novelty of this paper is the explicit taking into account the inter-cell interference impact on the relay performance. Thirdly, we compare AF and CF relaying which are two leading forms of relaying strategies in the case of a strong RS-to-BS link [2]. We show the advantage of CF in a situation where the bandwidth allocated to the RS-to-BS links is greater than or equal to twice the bandwidth of the MS-to-RS links. Moreover, we show the gains brought by relays when compared with the ideal DASs (more expensive and unrealistic), and the conventional cellular systems. Finally, it is shown that with the SIC decoder one can get inherent fairness in terms of achievable minimum user rates.

2. SYSTEM MODEL AND PROBLEM DEFINITION
In this section, a generalized signal model for multi-cell networks comprising infrastructure RSs will be given. We con-
sider UL (from MSs to the BS with the aid of RSs) communication in a hexagonal wrap-around cellular topology with $B$ cells. Each cell has $K$ MSs wishing to communicate with the BS through $N$ RSs which are placed at the corners of the cell. Each BS is assumed to have $N$ sectoral antennas. We consider a network scenario where the total number of RF chains (i.e., antennas) added due to the deployed RSs is equal to the number of RF chains in the BSs. Therefore for each of the considered $N$ sectoral BS antennas, one sectoral RS antenna is added into the network. The added RS antenna is assumed to be directed at the corresponding BS sectoral antenna, for a maximum power gain. Each MS has single omni-directional antenna. This system set-up is depicted in Fig. 1.

We index the BSs by $b = 1, 2, \ldots, B$. The system performance in terms of achievable sum of rates will be analyzed where we choose the central cell (cell-$c$) for performance evaluation. Unlike in single-cell networks [9], in this paper we want to see the effects of interference on the infrastructure relaying schemes. Let $h_{c,i,b,k} = \sqrt{\Phi_{c,i,b,k}} \tilde{h}_{c,i,b,k}$, $i = 1, 2, \ldots, N$, $k = 1, 2, \ldots, K$ and $\forall c, b \in \{1, \ldots, B\}$ denote the channel coefficient at the first hop (from the MSs to the RSs) between $k$-th MS in the $b$-th cell and $i$-th RS in the $c$-th cell where $\tilde{h}_{c,i,b,k}$ is an independent identically distributed (i.i.d.) zero mean circularly symmetric complex Gaussian (ZMCSGC) random variable unit variance and $\Phi_{c,i,b,k}$ is the channel gain from $k$-th MS in the $b$-th cell to the $i$-th RS in the $c$-th cell including path-loss, transmitter and receiver antenna gains and shadowing effects.

Similarly, let $g_{c,i,b,n} = \sqrt{\Psi_{c,i,b,n}} \tilde{g}_{c,i,b,n}$ denote the channel coefficient at the second hop (from the RSs to the BSs) between $n$-th RS in the $b$-th cell and $i$-th beam of the BS in the $c$-th cell, where $\tilde{g}_{c,i,b,n}$ is an i.i.d. ZMCSGC random variable with unit variance and $\Psi_{c,i,b,n}$ is the channel gain from $n$-th RS in the $b$-th cell to $i$-th beam of the BS in the $c$-th cell, $\forall i, n \in \{1, 2, \ldots, N\}$, including path-loss, transmitter and receiver antenna gains and shadowing effects. The specifications of channel gains are explained in details in Section 5. In what follows, we will consider ergodic rates and thus the single coefficient channel model is sufficient even to characterize wide-band channels (e.g. OFDM).

### 2.1. Problem Definition

It is clear that DAS are an idealized form of fixed RSs where there is no concern of backhauling the MS data to the BS through the RSs. However, with wireless connection from the RSs to the BS back hauling of the MS data becomes a bottleneck for the overall communication. Considering this problem in this paper we try to find answer to the following questions: what is the impact of the nonideal link between the RSs and the BS on overall capacity? How much bandwidth should be set aside for the RS-to-BS links? And which relaying strategy performs better?

The specific relaying strategy performs better? The RSs and the BS on overall capacity? How much band-

### 3. Transmission Strategies

One of the main objectives of the paper is to show that a small BW expansion on the second hop is sufficient to approach the performance of an ideal DAS. Let the BW allocated to the first hop be $W_1$ and to the second hop be $W_2$ where BW ratio is integer and given by $F = W_2/W_1 \in \mathbb{N}^+$. By controlling $F$ it will be possible to make second hop appear more ideal. We assume frequency division duplex (FDD) relaying, i.e., first and second hop communications take place on orthogonal frequencies. It is assumed that there is no direct-link between the MSs and the BSs which will serve as a lower bound for AF and CF schemes for cellular networks. The MSs communicate with the RSs in the first hop, while the RSs communicate with the BS in the second hop.

We assume that each RS knows its corresponding receiver CSI and each BS knows just the CSIs for the MSs that are located in its cell, and they treat the signals coming from surrounding cells as interference signals which are taken to be Gaussian random variables with zero mean and some variance depending on the channel gain between each BS and interfering nodes.

In the first hop, the received signal at the $i$-th RS in the $c$-th cell is given by

$$y_{c,i} = h_{c,i}^T s_e + \chi_{c,i} + n_{c,i}, \quad i = 1, 2, \ldots, N$$

where $s_e = [s_e,1, \ldots, s_e,K]^T$ and $s_e,k$ is the transmitted signal from $k$-th MS in the $c$-th cell with average power constraint $\mathbb{E}[|s_e,k|^2] = P_k$, $\forall k$, $h_{c,i} = [h_{c,i,1,1}, \ldots, h_{c,i,1,K}]^T$ is the channel vector from all MSs in the $c$-th cell to the $i$-th RS in the $c$-th cell. And $n_{c,i} \sim \mathcal{C}\mathcal{N}(0, \sigma_{c,i}^2)$ is the noise at $i$-th RS in the $c$-th cell where $\sigma_{c,i}^2 = N_0 W_1$ for $N_0$ representing the noise power spectral density, $\chi_{c,i} \sim \mathcal{C}\mathcal{N}(0, \sigma_{c,i}^2)$ with $\sigma_{c,i}^2$
\[
\sum_{b \neq c}^{B} \sum_{k=1}^{K} y_{c,i,b,k} P_{b}.
\]
In vector form, the received signals at the RSs, i.e., \( y_{c} = [y_{c,1}, \ldots, y_{c,N}]^{T} \), are given by
\[
y_{c} = H_{c,c} s_{c} + \chi_{c} + n_{c}
\]
where \( n_{c} \in C^{K \times 1} \) is the noise vector at the RSs, \( H_{c,c} \in C^{N \times K} \) is the channel matrix from all MSs in the c-th cell to all RSs in the c-th cell, and \( \chi_{c} = [\chi_{c,1}, \ldots, \chi_{c,N}]^{T} \) is the total interference vector at the RSs in the c-th cell.

At the second hop, assuming the BSs have \( N \) sectoral antennas, each directed to a unique RS, the received signal at the i-th sector antenna of the BS in c-th cell is given by
\[
y_{d,i} = g_{c,i,c}^{T} x_{c} + \chi_{c,i} + n_{d,i}, \quad i = 1, 2, \ldots, N
\]
where \( x_{c} = [x_{c,1}, \ldots, x_{c,N}]^{T} \) and \( x_{c,n} \) is the transmitted signal from n-th RS in the c-th cell with average power constraint \( \mathbb{E}[|x_{c,n}|^{2}] = P_{r}, \) and \( \chi_{c,i} \sim \mathcal{CN}(0, \sigma_{c,i}^{2}) \) with \( \sigma_{d,i}^{2} = \sum_{b \neq c}^{B} \sum_{k=1}^{K} \Phi_{c,i,b,k} P_{b}, \) and \( \sigma_{d,i}^{2} \sim \mathcal{CN}(0, \sigma_{d,i}^{2}) \) is the noise at the i-th sector antenna of the BS in c-th cell where \( \sigma_{d}^{2} = N_{0} W_{2}. \) Combining all of the received signals we get
\[
y_{c} = G_{c,c} x_{c} + \chi_{c} + n_{c}
\]
where \( G_{c,c} \in C^{N \times N} \) is the channel matrix from all RSs in the c-th cell to the BS in the c-th cell, \( \chi_{c} = [\chi_{c,1}, \ldots, \chi_{c,N}]^{T} \) the total interference vector at the RSs in the c-th cell.

### 3.1. Amplify-and-Forward Relaying

In AF the received signal at the RSs are scaled according to nodes’ power constraints and forwarded to the BSs. Though simple, the AF relaying strategy suffers from noise amplification. For AF the same signaling dimensions can be used in the first and second hop, i.e., \( W_{1} = W_{2}, \) hence \( \sigma_{d}^{2} = \sigma_{f}^{2} = N_{0} W_{1}. \) For fairness in comparison between the AF and CF schemes, the RS transmit power is increased by a factor \( F \) in the AF scheme.

According to the received signal at the RSs given in (1), the scaling factors are given by
\[
\alpha_{c,i} = \sqrt{\frac{P_{r}}{\mathbb{E}[|y_{c,i}|^{2}]}} = \sqrt{\frac{P_{r}}{\|h_{c,i,c}^{T}\|^{2} P_{s} + \sigma_{c,i}^{2} + \sigma_{f}^{2}}}
\]
The signal vector transmitted by the RSs on the c-th cell is
\[
x_{c} = D_{c} y_{c} = D_{c} H_{c,c} s_{c} + D_{c} \chi_{c} + D_{c} n_{c}
\]
where \( D_{c} = \text{diag}\{\sqrt{\mathcal{F}} \alpha_{c,1}, \sqrt{\mathcal{F}} \alpha_{c,2}, \ldots, \sqrt{\mathcal{F}} \alpha_{c,N}\}. \) The received signal vector at the BS on the c-th cell is given by
\[
y_{c} = G_{c,c} y_{c} + G_{c,c} D_{c} \chi_{c} + G_{c,c} D_{c} n_{c} + \sqrt{\mathcal{F}} \chi_{c} + n_{c}^{f}
\]
where \( G_{c,c} \in C^{N \times N} \) channel matrix from all RSs in the c-th cell to the BS in c-th cell, \( z_{c} \in C^{N \times 1} \) is the equivalent noise term which has the following covariance matrix
\[
\Lambda_{c} = \mathbb{E}[z_{c} z_{c}^{H}] = G_{c,c} D_{c} (\Lambda_{d} + \sigma_{c}^{2} \mathbf{1}_{N}) D_{c}^{H} G_{c,c}^{H} + \sqrt{\mathcal{F}} \Delta_{d} + \sigma_{c}^{2} \mathbf{1}_{N}
\]
where \( \Lambda_{c} = \mathbb{E}[x_{c} x_{c}^{H}] = \text{diag}\{\sigma_{c,1}^{2}, \sigma_{c,2}^{2}, \ldots, \sigma_{c,N}^{2}\} \) and \( \Delta_{d} = \mathbb{E}[x_{c}^{2} x_{c}^{H}] = \text{diag}\{\sigma_{c,1}^{2}, \sigma_{c,2}^{2}, \ldots, \sigma_{c,N}^{2}\}. \)

Then, the achievable ergodic sum-rates (in [bits/sec]) for UL communications in the c-th cell is given by
\[
\mathcal{R}_{AF} = W_{1} \mathbb{E}[r_{c,i}] \left[ \log_{2} \left| I_{N} + P_{d} D_{c} H_{c,c} H_{c,c}^{H} D_{c}^{H} \Omega \right| \right]
\]
where \( \Omega = G_{c,c}^{H} \Lambda_{c}^{-1} G_{c,c} \) and \( |.| \) stands for determinant.

### 3.2. Compress-and-Forward Relaying

In CF relaying strategy, the RSs compress their observations and send them to the BS. It has been shown in [2] that as the RS-to-BS links improve the system mimics single-input multiple-output (SIMO) performance. Due to no direct link assumption between the MSs and the BS, the RSs cannot facilitate from side information of the received signal seen at the BS. Hence, compression done at the RSs boils down to the standard rate-distortion scheme. Note that higher performance gains can be achieved by exploiting correlations between the compressed signals at the RSs (distributed source coding [10]).

The RSs first invert the channel gains to have unit-variance i.i.d. ZMCSHC source \( \tilde{y}_{c,i} \), i.e., \( \tilde{y}_{c,i} = y_{c,i}/v_{c,i}, \forall i, \) where \( v_{c,i} = \alpha_{c,i}/\sqrt{\mathcal{F}}. \) The RSs generate the quantized codewords according to the distribution \( f(v_{c,i}|\tilde{y}_{c,i}) \sim \mathcal{CN}(\tilde{y}_{c,i}, D_{c,i}). \) where \( D_{c,i} \) is the noise variance due to the distortion in reconstructing \( \tilde{y}_{c,i}, \) i.e.,
\[
v_{c,i} = \tilde{y}_{c,i} + n_{d,c,i}
\]
where \( n_{d,c,i} \sim \mathcal{CN}(0, D_{c,i}). \) Each RS sends the compressed signal to the BS with rate \( R_{c,i} \) which is (considering (9))
\[
R_{c,i} = W_{1} \log_{2} \left( 1 + \frac{1}{D_{c,i}} \right)
\]
or in terms of distortion
\[
D_{c,i} = \frac{1}{2^{2R_{c,i}/W_{1}}} - 1, \quad \forall i \in \{1, 2, \ldots, N\}.
\]

To be able to send compressed signals reliably to the BS, the RSs should select the compression rates, \( R_{c,i}, \) according to the MAC rate region on the second hop which is [11]
\[
\sum_{i \in S} R_{c,i} = 1 \left( X_{c,S}; Y_{c,S}^{H} X_{c,S} \right), \forall S \subseteq \{1, 2, \ldots, N\}.
\]
Assuming all RSs operate on the equal-rate point inside the achievable rate region, i.e. \( R_{c,1} = R_{c,2} = \ldots = R_{c,N} = R_{c}, \) we have the following rates for each RS
\[
R_{c} = W_{2} / N \left[ \log_{2} \left| I_{N} + P_{d} G_{c,c} G_{c,c}^{H} (\Delta_{d} + \sigma_{c}^{2} \mathbf{1}_{N})^{-1} \right| \right].
\]
Provided that we select the quantization rates according to MAC limits, we can represent the received signals of each RS with a certain fidelity at the BS. As our aim is to find the sum-of-rates from the MSs to the BS, having multiple independent representations of the received signals at the RSs will help us to improve the network capacity.

The BS jointly decodes the MSs messages using the quantized signals in (9) which have the following vector form

\[ \mathbf{v}_c = \mathbf{A}_c \mathbf{H}_{c,c} \mathbf{s}_c + \mathbf{A}_c \mathbf{x}_c + \mathbf{A}_c \mathbf{n}_c + \mathbf{n}_{d,c} \]

where \( \mathbf{A}_c = \text{diag}\{A_{c,1}, \ldots, A_{c,N}\} \) and \( \mathbf{D}_c = \mathbb{E}[\mathbf{n}_{d,c} \mathbf{n}_{d,c}^H] = \text{diag}\{D_{c,1}, \ldots, D_{c,N}\} \). Then the ergodic sum-of-rates for CF relaying is given by (in [bit/sec])

\[ \bar{R}_{CF} = \mathbb{E}[(\mathbf{H}_{c,c} \mathbf{g}_{c,c}) \cdot \left[ W_1 \log_2 (1 + \mathbf{H}_{c,c} \mathbf{H}_{c,c}^H \Gamma) \right] \] (14)

where \( \Gamma = \mathbf{P}_s \mathbf{A}_c^H \left( \mathbf{A}_c (\Delta_r + \sigma_r^2 \mathbf{I}_N) \mathbf{A}_c^H + \mathbf{D}_c \right)^{-1} \mathbf{A}_c \).

### 3.3. Comparison with Other Cellular Systems

In this section, we look at both conventional cellular system where the BS antennas are co-located, and ideal DAS where the antennas are distributed in space and connected to the BS via noiseless links. We note that for both cases there is no RS in the system anymore. The ideal DAS should provide better performance due to different shadowing and path-loss at each antenna [7]. These two schemes provide benchmark for relaying schemes considered above.

As we consider \( N \) RSs for the relaying schemes, to have fairness we assume for both of the conventional cases that there are \( N \) receive antennas at the BS which are either co-located at the BS or distributed in space (at the same locations as the RSs) and connected to the BS via noiseless links.

### 4. MINIMUM OF ACHIEVABLE USER RATES FOR SIC DECODERS

In this section we will look at the minimum of the achievable user rates which we believe is a good metric for fairness in cellular networks. We assume that the receiver is equipped with a SIC decoder where user signals are decoded by assuming undecoded user signals as noise and then subtracting the decoded user signal from the received signal. For this type of decoder the decoding order plays a crucial role. To find the best decoding order, one needs to look at all of the possible permutations. Note that there is no rate penalty for the achievable sum-of-rates in using SIC decoder.

Assuming \( K \) users, there are \( K! \) possible decoding orders which we will denote with the array \( \Pi_l = [\pi_l(1), \ldots, \pi_l(K)] \), \( \forall i \in \{1, 2, \ldots, K\} \). If we denote \( R_{\pi_i(l)}^* \) as the achievable user rate for the \( l \)-th user on the order set \( \Pi_l \), we can find the decoding order that gives the best minimum user rate as

\[ R_{\text{min}}^* = \max_{i \in \{1, 2, \ldots, K\}} \min_{i \in \Pi} R_{\pi_i(l)}. \] (15)

The SIC decoder provides a kind of degrees of freedom by controlling the decoding order which would be used in favor of increasing the achievable rate of the worst user. In other words, it would be possible to have a fairer cellular system by using the SIC decoder.

### 5. NUMERICAL RESULTS

In this section we give some numerical results for the achievable average sum-of-rates and minimum average user rates for the schemes described above. The cellular layout depicted in Fig.1 is used for the simulations where \( B = 19 \) cells are considered. The cell radius is taken to be \( R_{\text{cell}} = 2 \) km and RSs are placed at the corners of the cells. The BSs are assumed to have \( N \) sectoral antennas each directed to a unique RS, each having a single sectoral antenna. \( K \) MSs with single omni-directional antenna are randomly and uniformly distributed in each cell. All channel gains include path-loss, shadowing and antenna gain terms. We used parabolic antenna pattern for the BS and RS with 3dB beam-width, \( \theta_{3dB} \) in degrees, and maximum attenuation, \( A_{\text{max}} \) in [dBi]. All parameters used in the simulations are specified in Table 1.

In Fig.2, we plot the achievable average sum-of-rates in [bits/sec] for the MSs in the central cell with respect to the relay transmit power, \( P_r \) in [dBm] for fixed MS transmit power \( P_s = 30 \) [dBm] and for different BW ratios, \( F = 2, 3, 4 \). For high \( P_r \), we see that both AF and CF performances come closer to that of ideal DAS. Also, for all \( F \) values it can be seen that at \( P_r = 40 \) [dBm] the CF relaying performance comes closer to the ideal DAS performance. For moderate to high RS transmit powers just with a bandwidth expansion of two, the CF achieves the same performance as the ideal DAS.

In Fig.3, we plot the achievable average minimum user rates with respect to the relay transmit power, \( P_r \) in [dBm] for fixed MS transmit power \( P_s = 30 \) [dBm]. From the figure it can be seen that the ideal DAS is much more fair than the co-located antenna system. With the optimum ordering for SIC decoder, the AF and CF schemes also mimic the performance of the ideal DAS, i.e., they also have better performance in terms of fairness than the co-located antenna system.

### 6. CONCLUSIONS

In this paper, we consider relay-aided cellular UL communications under intercell interference. Assuming orthogonal frequencies for the MS-to-RS and RS-to-BS links, the achievable average sum-of-rates and minimum user rates are analyzed for the AF and CF relaying schemes and compared with two well-known cellular systems, namely the conventional cellular system and the ideal DAS. It is shown that a small BW
expansion on the RS-to-BS links is sufficient for the AF and CF to approach the performance of the ideal DAS. In addition we see that with the SIC decoder the ideal DAS outperforms the co-located antenna system in terms of the achievable average minimum user rates. Furthermore, it is shown that for moderate to high RS transmit powers the AF and CF schemes provide better fairness than the conventional cellular system.

7. REFERENCES


