Some Systems Aspects Regarding Compressive Relaying with Wireless Infrastructure Links

Erhan Yilmaz, Raymond Knopp and David Gesbert
Mobile Communications Department
EURÉCOM
Sophia-Antipolis, France
Email: \{yilmaz, knopp, gesbert\}@eurecom.fr

Abstract—In this paper, we consider single-cell cellular networks assisted with fixed relay station (RS), used by mobile stations (MS) to access the base station (BTS) via a relaying strategy. The RSs are positioned around the BTS, in such a way that wireless channels on the relay link (from RSs to the BTS) are line-of-sight, we analyze the achievable sum-of-rates for up-link communications. We compare two relaying strategies at the RSs, namely Amplify-and-forward (AF) and Compress-and-forward (CF). We show the gains brought by relays when compared with conventional cellular systems. One reference is an ideal relay-assisted system where the relays enjoy noiseless communications to the BTSs, i.e. a so-called distributed antenna system (DAS). The second reference is offered by a conventional cellular systems without relays, but same number of overall infrastructure antennas.

In this paper, it is demonstrated the surprising result that with a relay bandwidth just twice that of the mobile’s bandwidth, the system capacity approaches that of an ideal distributed antenna system, (while probably being much superior in practice in terms of ease of deployment and cost). The capacity gains of the relay-assisted network over a conventional network are also analyzed.

I. INTRODUCTION

Recently, cooperative diversity has captured significant attention in the research community due to its provision of spatial diversity gain for the wireless network in a distributed fashion. Simply stated, in cooperative communications, the source node cooperates with one or more partners to transmit its information to the destination node. This scheme provides a virtual transmit antenna array which results in a more reliable transmission toward the destination node compared to transmissions alone without any cooperation [1], [2], [3].

To increase the channel capacity, several cooperation modes involving relay nodes have been proposed in the literature [1], [2], [4]. Among them, amplify-and-forward (AF), decode-and-forward (DF) and compress-and-forward (CF) strategies are the fundamental ones. In AF, the relay nodes simply amplify the received signal according to a power constraint and forward the amplified version of the signal to the destination node. In DF, the relay nodes decode the received signals, re-encode them and transmit to the destination node. Note that, the performance ofDF strategy is limited by the source-to-relay links due to decoding process. In CF, the relay nodes compress (or estimate/quantize) the received signals with a certain fidelity and forward to the destination node. In this paper, we consider the AF and CF relaying strategies in order to benefit from joint processing at the BTS which is possible due to a strong relay-to-base link.

Recently, there has been a great deal of research on relay-assisted infrastructure based networks due to the potential improvements in system performance provided by relaying mechanism. The performance improvements take the form of reducing deployment cost, reducing terminal transmission power, enhancing network capacity, extending radio range, mitigating shadowing effect and providing spatial diversity [5], [6], [7]. The impact of limited-capacity backhaul on both multi-cell processing and MS cooperation for the up-link and the down-link for non-fading Gaussian scenarios have been studied in [8], [9], [10], [11].

In this paper, we analyze achievable up-link sum-of-rates of relay assisted single-cell cellular systems. We compare the two relaying schemes, AF and CF relaying strategies, with two well-known cellular systems where in the first case the BTS antennas are assumed to be co-located and in the second case they are assumed to be distributed in the cell and be connected to the BTS via wired links (which is hard to implement in practice and incurs a high-cost).

The contributions of this paper are:

- We propose a theoretical analysis of the gains brought by fixed relays in a single-cell scenario, exploiting the ability of the system designer to engineer near LOS links between the relays and the base at deployment time.
- We compare two leading forms of relaying strategies in the case of a strong relay-to-base link [4], namely AF and CF. We show the advantage of CF in a situation where the bandwidth allocated to the relays-to-base links is greater than or equal to two times the bandwidth of the mobiles-to-relays links.
- We show the gains brought by relays when compared with ideal (more expensive and unrealistic) DAS systems, and with conventional cellular systems.

II. SYSTEM MODEL OF RELAYING STRATEGIES FOR CELLULAR NETWORKS

A cellular network with relay deployment depicted in Fig. 1 is studied. We assume that \( N \) RSs are placed around the BTS. Note that although relay positioning has a great effect on the system performance, this issue is out of the scope of...
this paper. We assume $K$ MSs want to communicate with the BTS through the RSs. Although the possibility of direct-link between the MSs and the BTS, here we assume there is no direct-link which will serve as lower bound for CF and AF schemes for cellular networks. We consider AF and CF relaying strategies. It is well known that AF relaying performance is limited by noise power amplification and CF performance is limited by the relay-to-base link quality, and it is shown in [4] that when the relay comes closer to the BTS, the achievable rate by CF relaying approaches to the outer-bound. All terminals are assumed to be equipped with single antennas.

We denote $h_{ik}$, $i = 1, 2, \ldots, N$ and $k = 1, 2, \ldots, K$, as the channel coefficient at the first hop (from the MSs to the RSs) between MS $k$ and RS $i$, 

$$h_{ik} = \sqrt{\Upsilon_{ik}} \tilde{h}_{ik} \quad (1)$$

where each entry of $\tilde{h}_{ik}$ is an identically independent distributed (i.i.d) complex Gaussian random variable with zero mean and unit variance and $\Upsilon_{ik}$ is the parameter representing channel gain from $k$-th MS to $i$-th RS. $\Upsilon_{ik}$ includes path-loss, transmitter and receiver antenna gains and shadowing effects. We will explain in details what are the specifications for channel gains in Section-IV. In what follows, we will consider ergodic rates and thus the single coefficient channel model is sufficient even to characterize wide-band channels (e.g. OFDM).

In the second hop (from the RSs to the BTS), we denote $g_i = \sqrt{G_i}e^{j\Phi_i}$, $i = 1, 2, \ldots, N$, as the channel coefficient between relay node $i$ and BTS, where $G_i$ is channel gain (or path-loss component) and $\Phi_i$ is ergodic phase fading with uniform distribution over $[-\pi, \pi]$.

It is assumed that the communication between each of the $N$ relays to the BTS goes without cross interference. This assumption can be made realistic in several ways. For instance, a set of $N$ highly directive beams can be designed at the BTS, each one being directed to a distinct relay, over a (near-) LOS path. Another approach is to have $N$ regular antennas at the base and use multiple antenna combining to create non-interfering relays-to-base links. Although, in both cases, some residual inter-relay interference is predictable in practice, our main theoretical result will remain valid as long as the signal-to-interference ratio (SIR) exceeds 20 dB or so. This aspect is illustrated in simulation section. In practice this will assume relay-to-base spectral efficiencies on the order of 5-6 bps/Hz which is common for backhaul infrastructure links (e.g., WIMAX).

In the following for relaying cases we assume that the BW allocated to the first hop is $W_1$ and to the second hop is $W_2$ where we assume BW ratio is integer and given by $F = W_2/W_1 \in \mathbb{N}^+$. By controlling $F$ we will be able to make second hop appear more ideal. One of the main objectives of this paper is to show that a small BW expansion on the second hop is sufficient to approach the performance of ideal relay-to-base links.

We assume frequency division duplex (FDD) relaying, e.g., first and second hop communications appear on orthogonal frequencies. The MSs communicate with the RSs in the first hop, while the RSs communicate with the BTS in the second hop. In the first hop, the received signals at the RSs are given by

$$y_i = \sum_{k=1}^{K} h_{ik} s_k + n_i = \mathbf{h}_i^T \mathbf{s} + n_i \quad (2)$$

for $i = 1, 2, \ldots, N$ where $s_k$ is the transmitted signal from $k$-th MS with average power constraint $E[|s_k|^2] = P_s$ for $k = 1, 2, \ldots, K$, and $n_i \sim \mathcal{CN}(0, N_0 W_1)$ is zero-mean circularly symmetric complex Gaussian (ZMCSG) noise with variance $N_0 W_1$ at $i$-th RS. At the second hop, the received signal at the $i$-th beam of the BTS is given by

$$y_{d,i} = \sqrt{G_i}e^{j\Phi_i} x_i + n_{d,i} \quad (3)$$

where $x_i$ is the transmitted signal from $i$-th RS with average power constraint $E[|x_i|^2] = P_r$ for $i = 1, 2, \ldots, N$, and $n_{d,i}$ is ZMCSG noise with variance $N_0 W_2$ at the BTS.

A. Amplify-and-Forward Relaying

In AF relaying the received signal at the relay stations are scaled according to their power constraint and transmitted to the BTS. We assume that each RS knows its corresponding backward channel state information (CSI) and the BTS knows the full CSI of the system. Moreover, we assume that in the second hop, the relays’ transmissions do not interfere with each other, which can be achieved by utilizing directional antennas at the BTS with $N$ beams directed to each relay node.

According to the received signal at the relay nodes given in (2), the scaling factors are given by

$$\alpha_i = \sqrt{\frac{P_r}{|\mathbf{h}_i|^2 P_s + N_0 W_1}}, \quad i = 1, 2, \ldots, N \quad (4)$$

Fig. 1. Relay Stations deployment in Cellular Networks for circular cell layout.
and the signal transmitted by the $i$-th RS is given by
\[ x_i = \alpha_i y_i = \alpha_i (h_i^2 s + n_i) \]
for $i = 1, 2, \ldots, N$, which has the following compact form
\[ x = Dy = DHs + Dn \]
where $x = [x_1 \ x_2 \ \ldots \ x_N]^T$, $n = [n_1 \ n_2 \ \ldots \ n_N]^T$, $D = \text{diag}(\alpha_1, \alpha_2, \ldots, \alpha_N)$ and $H = [h_1 \ h_2 \ \ldots \ h_N]^T \in \mathbb{C}^{N \times K}$ channel matrix from the MSs to the RSs, where $h_i \in \mathbb{C}^{K \times 1}$ is the channel vector from the MSs to the $i$-th RS.

After phase compensation, the received signal at the BTS from the $i$-th beam is given by
\[ y_{d,i} = \sqrt{G} x_i + \tilde{n}_{d,i} \]
for $i = 1, 2, \ldots, N$ and $\tilde{n}_{d,i} \sim \mathcal{CN}(0, N_0 W_1)$. The received signals at the BTS have the following vector form
\[ y_d = Gx + \tilde{n}_d = GDHs + GDn + \tilde{n}_d \]
where $G = \text{diag}([\sqrt{G_1}, \sqrt{G_2}, \ldots, \sqrt{G_N}])$. With these settings, the achievable instantaneous sum-of-rates in [bits/sec] is given by
\[ R_{AF} = W_1 \log_2 \det \left( I + \frac{P_s}{N_0 W_1} GDHH^H D^H G^H A^{-1} \right) \]
where
\[ A = GDH^H H + I_{N \times N}. \]

After all the ergodic capacity is given by
\[ \overline{R}_{AF} = \mathbb{E}_H [R_{AF}] \].

**B. Compress-and-Forward Relaying**

In CF relaying strategy, the relay nodes compress their observations and send to the base station. It has been shown in [4] that as the relay-to-base links improve the system mimics single-input multiple-output (SIMO) performance. Hence, the scenario we consider here is well suited for usage of CF relaying. In our scheme as we assume there is no direct link between the sources and the destination, the relay nodes cannot facilitate from side information of the received signal seen at the destination node. Hence, compression done in the relay stations boils down to the standard rate-distortion scheme. Note that, although higher performance can be achieved by exploiting correlations between the compressed signals at the relay stations (distributed source coding [12]), we do not add this feature to the relay station and keep them as simple as possible. In [13] and [14], distributed source coding followed by compression is considered for parallel relay networks for Gaussian and phase fading cases, respectively.

We proceed as follow: due to CSI assumption at the receiving nodes, the relay nodes first invert the channel gains to have unit-variance i.i.d. ZMCSGC source $\tilde{y}_i$, i.e.,
\[ \tilde{y}_i = A_i y_i, \quad i = 1, 2, \ldots, N \]
where
\[ A_i = \sqrt{\frac{1}{||h_i||^2 P_s + N_0 W_1}}. \]

Then, the relay nodes generate the quantized codewords according to the distribution $f(v_i | \tilde{y}_i) \sim \mathcal{CN}(\tilde{y}_i, D_i)$, where $D_i$ is the noise variance due to the distortion in reconstructing $\tilde{y}_i$ as in [15], i.e.,
\[ v_i = \tilde{y}_i + n_{d,i} \]
where $n_{d,i} \sim \mathcal{CN}(0, D_i)$ for $i = 1, 2, \ldots, N$. Assuming each relay nodes wants to send the compressed signal to the destination with rate $R_i$, $i = 1, 2, \ldots, N$, with the above conditional distribution we have:
\[ R_i = W_1 I(\tilde{y}_i; v_i) = W_1 \log_2 \left( 1 + \frac{1}{D_i} \right), \]
which in turn means
\[ D_i = \frac{1}{2R_i/W_1 - 1}, \quad \forall i. \]

To be able to send compressed signals reliably to the destination node, the relay nodes should select the compression rates, $R_i$, according to an orthogonal multiple-access channel (MAC) rate region on the second hop. In this paper, the same assumption we did for AF relaying still holds for CF also, which is that each relay transmission is received by BTS without interfering to others which is possible with the usage of directional antennas at the BTS. We allocate $W_2$ [Hz] BW for the second hop. For the scheme considered here, the achievable rates in the second hop is given by
\[ R_i = W_2 \log_2 \left( 1 + \frac{G_i P_s}{N_0 W_2} \right), \forall i = \{1, 2, \ldots, N\} \]
By inserting (16) into (15) we get
\[ D_i = \frac{1}{\left( 1 + \frac{G_i P_s}{N_0 W_2} \right)^{F_i} - 1} \]

Assuming that we selected the quantization rates according to MAC limits, we can represent the received signals of each RS with a certain fidelity at the BTS. As our aim is to find the sum-of-rates from the MSs to the BTS, having multiple independent representations of the received signals at the RSs will help us to improve the network capacity.

The BTS processes the quantized signals, given in (13), which has the following compact form
\[ v_i = \sum_{k=1}^{K} A_{ik} h_{ik} s_k + A_i n_i + n_{d,i} \]
and putting all of the representations together, we have
\[ v = AHs + An + n_d \]
where $\mathbf{A} = \text{diag}(A_1, A_2, \ldots, A_K)$ and $\mathbf{H} \in \mathbb{C}^{N \times K}$ is defined in the previous section, $\mathbb{E}[\mathbf{m}\mathbf{n}^H] = \sigma^2 \mathbf{I}_{N \times N}$ where $\sigma^2 = N_0 W_1$ and $\mathbf{D} = \mathbb{E}[\mathbf{n}\mathbf{n}^H] = \text{diag}(D_1, D_2, \ldots, D_N)$.

Considering (19), the instantaneous sum-of-rates for CF relaying scheme is given by

$$R_{CF} = W_1 \log_2 \det \left( \mathbf{I} + P_N \mathbf{A} \mathbf{H}^H \mathbf{B} \mathbf{A}^H + \mathbf{D} \right)^{-1}$$

$$= W_1 \log_2 \det \left( \mathbf{I} + P_N \mathbf{H}^H \mathbf{B} \mathbf{H}^H \mathbf{B} \right)$$

(20)

where

$$\mathbf{B} = \mathbf{A}^H \left( \sigma^2 \mathbf{A} \mathbf{A}^H + \mathbf{D} \right)^{-1} \mathbf{A}$$

$$= \text{diag}\left( \frac{A_1^2}{\sigma^2 A_1^2 + D_1}, \ldots, \frac{A_K^2}{\sigma^2 A_K^2 + D_N} \right).$$

(21)

After all the ergodic capacity for CF relaying is given by

$$R_{CF} = \mathbb{E}[W_1 \log_2 \det \left( \mathbf{I} + P_N \mathbf{H}^H \mathbf{B} \mathbf{H}^H \mathbf{B} \right)].$$

(22)

### III. CONVENTIONAL CELLULAR SYSTEMS

In this section, we look at both conventional cellular system where the BTS antennas are co-located and ideal distributed antenna cellular system where the antennas are distributed in space and connected to the BTS via noiseless links. The distributed antenna cellular system should provide better performance due to different shadowing and path-loss at each antenna. These two schemes provide benchmark for relaying schemes considered above.

As we consider $N$ RSs for the relaying schemes, to have fairness we assume for both of the conventional cases there are $N$ receive antennas at the BTS which are either co-located at the BTS or distributed on space and connected to the BTS via noiseless links (or vie infinite BW links).

#### A. Co-Located Antenna Cellular System

As stated above, the BTS has $N$ co-located antennas. Considering up-link communication, the received signal vector at the BTS is given by

$$\mathbf{y} = \sum_{k=1}^{K} \mathbf{h}_k s_k + \mathbf{n}$$

$$= \mathbf{H} \mathbf{s} + \mathbf{n}$$

(23)

where $\mathbf{y} \in \mathbb{C}^{N \times 1}$ is the received signal at the BTS, $\mathbf{n} \in \mathbb{C}^{N \times 1}$ is the noise vector, $\mathbf{s} \in \mathbb{C}^{K \times 1}$ is the transmitted signal vector and $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_K]$ is $N \times K$ channel matrix where $k$-th column corresponds to channel vector from $k$-th MS to the BTS which has the following form

$$\mathbf{h}_k = \sqrt{\gamma_k} \tilde{\mathbf{h}}_k$$

(24)

for $k = 1, 2, \ldots, K$, where each entry of $\tilde{\mathbf{h}}_k \in \mathbb{C}^{N \times 1}$ is an i.i.d. complex Gaussian random variable with zero mean and unit variance and $\gamma_k$ is the channel gain from $k$-th MS to the BTS. Considering the above conventional cellular system, the MAC sum-of-rates is given by

$$\mathcal{R}_{\text{conv}} = \mathbb{E}[W_1 \log_2 \det \left( \mathbf{I} + \frac{P_N}{N_0 W_1} \sum_{k=1}^{K} \mathbf{h}_k \mathbf{h}_k^H \right)]$$

$$= \mathbb{E}[W_1 \log_2 \det \left( \mathbf{I} + \frac{P_N}{N_0 W_1} \mathbf{H}^H \mathbf{H} \right)].$$

(25)

#### B. Ideal Distributed Antenna System (DAS)

For this case, we assume that the antennas are placed the same locations as the relays. Note that optimal antenna placement is a tough issue that is out of scope of this paper. Comparing with conventional co-located antenna case, the only difference is in channel characteristics. Each channel element for this case is the same as in the relaying schemes and is given by (1). With this channel characteristics, corresponding sum rate expression is the same as the co-located antenna case.

### IV. NUMERICAL RESULTS

In this section, we give some numerical results for the achievable networks capacities for the schemes described above. The cell layout is assumed to be circular as depicted in Figure-1 and the cell radius is taken $R = 1$ km. Moreover, we assume $N = 3$ RSs and $K = 3$ MSs are present in the cellular system. RSs are places circularly and uniformly around the BTS at a distance of 0.5 km, i.e. half-way to the cell-edge. MSs are randomly distributed on the cell. As explained above all channel gains includes path-loss, shadowing and antenna gain terms which is given by

$$\mathcal{T}_{i,k} = -PL_{i,k} + G_{TX} + G_{RX} + \xi$$

where $G_{TX} = 16$[dB] is the transmit antenna gain, $G_{RX} = 4$[dB] is the receiver antenna gain, and the log-normal shadowing term, $\xi$, is a random variable with a normal distribution with mean of 0[dB] and standard deviation 8[dB]. It is assumed that each link undergoes path-loss according to simplified COST 231 model [16], given by

$$PL_{i,k} = 138 + 39.6 \log_{10}(d_{i,k})$$

where $d_{i,k}$ is the distance between $i$-th and $k$-th station. We assume operation temperature is $T = 290$ Kelvin and operation BW (first hop BW) is $W_1 = 20$ MHz.

In Figure-2 and Figure-3, we plot the achievable network capacities in [bits/sec] with respect to the average received signal-to-noise ratios (SNRs) at the BTS from a MS at the edge of the cell, $\gamma_e$, and from a RS on a circle 0.5 km away from the BTS, $\gamma_r$, respectively. For fairness in comparison between AF and CF schemes, we increase the relay transmit powers in AF relaying scheme on the order of $F = W_2/W_1$.

In Figure-2, for the average received SNR from a RS at a distance of 0.5 km is fixed to $\gamma_r = 20$dB, AF and CF schemes for different BW ratios, $F = 2$, are plotted. For the same network performance, there is 4 dB average received SNR difference between the ideal DAS and the co-located antenna cellular system. For BW ratio $F = 2$, it can be seen that CF relaying performance comes so closer to the ideal DAS while
AF relaying requires more relay power (comes closer to the ideal DAS. Equivalent performance with
relay positioning in cellular systems is also a crucial field to be explored. Other metrics for system analysis, such as
distribution of rates as opposed to average sum rates should also be investigated.

V. CONCLUSIONS

In this paper, we consider up-link for single-cell cellular systems with relay deployment where MSs access to the
BTS through intermediate relays placed around the BTS. We analyze the achievable sum-of-rates. Assuming FDD relaying,
we compare AF and CF relaying schemes with two well-known cellular systems which are cellular systems with co-
located antennas at the BTS and ideal DAS. It is shown that a small BW expansion on the relay-to-base links is sufficient
for CF relaying scheme to approach the performance of the ideal DAS. Moreover, CF scheme is more favorable than AF
scheme in terms of the sum-of-rates performance.

Fig. 2. The sum-of-rates [bits/sec] versus average received SNR at the BTS from a MS placed on the edge of the cell, γ_s.

AF relaying performance does not approach to the ideal DAS performance. With a BW ratio $F = 3$, CF achieves the same
performance of the ideal DAS. These results show that with $F ≥ 2$ we can achieve the same system performance as ideal
DAS by CF relaying.

In Figure-3, for the average received SNR from a MS from the edge of the cell is fixed to $γ_s = 10$dB. At high average
receiver SNR from RS to the BTS, $γ_r$, we see that both AF and CF performances come closer to the ideal DAS. The interesting
point here is that at $γ_r = 20$dB, CF relaying performance comes closer to the ideal DAS. Equivalent performance with
AF relaying requires more relay power ($15 – 20$ dB). These results show that with finite average received SNR from the
RSs, CF relaying scheme achieves the same performance as the ideal DAS.

Fig. 3. The sum-of-rates [bits/sec] versus average received SNR at the BTS from a RS placed 0.5 km away, $γ_r$.

In this paper, we consider up-link for single-cell cellular systems with relay deployment where MSs access to the
BTS through intermediate relays placed around the BTS. We analyze the achievable sum-of-rates. Assuming FDD relaying,