Abstract—In this paper we deliberate on channel coding for spatial data streams and focus on their equal-rate non-uniform power distribution in successive interference cancellation (SIC) detection algorithm. We focus on high spectral efficiency bit interleaved coded modulation (BICM) MIMO OFDM system where, after serial to parallel conversion, per antenna coding and antenna cycling, spatial data streams are simultaneously transmitted by using an antenna array. The reception is based on SIC detection algorithm. Standard receiver solutions for such schemes employ minimum mean square error (MMSE) successive stripping decoders. Application of MMSE filters combined with the Gaussian assumption of post detection interference institutes sub-optimality in the metrics and furthermore these equalizers are intricate in computation. We propose a novel near optimal demodulator based on match filter outputs for a $2 \times 2$ system which reduces the sub optimality of the metric resulting in an improved performance and a significant reduction in computational complexity with respect to the MMSE based solutions. We further extend the idea to higher-dimensional MIMO systems showing that there is a slight degradation in the performance with increase in dimensionality of the system but is concurrently coupled with a boost in complexity savings.

I. INTRODUCTION

Multiple antenna communication systems being capable of considerably increasing the capacity of a wireless link [1] are the focus of attention over the past few years. The requisite antenna spacing combined with the complexity constraints restrict future MIMO based communication systems to the maximum of 4 spatial streams. The existing and forthcoming MIMO based standards as IEEE 802.11n [2], IEEE 802.16m [3] and 3GPP LTE [4] substantiate this argument. Researchers persist to strive for better performance and reduced receiver complexity for such systems.

These communication systems need robust coding schemes and an appropriate solution in todays wireless world is bit interleaved coded modulation (BICM) [5]. The performance of BICM improves further through soft decision iterative decoding (BICM-ID) [6] at the cost of complexity. In order to benefit from the improvement, mappings other than the Gray mapping are used [7]. BICM MIMO OFDM therefore provides a promising choice for next-generation wireless networks where MIMO enhances the spectral efficiency, OFDM reduces the complexity of equalization and BICM stands as a robust coding scheme for fading channels.

We consider in this paper a low dimensional BICM MIMO OFDM system based on successive interference cancellation (SIC) detection algorithm i.e. sequential decoding and subtraction (stripping) of spatial streams. We propose a low complexity near optimal demodulator for a $2 \times 2$ system and extend it further to higher-dimensional systems. We focus on equal-rate non-uniform power distribution between these spatial streams for the proposed demodulator in view of successive stripping. This can be coarsely regarded as MMSE DFE as described in [8]. To the authors knowledge, the power distribution for spatial streams in SIC detection scheme for MIMO systems has not yet been investigated however literature discusses SIC and PIC detection schemes for CDMA systems in reference to different powers of received signals in multi user context [9][10]. Standard receiver solutions for such schemes including V-BLAST [11] use stripping decoders which incorporate minimum mean square error (MMSE) filters against the yet undecoded streams at each successive cancellation stage. It is optimum in the power constrained case with Gaussian inputs but practical systems make use of discrete small-size modulation alphabets. For such cases, application of these filters combined with the Gaussian assumption of post detection interference institutes sub-optimality in the metrics. The degradation of the performance due to the sub-optimality combined with the complexity in calculation of linear equalizers at each frequency tone (in OFDM based system) renders the real-time implementation of these algorithms difficult especially in fast fading wideband environments.

Our proposed demodulation algorithm for equal-rate non-uniform power distribution of spatial streams in BICM MIMO OFDM system exhibits considerable improvement in performance with respect to the existing sub-optimal approaches. While decoding the first stream, the proposed algorithm in a $2 \times 2$ system takes into account the yet undecoded second stream instead of using a MMSE filter against it. The algorithm successfully escapes the exponential complexity of MIMO detection and avoids calculation of MMSE filter coefficients. This leads to a performance (error rate) better than the standard MMSE based approaches and comparable to that of List Sphere detection (LSD) [12] coupled with a significant reduction in the complexity. The performance slightly degrades for higher dimensional systems but it is concurrently matched with a boost in complexity savings.

The paper is organized as follows. In section II we provide the system model and a review of existing approaches we used in this paper for comparison while in section III we propose
low complexity demodulation algorithm for a $2 \times 2$ system and subsequently extend it to higher dimensional systems. Section IV is dedicated to the simulation results while section V concludes the paper.

II. SYSTEM MODEL

We consider a MIMO system which is a $n_t \times n_r$ ($n_t \geq n_r$) BICM MIMO OFDM system with $n_r$ spatial streams. We effectively reduce this to $n_r \times n_r$ system by antenna cycling at the transmitter [1] with each stream being transmitted by one antenna in any dimension. The antenna used by a particular stream is randomly assigned per dimension so that each stream sees all degrees of freedom of the channel. The detection is based on stream by stream detection using successive stripping. The block diagram of the transmitter and receiver are shown in the figures 1 and 2 respectively. The well known baseband model of the system at $n$-th frequency tone is given as:-

$$y_n = h_{1,n} x_1 + h_{2,n} x_2 + \cdots + h_{n_r,n} x_{n_r} + z_n, \quad n = 1, 2, \cdots, N$$

where $N$ is the total number of frequency tones and $n_r$ is the total number of spatial streams/receive antennas. We can conveniently drop the frequency index and can rewrite the system equation as

$$y = h_1 x_1 + h_2 x_2 + \cdots + h_{n_r} x_{n_r} + z$$ (1)

where $y, z \in \mathbb{C}^{n_r}$ are the vectors of received symbols and circularly symmetric complex Gaussian noise with variance $N_0$ at the $n_r$ receive antennas. $h_1 \in \mathbb{C}^{n_r}$ is the vector characterizing flat fading channel response from first transmitting antenna to $n_r$ receive antennas and $x_1$ is the complex symbol of the first stream transmitted by first transmit antenna with $E[|x_1|^2] = \sigma_1^2$. It is assumed that each channel path between the transmitter and the receiver is independent. The complex symbols $x_1, \cdots, x_{n_r}$ of $n_r$ streams are also assumed independent.

A. Channel Capacity

For Gaussian inputs, the ergodic capacity of the system as per channel rule [1] is

$$I(x_1, x_2, \cdots, x_{n_r}; y|H) = I(x_1; y|H) + I(x_2; y|H, x_1) + \cdots$$

$$+ I(x_{n_r}; y|H, x_1, x_2, \cdots, x_{n_r-1})$$

where $H = [h_1, h_2, \cdots, h_{n_r}]$ is the channel matrix with complex channel gains $E[|h_{ij}|^2] = 1$. These terms are

$$I(x_1; y|H) = \log_2 \left\{ \det \left[ 1 + \sigma_1^2 h_1 h_1^\dagger \left( N_0 I + \sigma_1^2 h_2 h_2^\dagger + \cdots + \sigma_{n_r}^2 h_{n_r} h_{n_r}^\dagger \right)^{-1} \right] \right\}$$

$$I(x_2; y|H, x_1) = \log_2 \left\{ \det \left[ 1 + \sigma_1^2 h_2 h_2^\dagger \left( N_0 I + \sigma_1^2 h_3 h_3^\dagger + \cdots + \sigma_{n_r}^2 h_{n_r} h_{n_r}^\dagger \right)^{-1} \right] \right\}$$

and

$$I(x_{n_r}; y|H, x_1, x_2, \cdots, x_{n_r-1}) = \log_2 \left( 1 + \frac{\sigma_{n_r}^2}{N_0} \| h_{n_r} \|^2 \right)$$

where $\dagger$ indicates conjugate transpose and $I$ denotes the identity matrix. Under the power constraint $P_T$, the average SNR at each receiver branch is $\frac{P_T}{N_0}$. Key to the optimality of stripping is the use of Gaussian inputs as long as the stripping decoders incorporate MMSE filters against yet undecoded streams at each successive cancellation stage. Successive stripping requires that each stream must be transmitted at a different rate with equal power. We investigate a slightly sub-optimal solution where we guarantee equal rate with non-uniform powers on each stream. Numerical optimization revealed that equal-rate non-uniform power distribution leads to negligible sub-optimality as shown in Fig. 3a.

B. Review of Existing Schemes

We now review some existing schemes that we used for comparison with our proposed approach.
III. MATCH FILTER BASED DEMODULATOR

Our derivation of the demodulator is based on a $2 \times 2$ system and subsequently it is extended to higher dimensional systems. In a $2 \times 2$ system, the bit metric for bit $b$ at the $i$-th location of the first stream $x_1$ is given as

$$\lambda_i^b(y, b) = \log \sum_{x_1 \in \chi_{1,b}} \sum_{x_2 \in \chi_{2}} \frac{1}{\pi^2 N_0} \exp \left[ -\frac{1}{N_0} \| y - h_1 x_1 - h_2 x_2 \|^2 \right]$$

where $\chi_2$ denotes signal signal set of $x_2$. Let

$$y_1 = \frac{h_1^t y}{\|h_1\|}, \quad y_2 = \frac{h_2^t y}{\|h_2\|}, \quad h_{21} = \frac{h_1^t h_1}{\|h_2\|^2}, \quad y_2(x_1) = y_2 - h_{21} x_1$$

Ignoring $\|y\|^2$ and adding $|y_1|^2$

$$\lambda_i^b(y, b) = \log \frac{1}{\pi^2 N_0^2} \sum_{x_1 \in \chi_{1,b}} \exp \left[ -\frac{1}{N_0} \left( |y_1 - \|h_1\| x_1|^2 \right) \right] \times \sum_{x_2 \in \chi_2} \exp \left[ -\frac{1}{N_0} \left( |y_2(x_1) - \|h_2\| x_2|^2 \right) \right]$$

This equation effectively decouples the two streams. We propose that for each value of $x_1 \in \chi_{1,b}$, we retain in (4) one constellation point of $x_2$ which results in the most dominant exponential. To reduce the computational complexity of finding this constellation point, we decouple $x_2$ into its real and imaginary parts i.e.

$$|y_2'(x_1) - \|h_2\| x_2|^2 = \Re^2 (y_2'(x_1) - \|h_2\| x_2) + \Im^2 (y_2'(x_1) - \|h_2\| x_2)$$

where $\Im$ indicates the imaginary part. This decoupling combined with Gray labeling in BICM reduces the search space for $x_2 \in \chi_2$ to $\sqrt{M}/2$ points for $M$ ary QAM [13]. Quantization further reduces this to 1 to 6 operations by looking for the closest real and imaginary part of $y_2'(x_1)$ to those of $\|h_2\| x_2$. The constellation point of $x_2$ which minimizes (5) introduces little sub optimality in the bit metric which is now given as

$$\lambda_i^b(y, b) \approx \log \frac{1}{\pi^2 N_0^2} \sum_{x_1 \in \chi_{1,b}} \exp \left[ -\frac{1}{N_0} \psi_b(x_1) \right]$$

where

$$\psi_b(x_1) = |y_1 - \|h_1\| x_1|^2 + |\|h_2\| x_2|^2 - 2 \Re \left( x_2^t \|h_2\| y_2'(x_1) \right)$$

1) MMSE: Detection based on MMSE equalization [12] involves linear MMSE preprocessing i.e. applying a spatial filter $M$ to the received signal vector $y$, i.e., $\hat{x} = My$ where $\hat{x}$ is the biased estimate of $x = [x_1, x_2, \ldots, x_n]^T$. It is followed by an unbiased operation i.e. $\hat{x} = \Gamma^{-1} \hat{x}$ where $\Gamma = \text{diag}(\text{MH})$. Based on the Gaussian assumption of post detection interference, MMSE preprocessing decouples the spatial streams and the bit metric for the $i$-th bit for bit value $b$ of the symbol $x_k$ on $k$-th stream is given as

$$\lambda_i^k(y, b) = \max_{x_k \in \chi_{k,b}} \left[ -\frac{\gamma_k^2}{N_0} |\hat{x}_k - x_k|^2 \right]$$

for $k = 1, 2, \ldots, n_r$ where $\gamma_k$ is the $i$-th diagonal element of $\Gamma$. $\chi_{k,b}$ denotes the subset of the signal set $x_k \in \chi_k$ whose labels have the value $b \in \{0, 1\}$ in the position $i$.

2) MMSE SIC: It is based on the same stripping and decoding approach which would be optimal for Gaussian inputs. A specific ordering for stripping needs to be enforced. In this work, spatial streams are ordered based on their decreasing received power levels (induced by non-uniform transmit power). The contribution of the strongest stream is detected and subsequently canceled leading to the detection of residual streams. Equal-rate non-uniform power distribution is used for the desired SNR region as dictated by Fig. 3b. In almost all the forthcoming communication systems as 3GPP LTE [4], there would be a feedback for per stream power control to achieve performance goals. An analysis of MMSE SIC with different user power levels in multi user context for CDMA system can be found in [9][10].
Applying log sum approximation [5]

\[ \lambda_i^1(y, b) \approx -\frac{1}{N_0} \min_{x_1 \in \chi^1_n} \psi_b^1(x_1) \] (8)

This implies reduction in complexity to \( O(1) \) which is equivalent to the complexity of detection preceded by linear MMSE filter equalization [12]. In MIMO OFDM system, MMSE filter needs to be computed for each frequency tone thereby enhancing the computational complexity of the demodulator. Evading MMSE calculation in (8) causes reduction in complexity of demodulator as will be verified in the subsequent sections.

A. Extension to Higher Dimensionality

We propose the extension of proposed demodulator to higher dimensional systems basing on the Gaussian assumption for the undetected symbols. For the detection of stream-1 in \( n_r \times n_r \) system \((n_r > 2)\), it is reduced to \( 2 \times 2 \) system by Gaussian assumption for \( 3, \cdots, n_r \) streams. Subsequent to the stripping of stream-1 using the proposed demodulator, again the system is reduced to \( 2 \times 2 \) system for the detection of stream-2 by Gaussian assumption for \( 4, \cdots, n_r \) streams. The course continues until the detection of the last stream. The Gaussian assumption for undetected symbols and their integration in noise enhances sub optimality of the proposed metric. This detection scheme necessitates numerical optimization of power distribution between these spatial streams to equate the error rates at each decoding level in the desired SNR region. Another approach for extension to higher dimensionality which is not analyzed in this work is using MMSE SIC approach for decoding of initial streams and using the proposed demodulator approach for the detection of last two streams.

B. Complexity Analysis

So far we have not been specific about the complexity savings of our proposed demodulator with respect to MMSE and MMSE SIC based demodulators. The operations governing the overall computational cost are complex multiplications and to a much lesser extent complex additions. The complexity of calculating bit metric in \( n_r \times n_r \) BICM system is \( O(1) \) while in case of MMSE SIC based demodulator, it reduces to \( O(1) \) for the \( k \)-th spatial stream. This reduction is the result of computing and employing MMSE filter followed by the unbiased operation at each decoding level. Here we assume Gauss-Jordan elimination for matrix inversion.

The proposed demodulator is based on match filter outputs and reduces complexity to \( O(1) \) for the \( k \)-th spatial stream. In a \( 2 \times 2 \) system, this complexity reduction is the consequence of decoupling of \( x_1 \) and \( x_2 \). The minimum of (5) can be found by 1, 2, 4 and 6 operations if \( x_2 \) belongs to QPSK, QAM16, QAM 64 and QAM 256 respectively. This is realized by quantizing the constellation \( \| h_2 \| x_2 \). The sign and the magnitude of real and imaginary parts of \( \psi_b^2(x_1) \) specify the quantized region of \( \| h_2 \| x_2 \) in which it lies which leads to finding the minimum in 1 to 6 operations (comparisons referred as \( \eta_{b_2} \)). We consider one comparison as one complex addition. For higher dimensional systems, two spatial streams are considered at one time incorporating the remaining undetected streams in noise, therefore there is no significant increase in the complexity. Subsequently the complexity reduces to \( O(1) \) for the \( k \)-th spatial stream based on (8).

Table 1 shows the complexity for calculation of LLR for MMSE SIC approach and the proposed approach. I indicates the operations which need to be done once during the period the channel remains constant. \(|x|\) is the size of modulation alphabet. It is evident that complexity savings of proposed demodulator with respect to MMSE SIC shrinks as the alphabet size enhances while it expands with the increase in the size of system.

For complexity comparison with MMSE approach and List Sphere detection [14], readers are directed to [12] which gives detailed complexity assessment of these two approaches. However in this comparison, there is additional overhead in the proposed approach in the form of interleaving, encoding and multiplication with channel coefficients but the complexity of this overhead is small in comparison with that of computing MMSE filter coefficients and that of LSD.

<table>
<thead>
<tr>
<th>Demodulator Type</th>
<th>No. of Complex Additions</th>
<th>No. of Complex Multiplications</th>
</tr>
</thead>
<tbody>
<tr>
<td>MMSE SIC</td>
<td>(</td>
<td>4n_r^3 - 2n_r^2 - n_r</td>
</tr>
<tr>
<td>Proposed</td>
<td>(</td>
<td>5(n_r - 5)</td>
</tr>
</tbody>
</table>

C. Simulation Results

We now assess the performance of the proposed demodulator by means of simulation of the frame-error rates. We focus on target error-rates on the order of 10^-2. We consider 2 × 2 and 3 × 3 BICM MIMO OFDM systems using the de facto standard, 64 state, rate-1/2 convolutional encoder. The upcoming WLAN standard 802.11n [2] supports the codeword sizes of 648, 1296, and 1944 bits. For our purposes, we selected the codeword size of 1296 bits. The MIMO channel has iid Gaussian matrix entries with unit variance. The channel is independently generated for each time instant and perfect CSI at the receiver is assumed. Furthermore, all mappings of coded bits to QAM symbols use Gray encoding. We consider the MMSE based standard approach, MMSE SIC approach and the proposed approach. Spatial streams of equal rates are transmitted in a 2 × 2 and 3 × 3 system. These streams are of equal power for MMSE approach while equal-rate non-uniform power distribution is used for MMSE SIC (assuming Gaussian symbols). For 2 × 2 system, proposed approach uses the same power distribution as for Gaussian symbols while for 3 × 3 system, the error rates on 3 streams are equated by numerical optimization of power distribution in the desired
reduction in computational complexity especially in higher frequency tone in proposed approach leads to substantial that MMSE equalizer matrices need not be computed at each comparable with more complex LSD techniques. The fact standard linear equalizer based solutions as MMSE filter and complexity and in most cases better performance than the demodulation technique which has reduced computational with equal-rate non-uniform power distribution. Our proposed MIMO OFDM system based on SIC detection algorithm dimensional systems. The idea of spatial data streams with SNR region. Figures 4, 5 and 6 indicate much improved metric due to Gaussian assumption for the third stream. The attributed to increase in the sub optimality of the proposed IV. CONCLUSIONS

We have presented a novel optimal demodulator for BICM MIMO OFDM system based on SIC detection algorithm with equal-rate non-uniform power distribution. Our proposed demodulator is based on match filter outputs and results in a demodulation technique which has reduced computational complexity and in most cases better performance than the standard linear equalizer based solutions as MMSE filter and comparable with more complex LSD techniques. The fact that MMSE equalizer matrices need not be computed at each frequency tone in proposed approach leads to substantial reduction in computational complexity especially in higher dimensional systems. The idea of spatial data streams with equal rate power distribution has many potential applications with reference to application to 802.11n, prioritizing different data streams for different users in a broadcast scenario and interference cancellation in cellular environments which were not considered here. We used convolutional codes because of its widespread application in existing MIMO systems as IEEE 802.11n [2] but future work shall examine the influence of more powerful turbo codes on the proposed demodulator.

REFERENCES