ON LMMSE BIAS IN CDMA SIMO/MIMO RECEIVERS

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1. INTRODUCTION

We revisit CDMA downlink receivers (RX) based on linear Minimum Mean-Square Error (LMMSE) chip-equalizer front-end followed by a Walsh code correlator for Single-Input-Single (Multi-)Output (SISO or SIMO) channels with the purpose of highlighting the non-trivial question of bias in the output of the equalizer. In a linear time-invariant channel, this bias is constant at chip-equalizer output, but evolves over time at code correlator output impacting Signal-to-Interference-plus-Noise Ratio (SINR) and thus achievable rates in such receivers. In principle, this bias must be taken into account in further RX/decoding stages even if its impact is small. It is shown that a new class of Maximum-Likelihood (ML) RX leading to potential performance gains is obtained when properly accounting for symbol-level bias across a set of user codes. These results are extended to the Multi-Input-Multi-Output (MIMO) case of UMTS High-Speed Downlink Packet Access (HSDPA).

Index Terms— MIMO, CDMA, SINR, LMMSE Bias, Maximum Likelihood

1. INTRODUCTION

Most modern wireless data communications opportunistically schedule communications based upon knowledge of channel-state in some form or another. This information is usually made available through feedback over the reverse link.

The 3GPP standardization body has also gone a step further and recently introduced MIMO as enhancement of UMTS HSDPA [1]. The MIMO scheme selected is one of Per-Antenna Rate Control (PARC), namely D-TxAA for Dual-stream Transmit (TX) Diversity. Code reuse is employed across the two streams and the scrambling sequence is also common to both TX streams. All available (15) spreading codes (spreading factor 16) are allocated to the same user in the HSDPA MIMO context and Spatial Division Multiple Access (SDMA) is therefore ruled out in the interest of spatial multiplexing and multicarrier diversity (maximizing sum-rate).

A typical receiver structure for HSDPA and its MIMO extension is the linear MMSE chip-level equalizer followed by a per-Walsh code correlator presented a decade back. This receiver is one of the commonly accepted SISO/SIMO/MIMO receiver structures (see e.g., [2] and references therein). A Successive Decoding/Interference Canceling (SIC) receiver based on this LMMSE feedforward filter was shown to be mutual-information maximizing in [3] when operating at the chip-level (feeding back chip-sequence decisions). The authors of [3] translate chip-level SINR and symbol-level SINR through the spreading gain (L). Such an approach assumes treatment of scrambler as a random (white) sequence, and under this assumption, asymptotic analysis of the equalizer-correlator cascade (in number of codes and spreading factor as the ratio remains constant) indeed leads to the well-known SINR expressions [4].

In this paper, we expose a different point-of-view which ensures from a deterministic treatment of the scrambler and in which the desired signal contribution at the correlator output is not only concentrated in one tap of the channel-equalizer cascade but also contains a scrambler dependent time-varying component (thus not only a mean but also a variance). We derive the relationship linking LMMSE chip-equalizer output bias and correlator output (time-varying) bias. We subsequently derive the somewhat complicated analytical expression for the bias term and evaluate SINR including explicit contribution of this quantity. We evaluate the adjustment required to theoretical capacity (mutual information) in the SISO/SIMO and MIMO cases and present a class of ML RX based upon per-symbol ML detection across all (or a subset of) CDMA codes.

2. BIAS AND SINR IN SIMO CASE

Fig. 1 shows a Finite-Impulse Response (FIR) SIMO model of the CDMA downlink signal received at the q RX sensors of the MS. The signal is considered to be oversampled m times the chip rate

\[
\begin{align*}
\mathbf{c} &= [c_1, \ldots, c_K] \\
\mathbf{a} &= [a_1, \ldots, a_K] \\
\mathbf{h} &= [h_1, \ldots, h_s] \\
\mathbf{y} &= [y_1, \ldots, y_m] \\
\end{align*}
\]

Fig. 1. SIMO FIR downlink signal model.

which is 1/T_c. We can write the RX signal vector corresponding to contribution of K codes and Q symbols as

\[
\mathbf{Y}[n] = \mathbf{H} \mathbf{S}[n] \mathbf{C} \mathbf{A}[n] + \mathbf{V}[n],
\]

where \( \mathbf{H} \) is the standard channel convolution (block Toeplitz) matrix with \( \mathbf{H}_{m \times (Q-1)} \) as the first block row. \( \mathbf{H} = [h[n-M-1], \ldots, h[0]] \) is the \( m \times N \) channel matrix; \( L \) and \( N \) respectively being the spreading code and the channel (itself a cascade of TX filter, propagation channel, and the RX filter all sampled at rate \( m/T_c \) at the RX) length in chips. In the relation, \( \mathbf{S}[n] \) is the \( Q \times Q \) diagonal matrix containing the scrambler \( (s[n]) \) are zero mean, unit variance complex i.i.d. sequence for Q symbols at the time instant
$n$, $C = I_Q \otimes [c_1 \ldots c_K]$ is the spreading code matrix with $c_k = [c_k[0] \ldots c_k[L-1]]^T$ being the $k$th user’s unit-norm spreading code: $c_k^T c_k = \delta_{jk}$ and $A[n] = A'[n] \cdot A''[n-Q+1]]^T$ is the $n$th instant symbol vector for $Q$ symbols with $A[j] = [a_1[j] \ldots a_K[j]]^T$ the $j$th instant symbol vector for one symbol $(j)$ of each code. Note that the symbol sequence is zero mean with variance $E[|a_k[j]|^2] = \sigma^2_k$. The vector $V[n]$ is the $n$th instant noise with covariance matrix $R_{VV}$. A typical receiver structure is shown in Fig. 2.

![Fig. 2. LMMSE chip equalizer and correlator.](image)

### 2.1. Chip Equalizer Output

As is well known, the MISO LMMSE equalizer $f(z)$ of Fig. 2 operating at chip-rate will lead to estimates of the chip sequence $b[j]$ with a certain equalization delay $d$,

$$b[j-d] = f Y[n].$$

(2)

If we express the cascade of $1 \times mqE$ equalizer and the channel as

$$\alpha = f H E = \sigma^2_\alpha (\alpha_d + \bar{\alpha})$$

(3)

where $\sigma^2_\alpha = E[|b[j]|^2]$ is the variance of the chip sequence. In the above,

$$\alpha_d = \begin{bmatrix} a_0 & \ldots & a_{d-1} \end{bmatrix} E_{-N+2-d}$$

(4)

$$\bar{\alpha} = \begin{bmatrix} a_0 & \ldots & a_{d-1} & 0 & a_{d+1} & \ldots & a_{E+N-2} \end{bmatrix}$$

(5)

The SINR at the output of the chip equalizer is given as

$$\Gamma_{\text{chip \_eq}} = \frac{\sigma^2_\alpha |\alpha_d|^2}{\sigma^2_\alpha |\bar{\alpha}|^2 + |f R_{VV} f|^2}$$

(6)

$\alpha_d$ is the constant bias term in the LMMSE output.

### 2.2. Chip Equalizer/Correlator Cascade Output

The chip equalizer output is aligned and stacked in a length-$L$ vector which from (1) can be written as


(7)

where $T(\alpha)$ is the code-channel cascade (Toeplitz) convolution matrix with $[\alpha \ 0_{1\times(QL-E-N+1)}]$ being the first row. $T(\bar{\alpha})$ is defined similarly. $X[n]$ is later despread for the $k$th Walsh-Hadamard code as

$$z_k[n] = c_k^H \bar{S}^H[n] T(\alpha) S[n] C_k A_k[n] + c_k^H \bar{S}^H[n] \left\{ T(\bar{\alpha}) S[n] C_k A_k[n] + T(\bar{\alpha}) S[n] \bar{A}[n] + F V[n] \right\}.$$  

(8)

In (8), $\bar{S}[n]$ is the $L \times L$ scrambler matrix, $T(\alpha)$ is the $L \times L$ submatrix of $T(\alpha)$ with $[a_{d+1} \ldots a_{d+L-1]}$ as the first row, $T(\bar{\alpha})$ is the same as $T(\alpha)$ with the $\alpha$ submatrix replaced by $0_{L \times L}$ matrix and $C_k$ and $A_k[n]$ are the $QL \times Q$ and $Q \times 1$ spreading code matrix and the symbol vector for the $k$th user respectively. $\bar{C}$ and $\bar{A}[n]$ are the same as $C$ and $A[n]$ without the $k$th user’s contribution.

In this expression, the first term expresses the desired symbol contribution for the $k$th user, the second and third, the ISI and interference from other codes respectively, while the fourth term represents the noise contribution for symbol $n$.

#### 2.2.1. Signal Contribution - Bias

Let us first concentrate on the desired symbol contribution which can itself be split into two parts (a diagonal one and the rest) as

$$z_k[d] = c_k^H \bar{S}^H[n] \left\{ \alpha_d I_L + T(\alpha_{d,u}) + T(\alpha_{d,l}) \right\} S[n] C_k A_k[n].$$

(9)

where, $T(\alpha_{d,u})$ is the $L \times L$ upper triangular part of $T(\alpha)$ with the first row as $\alpha_{d,u} = [a_{d+1} \ldots a_{d+L-1}]$ and $T(\alpha_{d,l})$ is a lower triangular submatrix of $T(\alpha)$ with the last row as $\alpha_{d,l} = [a_{d-L+1} \ldots a_{d-1}]$. The expression (9) represents the bias term which is now symbol-dependent due to time-varying nature of the scrambling code. Due to deterministic treatment of the scrambler, the $n$th instant contribution of the desired term to RX output energy is

$$|z_k[n]|^2 = \sigma^2_\alpha \left\{ |\alpha_d|^2 + |\alpha_{d,u} u_{n,u}|^2 + |\alpha_{d,l} u_{d,l}|^2 \right\}.$$  

(10)

with $u_{n,u} = (C_k S[n])^H S[n] c_k$ and $C_k$ is a $L \times L$ upper triangular Toeplitz matrix with $0_{c_1[0]} \ldots c_k[L-2]$ being the first row. Likewise, $\bar{C}_k$ is a $L \times L$ lower triangular Toeplitz matrix with $c_k^\ast$ being the last row. $\bar{S}[n]$ have the same corresponding structure, and an average energy term expressed as

$$E|z_k[d]|^2 = \sigma^2_\alpha \left\{ |\alpha_d|^2 + L^{-1} \left( |\alpha_{d,u} D_u |^{\alpha_H} + |\alpha_{d,l} D_l |^{\alpha_H} \right) \right\}.$$  

(11)

In the above, $D_u$ is a diagonal matrix with $1-i/L$ as the $i$th diagonal term and $D_l$ is the same with $i/L$ as the $i$th diagonal term.

Comparing (6) and (11), it can be seen that while treating the scrambler as deterministic, the desired signal contributions at the output of the LMMSE chip equalizer and correlator cannot simply be related through $L$ (see e.g., [3]). Furthermore in (11), the first term is the mean value of the desired signal energy, while the second (set of terms) is the variance.

#### 2.2.2. Interference Contribution and SINR

It is easy to show that the two interference contributions (the first being the ISI and the second the MAI) in (8) are

$$E|c_k^H \bar{S}^H[n] T(\bar{\alpha}) S[n] C_k A_k[n]|^2 = \sigma^2_\alpha L^{-1} \left\{ |\alpha_u|^2 + \bar{\alpha}_u D_u |^{\alpha_{H}} + |\alpha_l|^2 + \bar{\alpha}_l D_l |^{\alpha_H} \right\},$$

(12)

and

$$E|c_k^H \bar{S}^H[n] T(\bar{\alpha}) S[n] C \bar{A}[n]|^2 = \sigma^2_\alpha |\bar{\alpha}|^2 (K-1) L^{-1}.$$  

(13)

In the above expressions, $\alpha_u = [a_0 \ldots a_{d-1}]$ and $\bar{\alpha}_u = [a_{d-L-1} \ldots a_{d-1}]$. Similarly, $\alpha_l = [a_{d+1} \ldots a_{d+L-1}]$ and $\bar{\alpha}_l = [a_{d+1} \ldots a_{d+L-1}]$. In a manner similar to (11), $D_u$ is a diagonal matrix with $1-(i-1)/L$ as the $i$th diagonal term and $D_l$ is the same with $i/L$ as the $i$th diagonal term.

It is clear that symbol level SINR is the ratio of quantities from (11), (12) and (13). The denominator further contains a noise contribution as $f R_{VV} f^H$. It is obvious that symbol-level SINR is not simply a scaled version of the chip-level SINR in (6) when a deterministic treatment of the scrambler is considered.

#### 2.2.3. Joint ML Detection for $K$ Users

In (8), we isolate the contribution of the desired user ($k$th user) matrix $T(\alpha)$. Assuming a similar treatment for all ($K$) user codes, from (7) as

$$z[n] = C^H \bar{S}^H[n] X[n].$$

(14)
which can be seen as a series of symbol rate contributions of the user symbols. Considering the same contributions as \( T(\alpha) \) for all users (thus not only mean but variance of the bias contribution), we can write in a similar fashion as in (9) as

\[

The ML cost function is \( \min_{\alpha} ||\bar{A}[n] - G[n]A[n]||^2 \). In this case the contribution of nth symbols from all other codes can be considered to be eliminated when detecting the corresponding symbol of kth user. Thus the SINR corresponds to an appropriately defined MFB that can be expressed as

\[
MF B = \frac{\sigma^2 \{\alpha_d^2 + L^{-1}(\alpha_d a_d + \alpha_d d_i a_{d_i} \alpha_{d_i}^H)\}}{\sigma^2 \{\| \alpha_u \|^2 + \| \alpha_d \|^2 + \| \alpha_d d_i \|^2 + \| \alpha_d d_i d_i \|^2 \} + f \sigma^2 V V^H}, \tag{16}\]

where all the quantities are given in previous sections.

In the interest of reducing the receiver complexity, any subset of codes can be also considered for ML detection at the cost of a reduced SINR gain.

2.2.4. Discussion

So far, for the SISO/SIMO case, we have shown that instead of treating symbol level LMMSE bias present at the output of correlator as a constant value equal to the mean of time-varying bias, the receiver can choose (by treating the scrambler to be deterministic) to consider bias as time-varying and treat it accordingly. In doing so, it benefits from a higher SINR which can be seen as addition of the contribution due to non-diagonal contribution of \( T(\alpha) \) to the numerator, which would otherwise be relegated to the denominator of the SINR expression.

In addition, if the receiver treats all codes in a similar fashion and jointly detects symbols on all codes, the receiver sees a much greater SINR gain; at the expense of increased complexity. The symbol level SINR for this type of a receiver is simply the MFB given in (16).

3. BIAS AND SINR IN MIMO

Fig. 3 shows 3GPP release-7 MIMO [1] without unitary precoding. A FIR MIMO model closely resembles SIMO. Like the SIMO case,

\[
Y[n] = \sum_{i=1,2} H_i S[n] C A_i[n] + V[n], \tag{17}\]

where, \( H_i \) and \( A_i[n] \) represent (FIR) channel and symbol vector from ith TX stream to the q RX antennas. Note that \( q = 2 \) in release-7 MIMO. The scrambler, channelization codes and \( K \), the number of users, as well as the TX power is the same for both streams. All other quantities are the same as the SIMO case.

3.1. LMMSE Extension for MIMO

A \( 2 \times 2 \) MIMO LMMSE chip-equalizer \( F = [f_1^T, f_2^T]^T \) cancels interchip as well as interstream interference [2]. Each component of the equalizer is given by the structure shown in Fig. 2 for the SISO/SIMO case. Let us define \( \alpha^{ij} = f_i H_j \). Then the chip-equalizer/correlator output can be written as

\[
z[n] = \begin{bmatrix} z_{1k}[n] \\ z_{2k}[n] \end{bmatrix}, \tag{18}\]

with

\[
z_{1k}[n] = c_k^H \bar{S}^H[n] \{ T(\alpha^{(1)}) S[n]C A_1[n] + T(\alpha^{(2)}) S[n]C A_2[n] + T(f_1) V[n] \}. \tag{19}\]

The \( 2 \times 2 \) covariance matrix of MIMO RX output is

\[
R_{zz} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix}, \tag{20}\]

where,

\[
r_{ii} = \sigma^2 \{ [\alpha^{(i)}]_d^2 + [\alpha^{(i)}]_d^2 \} + L^{-1} \{ \alpha^{(i)} d \alpha^{(i)} d^H + \alpha^{(i)} d_i \alpha^{(i)} d_i^H \}
+ \alpha^{(i)} d \alpha^{(i)} d^H + \alpha^{(i)} d_i \alpha^{(i)} d^H + \alpha^{(i)} d_i \alpha^{(i)} d_i^H + \| \alpha_u \|^2 + \| \alpha_d \|^2 + \| \alpha_d d_i \|^2 + \| \alpha_d d_i d_i \|^2 + (K-1) \| \alpha(i) \|^2 + K(\| \alpha(j) \|^2) \} + f_i \sigma^2 V V^H. \tag{21}\]

and

\[
r_{ij} = r_{j1} = \sigma^2 \{ KL^{-1} \| \alpha(i) \|^2 + \alpha^{(i)} d \alpha^{(i)} d^H \}
+ \{ KL^{-1} \| \alpha(i) \|^2 + \alpha^{(i)} d_i \alpha^{(i)} d_i^H \} + f_i \sigma^2 V V^H. \tag{22}\]

The quantities accounting for desired signal and inter-chip interference in the above expressions are the same as for the SISO/SIMO case. The terms \( \sigma^2 \| \alpha(i) \|^2 \) and \( \sigma^2 \| \alpha(j) \|^2 \) account for interstream interference. The above expressions give SINR adjustment made to equalizer/correlator RX for a single user (code). If the LMMSE equalizer/correlator output undergoes no further processing, the \( r_{ii} \) are simply the output energy of the receiver for the ith stream. The output SINR for this stream is then simply the ratio of the sum of \( [\alpha^{(i)}]_d^2 \) and \( L^{-1} \alpha^{(i)} d \alpha^{(i)} d^H \) and the sum of all other terms in (21). One could indeed consider taking the spatial correlation into account by a further processing stage at the output of chip-equalizer correlator RX, in which case the cross terms \( r_{ij} \) terms come into play [5].

From above developments, two versions of joint ML can be envisaged. In one version joint ML for all codes per stream could be considered while in a more sophisticated and complex version one could carry out joint ML across all codes and across spatial streams together. In both cases the SINR expression are simple extensions of the MFB in (16) and are omitted here for lack of space.

One may further remark that joint ML MIMO receiver presented here is an extension of the per code spatial joint ML [5] in that a deterministic treatment of the scrambler is considered and b) all codes are considered in the joint ML metric. Both these aspects improvements in RX design and performance with respect to the design of [5].
4. SIMULATIONS

We present here simulation results for the capacity bounds for the receiver where the scrambler is treated as random and compare it against the deterministic case where the time varying symbol-level bias is taken into account and ML detection is employed. For the SISO case, we plot the CDF of the capacity at the output of the equalizer-correlator pair (with all 15 codes) when only the mean of the bias is considered and compare it to the capacity at the output of a ML detector that jointly detects symbols on 15 codes and exploits time varying symbol-level bias. For the MIMO case, we compare capacity at the output of the equalizer-correlator pair when the bias is considered constant against capacity when the bias is considered time varying and ML detection is applied on a per-stream basis. For both cases, we use ITU Vehicular A channel model and a SNR of 10dB.

Fig. 4. SISO: Capacity bound for receiver with constant bias vs. time-varying bias and ML detection.

Fig. 5. MIMO: Capacity bounds for constant bias vs. time-varying bias and ML detection (per-stream).

We consider first the SISO/SIMO case. Fig. 4 shows gain in SINR when all the codes are used for ML detection. Alternatively, in the interest of decreasing receiver complexity, joint detection can be performed over a subset of codes. In Fig. 5 we present gains achieved for $2 \times 2$ MIMO Spatial Multiplexing case where all 15 codes are used for transmission on both antennas. We see an increase in SINR (and hence capacity) for both streams when we account for the variance of the symbol-level bias and use this while jointly detecting symbols on all codes of a particular stream. It can be seen while comparing the capacity plots for SISO and (PARC) MIMO case, that the latter takes a hit in SINR. This is due to residual spatial interference at the output of chip-equalizer correlator. ML detection across spatial streams recovers this loss due to residual interference albeit at a significant increase in complexity. Bounds for this are shown in Fig. 6.

Fig. 6. MIMO: Capacity bounds for constant bias vs. time-varying bias and ML detection across streams.

5. CONCLUSIONS

In this contribution, we argue that the LMMSE chip-equalizer correlator output in reality consists of time-varying bias and show that a deterministic treatment of the scrambler as opposed to it being considered (random) white, taps this bias in a more beneficial way. In doing so, signal energy contribution is given by a mean value that is the same as for the random scrambler case alongside a variance term which is the fruit of deterministic treatment of bias. In case of random scrambler, latter’s contribution is therefore lost and consigned to the interference term in the SINR expression. We also presented a class of SIS(M)O/MIMO receivers that consider time-varying bias for all codes and jointly detect user symbols over these codes.

6. REFERENCES