COCHANNEL INTERFERENCE CANCELLATION WITHIN THE CURRENT GSM STANDARD

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ABSTRACT

The performance of Maximum Likelihood Sequence Estimation (MLSE) is bounded (and often approximated well) by the Matched Filter Bound (MFB). In this paper, we first show how the GMSK modulation of GSM can be linearized and reformulated to have a real symbol constellation, leading to a two-channel aspect due to the in-phase and in-quadrature components of the received signal. More channels can be added by oversampling and/or the use of multiple antennas. We present the MFB for this model in the presence of colored noise (interferers) for MLSE that employs noise correlation information (that in general may differ from the true one). We show that properly taking into account the noise correlation leads to a much improved MFB.

Keywords: Mobile Communications, GSM, Matched Filter Bound, Interference Cancellation, Spatial filtering.

1. LINEARIZATION OF A GMSK SIGNAL

We analyse first GMSK signals. Let \( x(t) = e^{j\varphi(t)} \) be a baseband GMSK signal, where

\[
\varphi(t) = \sum_{k} a_k \int_{-\infty}^{t} \text{rect}(\frac{u-kT}{T}) h(u) du = \sum_{k} a_k \phi(t-kT)
\]

is the continuous modulated phase, \( \phi(t) \) is the ”phase impulse response”, \( \{a_k\} \) are the differentially encoded symbols from the original data \( d_k \in \{-1, 1\} \) and \( T \) is the symbol period. The phase impulse response is obtained by integrating a Gaussian filter \( h(t) = \frac{1}{\sqrt{\pi B T}} e^{-\frac{t^2}{B^2 T}} \), so \( \phi(t) = \frac{\pi}{T} \int_{-\infty}^{t} \text{rect}(\frac{u}{T}) h(u) du = \frac{\pi}{2} \left[ G(u+\frac{1}{2}) - G(u-\frac{1}{2}) \right] \)

where \( G(u) = \sigma^2 T h(uT) + u \int_{-\infty}^{\infty} h(t) dt \). It can be seen in figure 1a that \( \phi(t) \) can be approximated by zero for \( t < -\frac{3T}{2} \) and \( \frac{\pi}{2} \) for \( t > \frac{3T}{2} \). Interpreting this figure, we can conclude that one symbol \( a_k \) will have an influence on three symbol periods \( (k-1, k, k+1) \).

Figure 1: a) Phase impulse response of the GMSK modulation, b) GMSK pulse shape filter.

Figure 2: Signal flow diagram for the GMSK modulation.

Considering the data \( \{d_k\} \), the differential encoder (see figure 2) yields \( a_k = d_k d_{k-1} = \frac{d_k}{d_{k-1}} \). The relation between \( \{d_k\} \) and \( \{a_k\} \) is such that \( a_k = 1 \) if \( d_k = d_{k-1} \) and \( a_k = -1 \) if \( d_k = -d_{k-1}. \) This implies that the phase increases by \( \frac{\pi}{4} \) over three symbol periods if the symbols \( d_k \) at instant \( k \) and \( k-1 \) are equal and decreases by the same quantity in the other case. We have a modulation with memory in which some Inter-Symbol Interference (ISI) gets introduced by the modulation itself.

A sampled GMSK signal can be approximated by a linear filter with impulse response duration of about \( L T = 4T \) (see figure 1b), fed by \( \delta_{k} = j^k d_k \) \( j = \sqrt{-1} \), a modulated version of the transmitted symbols. Such a scheme (but with a shorter filter) holds exactly when sampled MSK signals are considered [1]. Exploiting the quasibandlimited character of the GMSK signal, we can even approximate the continuous-time GMSK signal by interpolating the output of a discrete-time linear system. To this end, we shall assume that sampling the GMSK signal at rate \( \frac{4f_s}{\pi} \) satisfies the Nyquist theorem. The linear system \( F \) that models the oversampled GMSK signal will be determined by least-squares estimation over a long stretch of signal, see figure 3.

Let \( \{f_{\nu} + u\} \sub 1 \leq \nu \leq L-1 \; ; \sub 1 \leq \nu \leq q-1 \) be the impulse re-
The propagation environment is described by a channel $c(t) = [c_1(t) \cdots c_N(t)]^H$. We consider GSM channel models which are taken as specular multipath channels with $L_p$ paths of the form $c_n(t) = \sum_{r=1}^{L_{p,n}} a_{r,n} \delta(t - \tau_{r,n})$ for the $n$th antenna. $a_{r,n}$ and $\tau_{r,n}$ are the amplitude and the delay of path $r$. The distribution of the amplitudes and the values of the delays depend on the propagation environment (urban, rural, hilly terrain). We consider independent channel realizations for the $K$ antennas. The received signal for antenna $n$ can be written as

$$y_n(t) = \sum_{k=-\infty}^{+\infty} h_n(t-kT)b_k, \quad h_n(t) = \sum_{r=1}^{L_{p,n}} a_{r,n} f(t - \tau_{r,n}).$$

(5)

The continuous-time channel $h_n(t)$ for the $n$th antenna when sampled at the instant $t_0 + (k + \frac{t}{T})T$ yields the $((n-1)p + l)^{th}$ component of the vector $\mathbf{h}'_k$. In order to obtain a causal discrete-time channel model, we choose $q$ as a multiple of $p$.

The constellation for the symbols $b_k$, the inputs to the discrete-time multichannel, is complex whereas the constellation for the symbols $d_k$ is real. It will be advantageous to express everything in terms of real quantities and in this way double the number of (fictitious) channels. To that end we demodulate the received signal by $j^{-k}$ [2]:

$$j^{-k} \mathbf{y}'_k = \sum_{i=0}^{N-1} j^{-k} \mathbf{h}'_{k-i} + j^{-k} \mathbf{v}'_k = \sum_{i=0}^{N-1} (j^{-k} \mathbf{h}'_{i})_k d_{k-i} + j^{-k} \mathbf{v}'_k$$

(6)

and then we decompose the complex quantities into their real and imaginary parts like

$$\begin{cases}
\mathbf{y}'_k^R &= Re(j^{-k} \mathbf{y}'_k) = \mathbf{H}'^R(q) d_k + \mathbf{v}_k^R \\
\mathbf{y}'_k^I &= Im(j^{-k} \mathbf{y}'_k) = \mathbf{H}'^I(q) d_k + \mathbf{v}_k^I
\end{cases}$$

(7)

where $q^{-1}$ is the delay operator: $q^{-1} \mathbf{y}_k = \mathbf{y}_{k-1}$ and

$$\mathbf{H}'^R(q) = \sum_{i=0}^{N-1} \mathbf{h}'^R_{i} q^{-i} = \sum_{i=0}^{N-1} Re(j^{-i} \mathbf{h}'_i) q^{-i}$$

and similarly for $\mathbf{H}'^I(q)$. We can represent this system more conveniently in the following obvious notation

$$\mathbf{y}_k = \begin{bmatrix} \mathbf{y}'_k^R \\ \mathbf{y}'_k^I \end{bmatrix} = \begin{bmatrix} \mathbf{H}'^R(q) \\ \mathbf{H}'^I(q) \end{bmatrix} d_k + \begin{bmatrix} \mathbf{v}_k^R \\ \mathbf{v}_k^I \end{bmatrix} = \mathbf{H}(q) d_k + \mathbf{v}_k.$$

(8)

3. MATCHED FILTER BOUND

The MFB bounds the probability of error ($P_e$) in the sense that the $P_e$ (e.g. MLSE) is bounded below by the $P_e$ of an AWGN channel with the MFB as SNR. MBFs in the presence of interferers are analyzed in [3]. The MFB expressions below will be derived in the case of burst (packet) transmission. Since in that case the MFB is symbol-dependent [4], we will consider the average MFB over the burst. Assume we receive $M$ samples:

$$\mathbf{Y}(M) = T \mathbf{M}(\mathbf{H}) D_{M+N-1} M^T \mathbf{V}(M) \quad \text{or} \quad \mathbf{Y} = T \mathbf{H} D + \mathbf{V}$$

(9)
where \( Y = [y_M^H \cdots y_1^H]^H \) and similarly for \( V \), and \( T(\mathbf{H}) \) is a block Toeplitz matrix with \( M \) block rows and \([h_0 \cdots h_{M-1}]^T \) as first block row. Let \( R_{VV} = \mathbf{E} \mathbf{V} \mathbf{V}^H \). We shall consider the MFB for MLSE in which the noise covariance matrix is assumed to be \( R_{VV} \) whereas its actual value is \( R_{yy} \). It can be shown that this MFB can be written as

\[
MFB = \frac{1}{M + N - 1} \sum_{j=1}^{M+N-1} MFB_j
\]

where \( T_j \) is the \( j \)th column of \( T_M(\mathbf{H}) \). Using Parseval’s equality, one can show that the equivalent continuous processing MFB can be written as (taking \( \lim_{M \to \infty} \) of the previous expression)

\[
MFB = \frac{\sigma_y^2 \left[ \frac{1}{2\pi j} \int \frac{dz}{z} \mathbf{H}^H(z) \tilde{S}_{VV}^{-1} \mathbf{H}(z) \right]^2}{\frac{1}{2\pi j} \int \frac{dz}{z} \mathbf{H}^H(z) \tilde{S}_{VV}^{-1} \mathbf{H}(z)}
\]

where \( \tilde{S}_{VV}(z) \) is the power spectral density matrix corresponding to \((z\text{-transform of})\) the correlation sequence of \( v_k \) and \( \mathbf{H}^H(z) = \mathbf{H}^H(1/z^*) \).

### 4. INTERFERENCE CANCELLATION PERFORMANCE BOUNDS

In this section we consider \( \tilde{S}_{VV}(z) = S_{VV}(z) \). We define a SNR w.r.t. the additive white noise in the data model (8) as the average SNR per channel:

\[
\text{SNR} = \frac{1}{m} \frac{\sigma_y^2}{\sigma_i^2} \frac{1}{2\pi j} \int \frac{dz}{z} \mathbf{H}^H(z) \mathbf{H}(z).
\]

We shall consider the additive noise to consist of \( d \) interferers plus spatially and temporally white circular noise:

\[
S_{VV}(z) = \sigma_y^2 \mathbf{G}(z) \mathbf{G}^H(z) + \sigma_i^2 \mathbf{I}_{2m}
\]

where \( \mathbf{G}(z) = [\mathbf{G}_1 \cdots \mathbf{G}_d] \) has dimensions \( 2m \times d \) and \( \mathbf{G}_k \) has the same structure as \( \mathbf{H} \). We can define the SIRs

\[
\text{SIR}_k = \frac{\int \frac{dz}{z} \mathbf{H}^H(z) \mathbf{H}(z)}{\int \frac{dz}{z} \mathbf{H}^H(z) \mathbf{G}_k(z) \mathbf{G}_k^H(z)}.
\]

With \( \tilde{S}_{VV}(z) = S_{VV}(z) \) and if all SIRs are equal (SIR), we then get from (11) \( \text{MFB}(\text{SNR}, \text{SIR}) = \frac{\sigma_y^2}{\sigma_i^2} \frac{1}{2\pi j} \int \frac{dz}{z} \mathbf{H}^H(z) \tilde{S}_{VV}^{-1} \mathbf{H}(z) \). If the multiple users would be detected jointly, a bound on the detection performance for the user of interest is obtained by assuming that the interferers are detected perfectly (in which case their signal contribution can be cancelled perfectly). This leads to \( \text{MFB}_{JD} = \text{MFB}(\text{SNR}, \infty) = \text{mSNR} \). Here we consider single-user detection, treating the interferers as colored Gaussian noise. MFB(SNR,SIR) is then the MFB for MLSE in which we take the correlation of the interferers and noise into account properly. By doing so, the multichannel aspect allows for some interference suppression. In order to get a feeling for how much interference suppression is possible, we compare to \( \text{MFB}_{JD} \) and introduce

\[
\text{MFB}_{rel} = \frac{\text{MFB}(\text{SNR}, \text{SIR})}{\text{MFB}(\text{SNR}, \infty)} = \frac{\sigma_y^2}{\sigma_i^2} \frac{1}{2\pi j} \int \frac{dz}{z} \mathbf{H}^H(z) S_{VV}^{-1} \mathbf{H}(z).
\]

It can be shown that in general

\[
\sin^2 \theta = 1 - \frac{\frac{1}{2\pi j} \int \frac{dz}{z} \mathbf{H}^H(z) P_{G(z)} \mathbf{H}(z)}{\frac{1}{2\pi j} \int \frac{dz}{z} \mathbf{H}^H(z) \mathbf{H}(z)} \leq \text{MFB}_{rel} \leq 1
\]

where \( P_{G(z)} = \mathbf{G}(z) \left( \mathbf{G}^H(z) \mathbf{G}(z) \right)^{-1} \mathbf{G}^H(z) \), and \( \theta \) is the angle between the (subspace spanned by the) interferers and the user of interest. The upper bound is attained as \( \text{SIR} \to \infty \) (even if \( \tilde{S}_{VV} \neq S_{VV} \), as long as \( \tilde{S}_{VV} \) also converges to a multiple of the identity matrix) whereas the lower bound is attained as \( \text{SIR} \to 0 \). In this latter case, we can only cancel the part of the interferers which is orthogonal to the user of interest. Nevertheless, this shows that when the user of interest is roughly orthogonal to the strong interferers, something that is more likely to happen when more channels are available, then multichannel MLSE which takes the correlation of the noise and interferers into account leads to limited performance loss compared to joint detection, regardless of the strength of the interferers.

### 5. OPTIMAL SPATIAL FILTERING

Consider now the problem of optimally combining the phases of the received signal by a purely spatial filter. For a real constellation, the classical \( 1 \times K \) filter \( \mathbf{W} \) should be replaced by a \( 1 \times 2K \) widely linear spatial filter \( \mathbf{W} = [\mathbf{w}^H \mathbf{w}^T] \) [5, 6]. One can design \( \mathbf{W} \) by maximizing the signal-to-interference plus noise ratio (SINR) of the resulting scalar output \( w^H y^R + w^T y^I \) where \( p = 1 \). Then the spatial filter is proportional to, the solution of the problem

\[
\max_{\mathbf{W}} \text{SINR} = \frac{\sigma_y^2}{\sigma_i^2} \frac{\mathbf{W} \left[ \frac{1}{2\pi j} \int \frac{dz}{z} \mathbf{H}(z)^H \mathbf{H}(z) \right] \mathbf{W}^H}{\mathbf{W} r_{YY}(0) \mathbf{W}^H} = \frac{\mathbf{W} r_{YY}(0) \mathbf{W}^H}{\mathbf{W} r_{YY}(0) \mathbf{W}^H} - 1,
\]

yielding the generalized eigenvector that is associated to the maximal generalized eigenvalue of the matrices \( r_{YY}(0) \) and \( r_{VV}(0) \). It follows that \( \text{SINR} = \lambda_{\text{max}}(r_{YY}(0) r_{VV}(0)^{-1}) = \sigma_y^2 \lambda_{\text{max}}(r_{YY}(0)^{-1}) \mathbf{H}_N \mathbf{H}_N^H \). In the case \( \text{SIR} \to \infty \), or if the channel of the user of interest is orthogonal to that of the interferers, we have the equality below and the bounds [4]:

\[
1 \leq \frac{\text{MFB}_{JD}}{\text{SINR}} \leq \frac{\mathbf{u}(\mathbf{H}_N \mathbf{H}_N^H)}{\lambda_{\text{max}}(\mathbf{H}_N \mathbf{H}_N^H)} \leq \min(2K, N)
\]

where the MFB_{JD} is the MFB for the case of joint detection (or absence of interferers). The lower bound can be reached when the spatio-temporal channel factors into a spatial filter \( \mathbf{h}_i \) and a scalar temporal filter \( c(z) = \sum_{i=0}^{N-1} c_i z^{-i} \); that is \( \mathbf{H}(z) = \mathbf{h}_i c(z) \). The upper bound is
attained when either $H_N H_N^H \sim I_{2K}$ or $H_N H_N^H \sim I_N$, whichever is of full rank. In that case, the individual channel impulse responses are orthonormal. In a statistical set-up, if the $2K$ channel impulse responses are i.i.d., then the upper bound is approached as the number of sensors grows.

6. SIMULATIONS

We consider three typical GSM propagation environments: Rural Area (RU), Hilly Terrain (HI) and Typical Urban (TU). For each environment, we consider the 6-tap ($L_c = 6$) statistical channel impulse responses as specified in the ETSI standard [7] and this for both the user of interest and the interferers. The channels for multiple antennas are taken independently. We show MFB curves as a function of SIR for a fixed SNR = 40dB. The MFB curves are averaged over 50 realizations of the channel impulse responses of the interferer(s), according to one of the three statistical channel models, with the channel response of the interferers being delayed independently w.r.t. to that of the user of interest by a delay that was taken uniform over $[0,T]$. Apart from the optimal way of taking the multichannel correlation into account, we also consider the performance of suboptimal MLSE receivers that only take ”spatial” correlation into account and neglect the temporal correlation of the interferer or that go as far as treating the interference plus noise as temporally and spatially white circular noise ($\hat{R}_{\mathbf{v} \mathbf{v}}$ is a multiple of identity, the multiple being the average value of the diagonal elements of $R_{\mathbf{v} \mathbf{v}}$). We also consider the MFB corresponding to the optimal Interference Canceling Matched Filter (ICMF [3]) followed by the Viterbi algorithm (equalization) assuming that the noise at the ICMF output is white (which corresponds to an approximation). We also show the SINR curve that corresponds to the performance of a purely spatial receiver. We show in solid line the MFB when $\hat{R}_{\mathbf{v} \mathbf{v}} = R_{\mathbf{v} \mathbf{v}}$ and respectively in dashline and dashdot the case where $\hat{R}_{\mathbf{v} \mathbf{v}}$ is diagonal or block-diagonal. The MFB after an optimal interference cancellation is shown by a triangle, and by a circle when we ignore the color of the residual noise. We consider one interferer and two combinations of multiple antennas and oversampling. We also consider the scenario of multiple interferers for a Typical Urban channel and two antennas.

7. CONCLUSIONS

The simulations show that whenever the number of channels is larger than the number of users, then the suboptimal single-user detector which takes the (spatio-temporal) correlation structure of the interferers correctly into account suffers a bounded MFB loss (as the interferers get arbitrarily strong) w.r.t. to the more complex joint detection scheme. The simulations show that the exploitation of the in-phase and in-quadrature components allows to double the number of channels, with full diversity. Multiple channels obtained by oversampling also lead to bounded loss, be it much larger though. Due to the bandwidth limited nature of GMSK signals, the diversity obtained by oversampling is limited. The simulations show that, in the case of one interferer, one antenna and twofold oversampling, on the average the loss is bounded by 5dB. This interference reduction capacity is due exclusively to the two-channel aspect created by exploiting the real aspect of the symbol constellation after an appropriate reformulation of GMSK. This loss can be reduced to 2dB by using two antennas (with complete spatial diversity). The results also show that not taking the noise correlation into account properly (esp. $\hat{R}_{\mathbf{v} \mathbf{v}} \sim I$, as is done in current GSM receivers) leads to the absence of robustness to interference, while taking only spatial correlation into account ($2K \times 2K$ blocks) leads to very limited robustness. The purely spatial approach performs as well as the optimal approach when the number of sensors is high. This result is expected since the channels of the user of interest and the interferers become almost orthogonal and then the loss in performance is shown to be bounded. It should be noted that the losses shown here are averaged over all possible interferer configurations. The actual loss can be quite a bit less most of the time.

8. REFERENCES


Figure 4: MFB vs SIR for SNR=40dB, one antenna, one interferer and twofold oversampling.

Figure 5: MFB vs SIR for SNR=40dB, two antennas, one interferer and no oversampling.

Figure 6: MFB vs SIR for SNR=40dB, two antennas, multiple interferers and no oversampling.

Figure 7: MFB vs SIR for SNR=40dB, multiple antennas, one interferer and no oversampling.