ABSTRACT
A general method for user dedicated downlink channel estimation in WCDMA receivers is addressed, particularly suited in the presence of dedicated channel transmit beamforming. A three-step dedicated channel estimation procedure is derived which exploits all the existing pilot sequences as well as the structured dynamics of the channel. In the first step, least squares (LS) estimates of the channels associated with dedicated and common pilots are built. In the second step, an improved unbiased minimum mean square error (UMMSE) estimate of the dedicated channel is obtained by optimally combining the initial LS estimates exploiting the correlation between dedicated and common pilot channels. In the last step, the improved dedicated channel estimate is further refined via Kalman filtering by exploiting the channel temporal correlation.

1. INTRODUCTION
The UMTS standard [1] user dedicated downlink physical channel (DPCH) consists of dedicated physical control channel (DPCCH), carrying user dedicated pilots, time multiplexed with the dedicated physical data channel (DPDCH) carrying dedicated pilots. In addition, common pilots are continuously provided over the common pilot channel (CPICH). Most channel estimation techniques proposed for WCDMA receivers are based on either the DPCCH (see e.g. [3, 4] and references therein), or on the CPICH (see e.g [5]). However, on the one hand, the accuracy of channel estimation approaches relying only on the DPCCH is limited by the reduced number of dedicated pilots per slot and by the lack of pilots during the DPDCH period that prevents effective tracking of fast fading channels. On the other hand, classical channel estimation approaches based on the CPICH can better adapt to fast fading conditions, but they are not suited for dedicated channel estimation in the presence of dedicated transmit beamforming. Both approaches remain sub-optimal though, due to the fact that they neglect the shared structure by the common and the dedicated propagation channels. There already exist some works for path-wise dedicated channel estimation which make use of both dedicated and common pilots [6], [7], under the assumption of perfect a priori knowledge of the path delays. Moreover they implicitly assume the channel associated with the DPCH to be identical to the one associated with the CPICH. However, as envisaged in the Release 5 of the UMTS standard, this assumption does not hold in the case when beamforming is employed for DPCH transmission. Indeed user dedicated transmit beamforming affects only the DPCH transmission while the CPICH is evenly broadcasted to all users in the cell. Hence, when dedicated beamforming is present one would be tempted to conclude that CPICH can no longer be used for dedicated channel estimation, while the dedicated pilots can still be exploited yet with all the previously described limitations. Actually in order to exploit the common pilots as well, the knowledge of the transmit beamforming parameters, i.e. the beamforming weight vector, antenna array responses corresponding to the excited angles and their related statistics should be known at the receiver. Furthermore, even in the absence of transmit beamforming, the offset between the transmit powers assigned to the DPCCH and CPICH needs to be estimated in order to properly form a combined estimate of the actual dedicated channel. In general, even in the presence of dedicated beamforming the DPCH and CPICH associated propagation channels are correlated to a certain extent, as it has been shown by field test measurements. A general dedicated channel estimation technique which optimally exploits both common and dedicated pilots based on a generic CPICH-DPCH channel correlation model was introduced for the first time in [2]. In addition to the correlation between dedicated and common channels, there is also the channel temporal correlation governed by the Doppler spread, which can be exploited to improve the channel estimation accuracy. To this end, by fitting the channel dynamics to an autoregressive model of sufficient order, Wiener filtering or Kalman filtering can be applied to refine the previously block-wise obtained estimates. Here we consider causal Kalman filtering which, as it is well known, corresponds to the causal Wiener filtering in the steady state. In this paper, we approach the problem of time-varying dedicated channel estimation by optimally combining all the known sources of information, i.e by exploiting the temporal and cross-correlations of common and dedicated pilots. Furthermore no a priori knowledge of path delays and the beamforming parameters is assumed. The performances are assessed in terms of normalized mean square error (NMSE) of the dedicated channel estimate via both analytical and simulation results. The impact of the channel estimation errors on the RAKE receiver performances are also addressed in terms of output signal-to-interference-plus-noise ratio (SINR).

2. CHANNEL AND SYSTEM MODEL
We assume the time-varying continuous time channels associated with dedicated and common pilots, \( h_d(t, \tau) \) and \( h_c(t, \tau) \) respectively, to obey the wide sense stationary uncorrelated scattering (WSS-US) model [8]

\[
\begin{align*}
\psi(t) &= \sum_{p=0}^{P-1} c_{d,p}(t) \psi(t - \tau_p) \\
\psi(t) &= \sum_{p=0}^{P-1} c_{c,p}(t) \psi(t - \tau_p)
\end{align*}
\]

where \( \psi(t) \) represents the pulse-shape filter, \( P \) denotes the number of significant paths, \( \tau_p \) represents the \( p \)-th path delay, \( c_{d,p}(t) \) and \( c_{c,p}(t) \) are time-varying complex channel coefficients associated with the \( p \)-th path of the dedicated and common channel respectively. In many practical circumstances, the two coefficients \( c_{d,p}(t) \) and \( c_{c,p}(t) \) result to be fairly highly correlated even in the presence of dedicated downlink beamforming. Notice that in (1) the coefficients \( c_{d,p}(t) \) for \( p = 0, \ldots, P-1 \) account also for the complete cascade of the beamforming weight vector, the antenna array response on the excited angles, as well as for the actual propagation channel between the transmitter and the receiver. The receiver is assumed to sample \( M \) times per chip period the low-pass filtered received baseband signal. Stacking the \( M \) samples per chip period in vectors, the discrete time finite impulse response (FIR) representation of both common and dedicated channels at chip rate takes the
form \( h_k = [h_1 \ldots h_M]^T \), which represents the vector of the samples of the overall channel, including the pulse shape, the propagation channel, the antialiasing receiver filter and, when applicable, the beamforming weighting. The superscript \( (\cdot)^T \) denotes the transpose operator. Assuming the overall channel to have a delay spread of \( N \) chip periods, the dedicated and common channel impulse responses take the form \( h(n) = \Psi \phi(n) \) where \( h = [h_1 \ldots h_N]^T \in \mathbb{C}^{N \times 1} \), \( \phi(n) = [c_1(n) \ldots c_p(n)]^T \in \mathbb{C}^{p \times 1} \) are the complex path amplitudes and the temporal index \( n \) relates to the time instant at which the time-varying channel is observed. The assumption of fixed delays \( \tau_p \)'s over the observation window, yields to a constant pulse-shape convolution matrix \( \Psi \in \mathbb{C}^{MN \times p} \) given by
\[
\Psi = \Psi(\tau_1, \ldots, \tau_p) = [\Psi(\tau_1), \ldots, \Psi(\tau_p)]
\]
where \( \Psi(\tau_p) \) represents the sampled version of the pulse shape filter impulse response delayed by \( \tau_p \). The complex path amplitudes variations are modeled as an autoregressive (AR) processes of order sufficiently high to characterize the Doppler spectrum. Matching only the channel bandwidth with the Doppler spread leads to a first-order AR(1) model of the form
\[
e(n) = \rho e(n-1) + \sqrt{1 - |\rho|^2} \Delta e(n) = \sqrt{1 - |\rho|^2} \frac{1}{1 - \rho^q} \Delta e(n)
\]
so that, \( \Psi \) being constant over the observation time interval, we obtain
\[
h(n) = \rho h(n-1) + \sqrt{1 - |\rho|^2} \Delta h(n) = \sqrt{1 - |\rho|^2} \frac{1}{1 - \rho^q} \Delta h(n)
\]
where \( q^{-1} \) denotes the delay operator such that \( q^{-1} y(n) = y(n-1) \) and \( \rho \) represents the AR process temporal coherence coefficient. Since the Doppler spread is assumed to be the same for both channels (1), the model (2) applies to both \( h_c(n) \) and \( h_d(n) \). The variance of \( k \)-th component \( h_c^k(n) \) or \( h_d^k(n) \) is
\[
\sigma_{c,d}^2 = \sigma_{h,c,d}^2 = \Psi_k^H D_k \Psi_k^T \text{ where } \Psi_k \text{ denotes the } k \text{-th line of } \Psi \text{ and } D_k = \text{diag}(\sigma_{c,d}^2, \ldots, \sigma_{c,d}^2).\]
Notice that \( \sigma_{c,d}^2 = \sigma_{h,c,d}^2 \). Similarly the variance of \( k \)-th component \( h_{c,d}^k(n) \) of \( h_{c,d}^n(n) \), is \( \sigma_{c,d}^2 = \sigma_{h,c,d}^2 = \Psi_k^H D_k \Psi_k^T \) where \( D_k = \text{diag}(\sigma_{c,d}^2, \ldots, \sigma_{c,d}^2) \).

3. THREE STEP DEDICATED CHANNEL ESTIMATION PROCEDURE

The proposed approach starts with block-wise dedicated and common channel least squares (LS) estimates \( \hat{h}_c(n) \) and \( \hat{h}_d(n) \) which are computed based on the a priori knowledge of the common and dedicated pilot chips. For the sake of simplicity, without loss of generality, we assume that block-wise corresponds to slot-wise estimates. In the second stage, for each \( k \)-th element of \( \hat{h}_c(n) \) and \( \hat{h}_d(n), \) \( k \in \{0, \ldots, MN - 1 \} \), a refined estimate \( \hat{h}_{c,d}^k(n) \) of \( h_{c,d}^k(n) \) is built by optimally combining the corresponding LS estimates \( \hat{h}_c^k(n) \) and \( \hat{h}_d^k(n) \) so as to obtain an unbiased minimum mean square error (UMMSE) estimate. Finally, successive estimates \( \hat{h}_{c,d}^k(n) \) of \( h_{c,d}^k(n) \) are temporarily Kalman filtered in order to generate an improved estimate \( \hat{h}_{c,d}^k(n) \) by exploiting the temporal correlation due to the finite Doppler spread.

3.1 LS Estimations of Common and Dedicated Channels

We assume that dedicated pilot chips are sent in every slot. Let \( S_d(n) = S_d(n) \otimes I_M \), where \( \otimes \) denotes the Kronecker product, represent the block Hankel matrix comprising the dedicated pilot chip sequence intended for the user of interest in slot \( n \). Similarly we refer to \( S_c(n) = S_c(n) \otimes I_M \) as the block Hankel matrix containing the common pilot chip sequence in slot \( n \). Let \( Y(n) \) be the received signal samples vector corresponding to slot \( n \). The LS unstructured FIR common and dedicated channel estimates FIR are given by
\[
\hat{h}_d(n) = \arg\min_{h_d} \| Y(n) - S_d(n) h_d(n) \|^2 \quad \hat{h}_c(n) = \arg\min_{h_c} \| Y(n) - S_c(n) h_c(n) \|^2
\]

(3)
The exact LS solutions of problems (3) are readily given by
\[
\hat{h}_d(n) = (S_d^H(n) S_d(n))^{-1} S_d^H(n) Y(n) \quad \hat{h}_c(n) = (S_c^H(n) S_c(n))^{-1} S_c^H(n) Y(n)
\]
where \( (\cdot)^H \) denotes Hermitian transpose. Note that the equations (4) reduce to
\[
\hat{h}_d(n) \approx \beta_d^{-1} S_d^H(n) Y(n) \quad \hat{h}_c(n) \approx \beta_c^{-1} S_c^H(n) Y(n)
\]
if the pilot chips can be modeled as i.i.d. random variables, where \( \beta_c \) and \( \beta_d \) represent the dedicated and common pilot chip sequences total energies respectively.

We observe that the LS channel estimation error variances are equal to \( \sigma_{c,d}^2 = \sigma_{c,d}^2 = \sigma_{c,d}^2 = \sigma_{c,d}^2 = \sigma_{h,c,d}^2 \). in channel taps \( k > MN - 1 \), at which \( h_{d,c}(n) \approx 0, h_{c,d}(n) \approx 0 \). Hence \( \sigma_{c,d}^2 \) and \( \sigma_{h,c,d}^2 \) can be estimated from \( \hat{h}_{d,c}(n) \) and \( \hat{h}_{c,d}(n) \) at delays \( k \) where we expect the channel not to carry any energy. That can be achieved by, e.g., overestimating the channel delay spread, and using the tails of the channel estimates to obtain unbiased estimates \( \sigma_{c,d}^2 \) and \( \sigma_{h,c,d}^2 \).

3.2 Unbiased MMSE Combining of LS Estimates

Let \( \hat{h}_d(n) = [\hat{h}_{d,c}(n), \hat{h}_{d,d}(n)]^T \) denote the vector of the LS estimates of the \( k \)-th elements of the dedicated and common pilot channel FIR responses at slot \( n \), i.e.,
\[
\hat{h}_d(n) = \begin{bmatrix} \hat{h}_{d,c}(n) \\ \hat{h}_{d,d}(n) \end{bmatrix} = \begin{bmatrix} \hat{h}_{d,c}(n) \\ \hat{h}_{d,d}(n) \end{bmatrix} + \begin{bmatrix} \hat{\epsilon}_{d,c}(n) \\ \hat{\epsilon}_{d,d}(n) \end{bmatrix}.
\]
In order for our derivation to be fully general, we introduce the following dedicated and common channel correlation model
\[
h_{c,d}(n) = \alpha_k h_{c,d}(n) + x_{c,d}(n)
\]
where \( \alpha_k h_{c,d}(n) \) represents the short-term UMMSE estimate of \( h_{c,d}(n) \) on the basis of \( h_{c,d}(n) \), and \( x_{c,d}(n) \) represents the associated estimation error. Then, a refined estimate can be obtained as \( \hat{h}_{c,d}(n) = f_1(\hat{h}_d(n)) \) by optimal combining of common and dedicated LS channel estimates. In order not to introduce bias for the processing in the next estimation step, we shall determine \( f_1 \) as the UMMSE filter, i.e., by solving for all \( k \)’s the optimization problem
\[
\min \mathbb{E}[h_{c,d}(n) - f_1(\hat{h}_d(n))^2] \quad \text{s.t. } f_1(1) = 1
\]
The optimal UMMSE filter \( f_1 \) is obtained as
\[
f_1_{\text{UMMSE}} = (\alpha_k^2 |R_{h,c,k}|^{-1} [1 |\alpha_k|^2])^{-1} [\alpha_k^2 |R_{h,c,k}|^{-1} R_{h,c,k}] \]
where \( R_{h,c,k} = \mathbb{E}[h_{c,d}(n) h_{c,d}^H(n), R = \text{diag}(\sigma_{c,d}^2, \sigma_{c,d}^2, \sigma_{c,d}^2, \sigma_{c,d}^2)] \), with \( \sigma_{c,d}^2 = \mathbb{E}[h_{c,d}(n)]^2 \). Notice that the covariance matrix \( R_{h,c,k} \) is equal to
\[
R_{h,c,k} = \begin{bmatrix} \tau_{11} & \tau_{12} \\ \tau_{12} & \tau_{22} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} \sigma_{c,d}^2 \\ 0 \end{bmatrix} \begin{bmatrix} \alpha_k \\ \alpha_k \end{bmatrix} + \begin{bmatrix} \sigma_{c,d}^2 + \sigma_{c,d}^2 \end{bmatrix}
\]
Having an estimate of the matrix $R_{h,k}^c$, e.g. by temporal averaging, we can apply the covariance matching criterion so that $\sigma^2_{d,k} = \frac{\sigma^2_{\hat{d},k} \alpha_h^2}{\alpha_h^2 + \sigma^2_{\hat{d},k}}$, (i.e. $\alpha_h$ has the same phase as $\sigma_h^2$), where the following bound $|\alpha_h| \leq \sigma_{h,k}/\sigma_h = \sqrt{(r_{22} - \sigma^2_{\hat{d},k})/(r_{11} - \sigma^2_{\hat{d},k})}$ can be used in actual estimation. Furthermore, since $\sigma^2_{\hat{d},k} = r_{22} - \sigma^2_{\hat{d},k} - r_{21}^2/(r_{11} - \sigma^2_{\hat{d},k})$. A similar covariance matching criterion can be applied to estimate the temporal correlation coefficient $\rho$.

Finally, the variance of the estimation error $\hat{\hat{d}}_{d,k}(n)$ after UMMSE combining, is given as

$$\sigma^2_{\hat{\hat{d}},k} = \frac{\sigma^2_{\hat{d},k} (\sigma^2_{\hat{d},k} + \sigma^2_{\hat{d},k})}{\sigma^2_{\hat{d},k} |\alpha_h|^2 + \sigma^2_{\hat{d},k} + \sigma^2_{\hat{d},k}}$$

(7)

The dedicated channel estimate after UMMSE combining, $\hat{h}_{d,k}(n) = h_{d,k}(n) + \hat{\hat{d}}_{d,k}(n)$, is such that the post-combining estimation error $\hat{e}_{d,k}(n)$ is mutually uncorrelated with $h_{d,k}(n)$, $\hat{e}_{d,k}(n)$ and $\hat{e}_{d,k}(n)$ are mutually uncorrelated for any $k \neq j$, and the variance of $\hat{e}_{d,k}(n)$ is independent of $k$ while it depends on the Doppler spread, on the channel power, and on the SINR.

3.3 Kalman Filtering of UMMSE Combined Estimates

Once the UMMSE dedicated channel estimates are obtained, we apply optimal Kalman causal filtering to exploit the channel temporal correlation. Since we adopted the channel statistic model (2), the optimal causal filter is the well-known first order scalar Kalman filter consisting of a prediction and a correction step. For $n > 0$ the prediction step yields to

$$\hat{h}_{d,k}(n | n-1) = \hat{h}_{d,k}(n-1 | n-1) + \hat{\hat{d}}_{d,k}(n-1 | n-1)$$

(8)

The associated prediction MSE is given by

$$\sigma^2_{\hat{h},k}(n | n-1) = |\rho|^2 \sigma^2_{\hat{d},k}(n-1 | n-1) + (1 - |\rho|^2) \sigma^2_{\hat{h},k}$$

Then the Kalman gain for the correction step is given by

$$g(n) = \frac{\sigma^2_{\hat{d},k}(n | n-1)}{\sigma^2_{\hat{h},k}(n-1 | n-1) + \sigma^2_{\hat{d},k}}$$

so that the correction step equation is readily found as

$$\hat{h}_{d,k}(n) = \hat{h}_{d,k}(n-1) + g(n)(\hat{\hat{d}}_{d,k}(n) - \hat{h}_{d,k}(n-1))$$

(9)

and the associated MSE is given by

$$\sigma^2_{\hat{h},k}(n | n) = (1 - g(n)) \sigma^2_{\hat{d},k}(n | n-1)$$

Finally, the steady state MSE is given by the Riccati equation

$$\sigma^2_{\hat{h},k}(\infty) = \frac{\sigma^2_{\hat{d},k} [|\rho|^2 \sigma^2_{\hat{d},k} (\infty) + (1 - |\rho|^2) \sigma^2_{\hat{h},k}]}{\sigma^2_{\hat{d},k} |\rho|^2 \sigma^2_{\hat{d},k} (\infty) + (1 - |\rho|^2) \sigma^2_{\hat{h},k} + \sigma^2_{\hat{d},k}}$$

(10)

and the steady state overall channel MSE is given by

$$\text{NMSE} = \frac{\sum_{k=0}^{MN-1} \sigma^2_{\hat{h},k}(\infty)}{\sum_{k=0}^{MN-1} \sigma^2_{\hat{h},k}}$$

(11)

4. Simulations and Conclusions

The performances of the presented channel estimation methods in the presence of dedicated transmit beamforming are presented in figures 1 to 6 in terms of the channel estimate NMSE and SINR at the RAKE receiver output. We assume the DPCCH to occupy 20% of the UMTS slot, and the DPCH spreading factor to be equal to 128. We define the normalized correlation factor $\rho_k = |\alpha_h| \sigma_{h,k}/\sigma_h \leq 1$. Being interested in the impact of dedicated and common channel correlation we set, for the sake of simplicity, $|\alpha_h| = |\alpha_0|$ constant $\forall k$. We initially assume the DPCCH and the CPICH, to be respectively assigned to 5 and 10 % of the whole base station transmitted power. We also assume an additional DPCH beamforming gain of 6dB, yielding to a power offset between DPCCH and CPICH equal to $\sigma^2_{\hat{h},k} / \sigma^2_{h,k} = 0.5$ for all $k$’s, so that $r = r_k = \sqrt{2}|\alpha_0|$. Channels are randomly generated from the power delay profile of the UMTS Pedestrian A channel [1] with a Doppler effect such that $|\rho| = 0.99$ between consecutive UMTS slots. We also assume that we a priori know the quantities $|\alpha_0|$, $\sigma^2_{\hat{h},k}$, $\sigma^2_{h,k}$, $\sigma^2_{\hat{d},k}$, and all the needed error variances. Methods for all the unknown parameters estimation and the impact of the required parameters estimation errors are not addressed here because of lack of space. By inspecting the plotted results, we conclude that, as expected, Kalman filtering of combined UMMSE LS estimates (denoted as “Kalman over joint UMMSE” in the figures) outperforms all the other methods, namely mere LS estimation, Kalman filtering of dedicated LS estimates, Kalman of common channel LS estimates, and simple UMMSE combining of LS estimates (denoted as “LS dedicated” and “LS common”, “Kalman over dedicated LS”, “Kalman over common LS”, “Joint UMMSE” respectively) in all circumstances, and approaches the perfect channel state information (CSI) performance. Moreover the steady-state analysis results (denoted as “Kalman over joint UMMSE (ideal)” in the figures) perfectly match the simulation results. Finally one may observe that when the correlation coefficient $r$ decreases (e.g. $r = 0.75$ as in figure 3) the contribution from the common channel estimation quickly becomes negligible compared to the benefit still provided by Kalman filtering alone over dedicated LS channel estimates. In this case the UMMSE combining step can be skipped to reduce complexity.

REFERENCES
