A Game Theoretical Approach to Evaluate Cooperation Enforcement Mechanisms in Mobile Ad hoc Networks

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Abstract. *This paper focuses on the formal assessment of the properties of cooperation enforcement mechanisms used to detect and prevent selfish behavior of nodes forming a mobile ad hoc network. Taking as a reference the CORE mechanism introduced in [9], we present two alternative approaches based on game theory that provide a powerful analytical method to study cooperation between self-interested players. We demonstrate that the formation of large coalitions of cooperating nodes is possible only when a mechanism like CORE is implemented in each node. Game theory also provides further insight to features of CORE such as the convergence speed to a cooperative behavior*.

1. Introduction

An *ad hoc* network is a collection of wireless mobile hosts forming a temporary network without the support from any established infrastructure or centralized administration. In such an environment, it may be necessary for one mobile host to enlist the aid of other hosts in forwarding a packet to its destination, due to the limited range of each mobile host's wireless transmissions. Indeed, as opposed to networks using dedicated nodes to support basic networking functions like packet forwarding and routing, in ad hoc networks these functions are carried out by all available nodes in the network.

However, there is no good reason to assume that the nodes in the network will eventually cooperate, since network operation consumes energy, a particularly scarce resource in a battery powered environment like MANET. The lack of cooperation between the nodes of a network is a new problem that is specific to the ad hoc environment and goes under the name of *node selfishness*. A selfish node does not directly intend to damage other nodes by causing network partitioning or by disrupting routing information (mainly because performing these kinds of attacks can be very expensive in terms of energy consumption) but it simply does not cooperate to the basic network functioning, saving battery life for its own communications. Damages provoked by a selfish behavior can not be underestimated: a simulation study available in the literature [8] shows the impact of a selfish behavior in terms of global network throughput and global communication delay when the DSR [7] protocol is used. The simulation results show that even a little percentage of selfish nodes leads to a severe degradation of the network performances.

Several mechanisms that detect and prevent a selfish behavior are available in the literature [10, 11, 12, 13, 14]: we take as a reference the CORE [9] mechanism. In CORE, node cooperation is stimulated by a collaborative monitoring technique and a reputation mechanism. Each node of the network monitors the behavior of its neighbors with respect to a requested function and collects observations about the execution of that function. If the observed result and the expected result coincide, the observation takes on a positive value, otherwise it takes on a negative value. Based on the collected observations, each node computes a reputation value for every neighbor. The formula used to evaluate the reputation value avoids false detections (caused for example by link outage) by using an aging factor that gives higher weight to past observations: frequent variations on a node behavior are filtered out. Furthermore, an indirect reputation value can be granted to those nodes that are not within the radio range of the monitoring node and whose contribution to the network operation can be verified based on an acknowledgement mechanism such as the Route Reply message of the DSR protocol. Only positive ratings are assigned as part of the indirect reputation mechanism. The CORE mechanism resists to attacks performed using the security mechanism itself: no negative ratings are spread between the nodes so that it is impossible for a node to maliciously decrease another node's reputation. The reputation mechanism allows the nodes of the MANET to

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gradually isolate selfish nodes: when the reputation assigned to a neighboring node decreases below a pre-defined threshold, service provision to the misbehaving node will be interrupted. Misbehaving nodes can, however, be re-integrated in the network if they increase their reputation by participating in the network operation.

We suggest an original approach based on an economic model in order to formally assess the security features of a cooperation enforcement mechanism such as CORE. In this model, service provision (e.g. the execution of the packet forwarding function) preferences for each node are represented by a utility function. The utility function quantifies the level of satisfaction a node gets from using the network resources. Game-theoretic methods are applied to study cooperation under this new model. Game theory is a powerful tool for modeling interactions between self-interested users and predicting their choice of strategy. Each player in the game maximizes some function of utility in a distributed fashion. The games settle to a Nash equilibrium if one exists, but, since nodes act selfishly, the equilibrium point is not necessarily the best operating point from a social point of view.

In this paper we propose two methods to evaluate the effectiveness of the CORE mechanism based on a cooperative game approach (presented in section 2, also adopted in [5]) and a non-cooperative game approach (presented in section 3).

2. Cooperative games approach

In an attempt to explain cooperation and coalition formation, most theoretical models use a two-period structure as introduced in [5]. Players must first decide whether or not to join a coalition. In a second step, both the coalition and the remaining agents choose their behavior non-cooperatively. A coalition is stable if no agent has an incentive to leave[2]. Simulations presented in [23, 26, 27] have shown that, although there is cooperation, the coalition size is rather small.

In this paper we suggest an approach based on a preference structure as defined by the ERC-theory [4]. This theory explains most of the behavior of agents observed in diverse experiments but deviates little from the traditional utility concept. The utility of an agent is not solely based on the absolute payoff but also on the relative payoff compared to the overall payoff to all agents. Given a certain relative payoff share, the utility is strictly increasing in the own absolute payoff of the agent. Given a fixed absolute payoff, the agent is best off when receiving just the equal (fair) share. To both sides of this equal share, i.e. when receiving less or more than the fair amount, utility is lower, even if the absolute payoff does not change[3].

We first study a symmetric N-node prisoner's dilemma (PD) game in which the agents have only two options available — cooperate or defect. We analyze Nash-equilibrium when agents' preferences can be described by ERC, i.e. players value both their absolute and their relative payoff. In particular, we look at the number of agents who play cooperatively. We show that non-cooperation is always an equilibrium, since — if no other node cooperates — a node would maximize its absolute payoff and receive the equal share by choosing to defect. Additionally, however, there may be Nash-equilibrium in which nodes cooperate: if, for example, the rest of the agents play cooperatively, a player can get the equal share by choosing to cooperate as well. Hence, if it values its relative payoff being close to the equal share more than its absolute payoff, it will choose to complete the grand coalition. Clearly, partial cooperation can also occur, whereby some nodes cooperate while others defect. For such equilibrium, we show that the number of cooperating nodes is rather large: since cooperation leads to a lower absolute payoff, for a node to choose to cooperate, playing cooperatively must move it closer to the equal share than defecting would. As we show, this can only be the case if at least half of the nodes cooperate. This result contrasts with the standard result presented in [23] which states that the coalition size is rather small.

Note, however, that in the prisoner's dilemma, the nodes have only the discrete choice of cooperating or defecting, but with respect to the cooperation enforcement problem, the nodes of an ad hoc network might choose their cooperation level[4] continuously. We therefore introduce a symmetric continuous cooperation game based on the ERC preference structure. An interesting finding of this analysis is that ERC alone cannot improve upon the non-cooperative Nash-equilibrium with standard preferences in which only the absolute payoff matters to a node.

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[2] The definition of stability also implies that no agent wants to join the coalition.

[3] Note that such a preference for equity is self-centered only and is distinct from altruism.

[4] The definition of cooperation level will be given in section [2.6]: here it is sufficient to know that cooperation level stands for the fraction of packets (data or routing) that are forwarded by a node of the network playing the cooperation game.

A further refinement of the cooperative approach consists of a combination between the ERC preference structure and the two-stage coalition formation method. In contrast to the traditional models from the game theory literature, the ERC preference structure allows coalitions to involve a rather large fraction of players. Furthermore, this model allows for a precise characterization of conditions under which even a grand coalition can be obtained.

2.1. The preference structure

Our analysis relies on a preference structure in which players, along with their own absolute payoff, are motivated (non-monotonously) by the relative payoff share they receive, i.e. how their standing compares to that of others. We use the ERC model presented in [4] and enhance it with a complete information framework. Let the (non-negative) payoff to node *i* be denoted by y_i , *i*, . . . , *N*, and the relative share by $\sigma_i = \frac{y_i}{\sum_j y_j}$.

The utility function is given by: $\alpha_i u(y_i) + \beta_i r(\sigma_i)$ where $\alpha_i, \beta_i \geq 0$ and *u()* is differentiable, strictly increasing and concave, and r*()* is differentiable, concave and has its maximum in $\sigma_i = \frac{1}{N}$. Throughout this paper we assume that nodes' disutility from disadvantageous inequality is larger if the node is better off than average, i.e. $r(\frac{1}{N} - x) \leq r(\frac{1}{N} + x), \forall x \in \left[0, \frac{1}{N}\right]$. The types of nodes are characterized by the relative weights α_i, β_i .

2.2. The prisoner's dilemma

In this section we study a simple symmetric *N*-node prisoner's dilemma where each mobile node can cooperate, 'c', or defect, 'd'. In terms of the node misbehavior problem, this means that the node either correctly executes the network functions or it doesn't.

Let the total number of cooperating nodes be denoted by *k*. For any given *k*, the payoff to a node is given by *B*(*k*) if the node defects (tries to free-ride). If a node plays cooperatively, it must bear some additional costs *C*(*k*). Its payoff is therefore given by *B*(*k*) − *C*(*k*). We assume decreasing marginal benefits for a node if the number of mobile nodes rises, i.e. *B*(*k*) is increasing and concave. Furthermore, the total cost of cooperation, *kC*(*k*), increases in *k*.

In order to generate the standard incentive structure of a PD game, we make the following assumption.

Assumption 1. PD structure: $B(k+1) - B(k) < C(k+1)$

Assumption 1 implies that playing cooperatively reduces the absolute payoff, given an arbitrary number of 'c'-nodes. To make cooperation more attractive from both the social and the individual point of view, we make the following assumptions:

Assumption 2. "Socially desirable": $N \cdot B(k+1) - (k+1)C(k+1) \geq N \cdot B(k) - kC(k)$ (1)

Assumption 3. "Individually desirable": $B(k+1) - C(k+1) \geq B(k) - C(k)$ (2)

Furthermore, we assume that payoffs for both cooperating and defecting nodes are non-negative for all *k*.

The reputation measure introduced in [9] is compliant with the incentive structure given by (1) and (2). Cooperation is made attractive from an individual point of view because the cost of participating to the network operation is compensated with a higher reputation value, which is the pre-requisite for a node to establish a communication with other nodes in the network. On the other hand, when the number of cooperating nodes increases, the cost for participation is compensated by a more connected network that in turn increases the benefit of cooperation.

Section 2.6 provides a detailed description of the interactions between the reputation mechanism implemented in CORE and the cooperative game model presented throughout this paper.

2.3. The Nash equilibrium

In the following section we analyze the Nash equilibria in the one shot PD game under the assumption that all the nodes joining an existing network choose simultaneously. Assume that *k* nodes, aside from node *i*, play cooperatively. Then node *i* chooses to play 'c' if and only if:

$$\alpha_i u[B(k+1) - C(k+1)] + \beta_i r\left[\frac{B(k+1) - C(k+1)}{N \cdot B(k+1) - (k+1)C(k+1)}\right] \geq \alpha_i u[B(k)] + \beta_i r\left[\frac{B(k)}{N \cdot B(k) - kC(k)}\right] \qquad (3)$$

This is equivalent to node *i* playing 'c' if and only if:

$$\frac{\alpha_i}{\beta_i} \leq \delta(k) \text{ where } \delta(k) = \frac{r\left[\frac{B(k+1) - C(k+1)}{N \cdot B(k+1) - (k+1)C(k+1)}\right] - r\left[\frac{B(k)}{N \cdot B(k) - kC(k)}\right]}{u[B(k)] - u[B(k+1) - C(k+1)]} \qquad (4)$$

In order to choose 'c' the node must be overcompensated for the loss in absolute gain by moving closer to the average gain.

The general conditions for a Nash equilibrium of a ERC-PD game [4] of *N* nodes whereby the number of cooperating nodes is k* can be used to study expression (4):

$$\frac{\alpha_i}{\beta_i} \leq \delta(k^*-1) \quad \text{for k* nodes (playing 'c')} \qquad (5)$$

$$\frac{\alpha_i}{\beta_i} \geq \delta(k^*) \quad \text{for the remaining N-k* nodes (playing 'd')} \qquad (6)$$

Conditions (5) and (6) can be used to evaluate the number of nodes *k** that may possibly cooperate in a Nash equilibrium. On one hand, as long as $\delta(k^*-1) < 0$, there is no chance of having a coalition of size *k** because $\frac{\alpha_i}{\beta_i} > \delta(k^*-1)$ for all types and condition (5) cannot hold for any node. On the other hand, the conditions for a Nash equilibrium given by (5) and (6) imply that if $\delta(k^*-1) > 0$ then there are types $\left[\left(\frac{\alpha_i}{\beta_i}\right)_{i=1,\ldots,N}\right]$ of nodes such that *k** nodes cooperate and N-*k** nodes free-ride. Note that for a given distribution of ERC-types, $\delta(k^*-1) > 0$ is a necessary condition but it is not sufficient to get a coalition size of *k**. For a given payoff structure with $\delta(k^*-1) > 0$, however, there exist ERC-types such that *k** is the equilibrium for any coalition size.

In order to find feasible coalition sizes, we must therefore study conditions under which $\delta(k)$ is positive.

Note that in (4) the denominator of $\delta(k)$ is positive due to assumption 1. The sign of the numerator, however, depends on the number *k* of cooperating nodes.

For *k*=0 the sign of the numerator is negative, since $r\left(\frac{B(1) - C(1)}{NB(1) - C(1)}\right) = r\left(1 - \frac{(N-1)B(1)}{NB(1) - C(1)}\right) < r\left(\frac{B(0)}{NB(0)}\right) = r\left(\frac{1}{N}\right)$

For *k*=N-1 the sign of the numerator is positive, since

$$r\left(\frac{B(N) - C(N)}{NB(N) - NC(N)}\right) = r\left(\frac{1}{N}\right) > r\left(\frac{B(N-1)}{NB(N-1) - (N-1)C(N-1)}\right) = r\left(1 - \frac{(N-1)B(N-1) + (N-1)C(N-1)}{NB(N-1) - (N-1)C(N-1)}\right)$$

Therefore, $\delta(0) < 0 < \delta(N-1)$ and no nodes unilaterally cooperate whereas all nodes playing 'c' can establish an equilibrium, provided that all nodes' types $\left(\frac{\alpha_i}{\beta_i}\right)$ are smaller than $\delta(N-1)$.

In general, there are equilibria where only a certain number k^* of nodes cooperate. The crucial point is to find whether or not the numerator is positive. Remember that we previously assumed that $r(\frac{1}{N}-x)\le r(\frac{1}{N}+x), \forall x\in\left[0,\frac{1}{N}\right]$.

It is necessary, in order to obtain $\delta(k)>0$, that a node choosing 'd' further deviates from the equal share (1/N) than by playing 'c', i.e.:

$$\frac{1}{N}-\frac{B(k+1)-C(k+1)}{NB(k+1)-(k+1)C(k+1)}>\frac{1}{N}-\frac{B(k)}{NB(k)-kC(k)} \tag{7}$$

It is possible to show that inequality (7) is satisfied for $k>N/2$ [5].

Assumption (1) and (2) imply that the condition $\delta(k)>0$ is necessary (but not sufficient) to state that, for any given vector of types, if a node plays 'c' at the equilibrium, then at least half of the nodes cooperate.

Proposition 1. *For any given payoff structure of the PD game with ERC preferences, there is always an equilibrium in which all nodes defect.*

Proposition 2. *Given Assumption 1 and Assumption 2, there is a Nash equilibrium where at least N/2 nodes cooperate.*

Based on proposition 2, if there is a coalition of cooperating nodes then it is rather large.

2.4. The cooperation game

In section 2.3, we assumed that nodes only have only a discrete option as to whether to cooperate or not. Now, we turn to cooperation games where nodes can continuously choose their cooperation levels. ERC alone cannot improve upon the non-cooperative Nash-equilibrium with standard preferences whereby only the absolute payoff matters. However, introducing more structure to the game, i.e. if nodes play a coalition game, ERC may yield a rather large coalition size or even support the grand coalition.

Let the number of nodes again be denoted by N. We define the cooperation level q_i ($\in[0,1]$) as the fraction of packets (both data and routing packets) that node i forwards to its neighboring nodes or to the destination node. Each node must choose its cooperation level q_i (i = 1,…, N). Cooperation induces some costs $C(q_i)$ that are assumed to be increasing and convex in the cooperation level ($C'(\ )>0, C''(\ )>0$). Cooperation also yields some benefit $B(Q)$ in terms of network connectivity and aggregate cooperation effort made available by cooperating nodes, where $Q=\sum_i q_i$ denotes the aggregate cooperation level.

Benefits from cooperation are increasing and concave, $B'(\ )\ge 0, B''(\ )<0$. The payoff to a node is therefore determined by: $B(Q)-C(q_i)$.

2.4.1. Nash equilibrium in the one shot cooperation game

We again analyze the Nash equilibrium when nodes act simultaneously. Node i chooses q_i to maximize its utility function $\alpha_i u(y_i)+\beta_i r(\sigma_i)$, where:

$$y_i=B\left(\sum_{j\ne i} q_j+q_i\right)-C(q_i)$$

$$\sigma_i=\frac{y_i}{\sum_j y_j}$$

By choosing q_i, each node determines its own cooperation costs and the benefits from cooperation. The choice of q_i also impacts the payoff of the remaining nodes that in turn is fed back to the node's own utility through the relative payoff. The first order condition[6] is therefore given by:

[5] The proof of this affirmation is given in Appendix 1.

$$\left[\alpha_i u'(\ ) + \beta_i r'(\ ) \frac{\sum_{j \neq i} y_j}{\left(\sum_j y_j\right)^2}\right] [B'(Q) - C'(q_i)] - \beta_i r'(\ ) \frac{y_i}{\left(\sum_j y_j\right)^2} (N-1) B'(Q) = 0$$

The first order condition can rewritten as:

$$\alpha_i u'(\ )[B'(Q) - C'(q_i)] + \beta_i r'(\ ) \left[\frac{\sum_j y_j - y_i}{\left(\sum_j y_j\right)^2} B'(Q) - \frac{\sum_{i \neq j} y_j}{\left(\sum_j y_j\right)^2} C'(q_i)\right] = 0 \qquad (8)$$

or

$$\alpha_i u'(\ )[B'(Q) - C'(q_i)] + \beta_i r'(\ ) \left[\frac{1 - N\sigma_i}{\sum_j y_j} B'(Q) - \frac{\sum_{j \neq i} \sigma_j}{\sum_j y_j} C'(q_i)\right] = 0 \qquad (9)$$

The reaction of node i to a given cooperation strategy for the rest of the network can be calculated from this first order condition. Let us first study the two extreme cases, α_i=0 and β_i=0, respectively.

- For β_i=0, i.e. an absolute payoff maximizer, the first order condition reduces to: $B'(Q) - C'(q_i) = 0$.
- For α_i=0, the node is solely interested in getting the equal payoff share. Thus, it would choose qi to satisfy: $NC(q_i) = \sum_j C(q_j)$.
- For α_i, β_i≠0 the chosen cooperation level is between the levels for those extreme cases.

In the Nash equilibrium, the first order condition must be satisfied for all nodes simultaneously. Since $r'(1/N) = 0$, it follows that there is a *symmetric equilibrium* where all nodes choose the same cooperation level, i.e. σ_i = 1/N for all types α_i/β_i, for i = 1, . . .,N. The resulting cooperation level q• is given by the condition: $B'(Nq^*) - C'(q^*) = 0$.

This situation is equivalent to a Nash equilibrium whereby agents are only interested in their absolute payoff (β_i=0) such as the Nash equilibrium in the PD game where all nodes defect. This is the unique equilibrium, assuming that, for at least one node, α_i is greater than 0.

We can summarize this result in the following proposition:

Proposition 3. *(Cooperation game) In the cooperation game for ERC preferences, the equilibrium is given by* $B'(Nq^*) - C'(q^*) = 0$. *It is unique as long as at least one node draws utility from its absolute payoff (αi>0).*

Sketch of proof. Let us prove by contradiction (*reductio ad absurdum*) that there is an *asymmetric equilibrium*, i.e. some nodes receive less, and others more than the equal share. In this case, on the one hand, σ_i < 1/N implies that $r'(1/N) > 0$, so from equation (8), we obtain $B'(Q) - C'(q_i) > 0$ (8a). On the other hand, for σ_i > 1/N we have $r'(1/N) < 0$, and therefore equation (9) implies $B'(Q) - C'(q_i) < 0$ (9a). Inequalities (8a) and (9a) imply that a node which gets more than the equal share has larger marginal cooperation costs ($C'(q_i)$) than nodes that receive less, which contradicts the assumed payoff distribution.

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[6] The first order condition corresponds to the identification of the singularity points of the utility function, i.e. finding the roots of the first order derivative of the utility function.

Hence, only *symmetric equilibrium* exists. If $\alpha_i>0$ for at least one node, we get $B'(Nq)-C'(q)=0$ from equation (8).

Introducing ERC preferences, therefore, does not increase the cooperation effort chosen by the nodes. It does not even change the equilibrium cooperation levels. In contrast to the (discrete) prisoner's dilemma, ERC does not add any equilibrium in which there is more cooperation effort. The existence of equilibrium in the PD game that mimics cooperative behavior, therefore, only arises in the presence of discrete action sets. Having a continuous decision variable, ERC does not change the set of equilibrium. The reason is that ERC does not establish a preference for being cooperative, but for being similar to other nodes with respect to the payoff.

2.5. Coalition formation in the cooperation game

We now turn to the two-stage game as introduced in [5]. Let us again assume that all nodes are identical with respect to their payoff function (i.e. they use the same definition of utility function). In a first stage, nodes decide whether or not to join the coalition. By the principle of "rationality", each node is assumed to know the decisions of the other nodes. The cooperation levels (i.e. the strategy) that will be chosen in the second stage depend on whether the nodes take part in the coalition or not. The coalition thereby maximizes its collective benefits and plays against the nodes that don't take part in the coalition, which simultaneously maximize their individual utility.

We first study the case of nodes that have identical ERC-types. We demonstrate that within the coalition formation game, ERC-preferences can enforce cooperation and even result in the grand coalition. We then look at the case of heterogeneous ERC-types. By studying the extreme scenario of nodes that are solely interested either in their absolute payoff or in equity, we will explore the effects of the existence of some equity-oriented nodes in the network.

2.5.1. Cooperation of identical ERC-types

We will now solve the coalition formation game backwards, that is, for any coalition size *k*, we first study the first order conditions for the choice of the cooperation level inside and outside the coalition. Then, in the second step, the equilibrium coalition size is determined by a stability condition. This means that in the equilibrium, *k* must satisfy the condition that there is no incentive to leave the coalition[7].

For standard preferences (using ERC-preferences this results in the special case $\beta=0$), the game theory literature shows that the coalition size is rather small. Using ERC preferences, however, the number of nodes within a coalition can be much higher in equilibrium.

Instead of solving the game in general, we will show that if nodes only value the relative payoff high enough, i.e. α/β is below a certain bound then even the grand coalition can be stable.

The first order condition for nodes outside the coalition (*S*) is given by (10), whereas the cooperation strategy of nodes that take part in the coalition is chosen by maximizing the utility function of a representative member: indeed all nodes within the coalition *S* select the same strategy q_s since they are assumed to be of the same type. This implies that all members of the coalition have identical absolute payoff ($y_S = B(Q)-C(q_S)$) and relative payoff ($\sigma_S = \dfrac{y_S}{ky_S + \sum_{j\notin S} y_j}$).

The first order condition is given by:

$$[\alpha u'(\ )+\beta r'(\ )]\frac{\sum_{j\notin S}\sigma_j}{\sum_j y_j}[kB'(Q)-C'(q_S)]-\beta r'(\ )\frac{\sigma_S}{\sum_j y_j}(N-k)kB'(Q)=0 \qquad (10)$$

[7] The original work introduced in [23] states that the stability condition is such that there is an incentive to neither leave nor join the coalition.

$$\alpha u'(\ )[kB'(Q)-C'(q_S)]+\beta r'(\ )\left[-\frac{\sum_{j\notin S}\sigma_j}{\sum_j y_j}C'(q_S)+kB'(Q)\frac{1-N\sigma_S}{\sum_j y_j}\right]=0 \qquad (11)$$

- For nodes that do not belong to the coalition S we know from section 2.4 that if $\sigma_j < (>) 1/N$ for $j \notin S$ then $B'(Q) > (<) C'(q_j)$.
- Analogously, for the coalition, we obtain from (10) and (11) that if $\sigma_S < (>) 1/N$ then $kB'(Q) > (<) C'(q_S)$[8]. Since $B'(Q) > kB'(Q)$[9], the first order conditions imply that for nodes within the coalition $\sigma_S \le 1/N$ and thus: $kB'(Q) \ge C'(q_S)$. To prove that inside the coalition $\sigma_S \le 1/N$, assume to the contrary that $\sigma_S > 1/N$ and that $\sigma_j < 1/N$ for some nodes j outside the coalition. Inequalities (10) and (11) imply that $C'(q_j) < B'(Q) < kB'(Q) < C'(q_S)$ which contradicts the assumption of increasing and convex cooperation costs.

Inequalities (10) and (11) can be used to show the following proposition:

Proposition 4. *(Coalition game) In the symmetric coalition game for identical ERC preferences (type α/β), the grand coalition is stable if α/β is sufficiently small, i.e. nodes are interested enough in being close to the equal share.*

Note first, that within the grand coalition, the cooperation level satisfies the condition $NB'(Nq^*) = C'(q^*)$, independently of the ERC-types and nodes that receive the equal share.
If node i leaves the coalition ($k = N-1$), then from the first order conditions we obtain:

$$(N-1)B'[(N-1)q_S + q_i] \ge C'(q_S) \ge C'(q_i) > B'[(N-1)q_S + q_i] \qquad (12)$$

Let us now look at the cooperation levels that would result if the ERC-type α/β goes to zero. In this case, nodes get more and more interested in getting their equal share, and their cooperation levels will converge: in the limit $\tilde{q} = q_S = q_i$. However, in the limit, inequality (12) still must hold, i.e. $(N-1)B'(Nq) \ge C'(q)$.
In the limit the absolute payoff of a node leaving the coalition is smaller than within the grand coalition, whereas the relative payoff is the same, i.e. $NB'(\tilde{Q}) > C'(\tilde{q})$.
Therefore, as long as α/β is small enough, the absolute payoff remains lower and the utility derived from the relative payoff is also smaller than in the grand coalition. Thus, no node has an incentive to leave the grand coalition if α/β is small enough.

2.5.2. Coalition of heterogeneous ERC-types

When nodes with heterogeneous ERC-types are allowed to take part in the coalition (S), those nodes that have the largest α_i/β_i will have the greatest interest to leave the coalition in order to obtain a larger absolute payoff.
We will now concentrate on the extreme case in which nodes are either interested in their absolute payoff ($\beta_i = 0$) or in equity ($\alpha_i = 0$). The former are referred to as A-nodes, the latter as B-nodes. In total, there are N_a A-nodes and N_b B-nodes; k_a of these A-nodes and k_b B-nodes form the coalition. The cooperation levels are denoted by q_{as}, q_{bs} for nodes inside S, q_{an} and q_{bn} for nodes outside the coalition.

Let us first look at the behavior of B-nodes.
Outside the coalition, any B-nodes can arrive at the equal share by choosing the average cooperation cost level. Thus,

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[8] Assuming that $\sigma_S > 1/N$ then $r'(\ ) < 0$ and (11) implies $kB'(Q) < C'(q_S)$.

[9] *For k≥2.*

$$C(q_{bn})=\frac{1}{N_a+k_b}\left[k_a C(q_{as})+k_b C(q_{bs})+(N_a-k_a)C(q_{an})\right] \qquad (13)$$

A B-node inside the coalition has no incentive to leave if it also receives the equal share:

$$C(q_{bs})=\frac{1}{N_a-k_b}\left[k_a C(q_{as})+(N_b-k_b)C(q_{bn})+(N_a-k_a)C(q_{an})\right] \qquad (14)$$

In equilibrium, all B-nodes choose the same cooperation level, $q_b \hat{=} q_{bn}=q_{bs}$ and receive the equal share:

$$C(q_b)=\frac{1}{N_a}\left[k_a C(q_{as})+(N_a-k_a)C(q_{an})\right] \qquad (15)$$

A-nodes outside the coalition maximize their absolute payoff, $B(Q)-C(q_{an})$. The first order condition is given by: $B'(Q)=C'(q_{an})$. (16)

Within the coalition, the utility of a representative A-type-member is maximized by guaranteeing that the B-members get the equal share, i.e. $C(q_{bs})$. The first order condition for choosing q_{as} is given by:

$$B'(Q)\left[k_a+k_b\frac{\partial q_{bs}}{\partial q_{as}}\right]-C'(q_{as})=B'(Q)k_a\left[1+\frac{k_b}{N-k_b}\frac{C'(q_{as})}{C'(q_{bs})}\right]-C'(q_{as})=0 \qquad (17)$$

By construction, for any given k_a and k_b, every B-node is indifferent to being either inside or outside the coalition. For a coalition to be stable, an A-node must not have an incentive to leave the coalition. In general, for any k_b there will be a certain number of A-nodes, k_a, that will join the coalition. We have multiple equilibria.

Inequalities (13) - (17) can be used to infer the following results:

Result 5. *The larger the total number of equity-oriented nodes (N_b), the higher the incentives for A-nodes to join the coalition. Hence, for a given k_b, the number of cooperating A-nodes k_a increases in N_b.*

Result 6. *The more B-nodes join the coalition, the smaller the incentive for A-nodes to do so. In equilibrium, k_b and k_a are negatively correlated.*

Result 7. *The total cooperation level increases with the number of B-types outside the coalition. A joining B-node improves the payoffs only if it does not drive out an A-node.*

The rationale of results 5 and 6 is the following: if an A-node enters the coalition and the coalition increases its cooperation efforts, B-nodes outside the coalition increase their cooperation activities as well and thereby additionally reward the entering node.

If the number of such equity-oriented B-nodes outside the coalition gets larger, this external reward for joining a coalition increases and, therefore, the equilibrium coalition size increases. Analogously, if B-nodes join the coalition, fewer nodes outside the coalition reward the entering A-node by an increase of their cooperation activities. Hence, the incentives for A-nodes to enter the coalition decrease and the number of A-nodes that are inside the coalition in equilibrium gets smaller.

Result 7 reflects the fact that the more nodes cooperate, the higher the efficiency gains are and the closer the aggregate cooperation level is to the efficient one. The impact of A- and B-nodes on the decision of the coalition, however, differs in the following way: A joining A-node is interested in the absolute payoff and, consequently, the re-optimizing coalition increases its cooperation effort because the positive effect on one more node is now taken into account. A joining B-node, however, is not primarily interested in the absolute payoff, but in the equal share. Therefore, the coalition will not increase the total cooperation level that much because the B-node refrains from deviating from the cooperation level of non-cooperating nodes. Consequently, the efficiency gains are larger if an A-node enters the coalition than if a B-node joins. Therefore, B-nodes are welcome inside a coalition only if their entering does not drive out an A-node.

2.6. Reputation mechanism and coalition formation

Self-interested, autonomous mobile nodes of an ad hoc network may interact “rationally” to gain and share benefits in stable (temporary) coalitions: this is to save costs by coordinating activities with other nodes of the network. For this purpose, each node determines the utility of its actions in a given environment by an individual utility function. In section 2.1 we introduced a more sophisticated model in which not only self-centered preferences are taken into account to derive the individual payoff of an action but also relative information is used in order to find an extended set of possible equilibrium points.

Results obtained with the proposed model are promising: in a dynamic network formed by nodes that follow the definition of utility given by the ERC theory, depending on the node types, it is possible to obtain stable coalitions of a relatively large size and under certain circumstances, even the grand coalition becomes feasible. Node types are determined by the two parameters α and β which represent the key factor of the coalition formation process.

We believe that the reputation technique implemented in CORE can be used as an effective mechanism to impose a specific identical ERC type for every node participating in a cooperative setting as an ad hoc network. If the two parameters α and β are represented as functions of the reputation r_{ni} defined in [9], then it is possible to enforce a particular value to the α/β ratio: specifically it is possible to dynamically adjust the α/β ratio in order to be compatible with **proposition 4**. Thus, even the grand coalition is stable, every node of the network cooperates bearing the same costs and getting equal benefits by choosing a fair operating point in which no one deviates from the average cooperation level chosen by the coalition.

The relation between α, β and r_{ni} is indirectly proportional: the lower the reputation value (meaning that the past strategy selected by the node has been to reduce the cooperation level) the higher will be factor β and the lower will be factor α thus reducing the α/β ratio, and vice-versa.

The relation between the reputation value and the ERC type of a node becomes more complicated if we allow the presence of nodes with different ERC types: modeling a network that allows different ERC types is interesting when considering mobile nodes with different capabilities such as different battery power and different computational power.

However, in order to provide a formal assessment of the efficiency of the reputation mechanism proposed in CORE it is necessary to evaluate the node model presented in the previous sections in a dynamic setting: the reputation value is computed based on the past strategies selected by the nodes of the network and have an influence on those nodes' future actions. Furthermore any variation on the strategy selection phase of a node has an impact on the strategies selected by neighboring nodes: solutions to the dynamic coalition formation process have still to be examined.

We believe that the research we have conducted so far has given some interesting results and proposes a useful basis to study the coalition formation process of autonomous self-interested mobile nodes by means of reputation mechanisms which is, to the best of our knowledge, a rather unexplored domain. However, we think that it is possible to express the dynamic coalition formation process using a more elegant and simple methodology, which is a key requirement for studying dynamic games. The relatively recent literature on the subject states that the models of coalition formation may be classified into two main categories: utility-based models, as it is largely favored by game theory, and complementary-based models [reference klush]. Up to now, most classic methods and protocols for the formation of stable coalitions among rational agents follow the utility-based approach and cover two main activities which may be interleaved: the generation of coalition structures, that is partitioning or covering the set of agents into coalitions, and the distribution of gained benefit among the participants to each of the coalitions. The future research direction we will take is to prove that reputation mechanisms in general are compliant to the so called *Coalition Formation Algorithm*. Coalition formation algorithms are those mechanisms that provide a feasible solution to a cooperative game in coalitional structure: there are several solution concepts and we will focus on the so called *Kernel-oriented* solutions. Kernel-oriented coalitions are the most suitable for our purpose because the related literature gives precise conditions for a coalition formation algorithm to be kernel-stable with a polynomial complexity, as opposed to other solution/algorithms that are only of theoretical relevance since they have exponential complexity.

3. Non-cooperative games approach

In an alternative approach, our analysis focused on the identification of preference relations specific to the selfishness problem and the design of a utility function that satisfies this structure. The utility function used to model the selfishness problem takes into account the energy that a node spends for the purpose of its own communications and the energy that the node has to use when participating in the routing protocol and when relaying data packets on behalf of other nodes. Node behavior is represented by the percentage of energy a node dedicates for its own communications and the percentage of energy spent for network operation. Under these assumptions the utility function used to study the strategy chosen by a node is the following:

$$u_{ni}(bi,bj) = E_{self} \cdot (1-b_i) - b_i \cdot f \cdot (E_R + E_{PF}) \tag{18}$$

where b_i corresponds to the strategy (behavior) adopted by node n_i, and b_j is the common strategy selected by n_i's neighboring nodes: b_i is the variable of equation (18). b_i and b_j represent the percentage of energy consumed by a node and range from 0 to 1: when a node selects b=0 it will use all the available energy for its own communications. The other factors that appear in (18) are respectively:

$E_{self} = n \cdot (E_{send} + E_{recv}) = n \cdot (k+1)E_{recv}$, energy spent for a node's own communications

$E_R = (1-b_j)\frac{n \cdot t}{m}(E_{send} + E_{recv})$, energy spent for participating in the routing protocol

$E_{PF} = (1-b_j) \cdot t \cdot n \cdot (E_{send} + E_{recv})$, energy spent to relay packets for neighboring nodes

$E_{send} = k \cdot E_{recv}$, respectively the energy spent for sending and receiving one packet

n , the number of packets to send
t , the number of neighboring nodes of node n_i
m, the average number of messages after which a new route discovery phase is needed
f , is a multiplicative factor that models the non-linearity of the second summand of (18)

A "rational" selfish node always tries to maximize equation (18): the maximum determines the strategy b_i chosen by that node, which is always to defect, by selecting the total amount of energy dedicated to other nodes close to zero. The equilibrium point obtained using (18) has to be considered as *static* Nash equilibrium point. Indeed, the strategy selection phase of a player is determined based only on the maximization of the self-centered utility function (18): neither past nor future strategies have an influence on the choice of the player.

Since nodes act selfishly, the equilibrium point is not necessarily the best operating point from a social point of view and pricing emerges as an effective tool to enforce the cooperation among the nodes because of its ability to guide node behavior toward a more efficient operating point. The pricing factor that has been chosen to settle the game at a more socially desirable operating point is the reputation value calculated within the execution of the CORE mechanism. The utility function presented in (18) is modified as follows:

$$u_{ni}(bi,bj) = E_{self} \cdot (1-b_i)b_j - b_i \cdot f \cdot (E_R + E_{PF}) \cdot (-r_{ni}) \tag{19}$$

where the term r_{ni} corresponds to the normalized reputation value assigned to node n_i and dynamically evaluated by its t neighbors depending on the past strategy adopted by node n_i. The use of a pricing factor modify the position of the maximum of equation (19) with respect to equation (18) evaluated in the same circumstances. By dynamically modifying the position of the maximum, it is possible to impose a selfish node to change its strategy to a fair behavior, as shown in Figure 1.

The identification of the equilibrium point when using the utility function defined in (19) depends on past strategies chosen by player n_i (due to the reputation factor r_{ni}) and has an influence on the selection of future strategies by n_i's neighboring nodes (which will increase or decrease their cooperation level according to the CORE mechanism). In order to find a feasible solution it is then necessary to consider the game as a *dynamic non-cooperative game*. We proceeded with a numerical analysis of the dynamic game using the MATLAB suite, considering the option of an analytical solution as part of our future work.

The model developed in MATLAB implements the CORE mechanism following faithfully the definitions given in [9]: however the reputation value is computed under the hypothesis that neighbors of node n_i have identical representation of n_i's behavior. This hypothesis holds if the traffic flow between neighboring nodes is uniform, that is, every node sends and receives the same amount of packets. Moreover, simulations made with MATLAB also take into account the influence of the reputation value on the cooperation effort of the monitoring entities: when reputation decreases the percentage of the remaining battery life available for cooperating with node n_i decreases.

The main results obtained with the numerical approach are depicted in Figure 1. The first graph shows the reputation evaluated by the t neighboring nodes of node n_i: the reputation value depends on the behavior of node n_i in the past observations. The second graph depicts the strategy chosen by the selfish node versus time: at the beginning, the node selfishness is not compensated by the reputation mechanism and the strategy chosen by the node falls to zero (i.e. a pure selfish behavior). However, as soon as the node behavior is detected to be selfish the node reputation starts to fall: a “rational” selfish node will then chose a new strategy that issues from the maximization of equation (19) and that tries to compensate the loss in the reputation factor.

The strategy selection phase stabilizes asymptotically to a fair position where half of the nodes’ energy is used to cooperate with other nodes in the network operation.

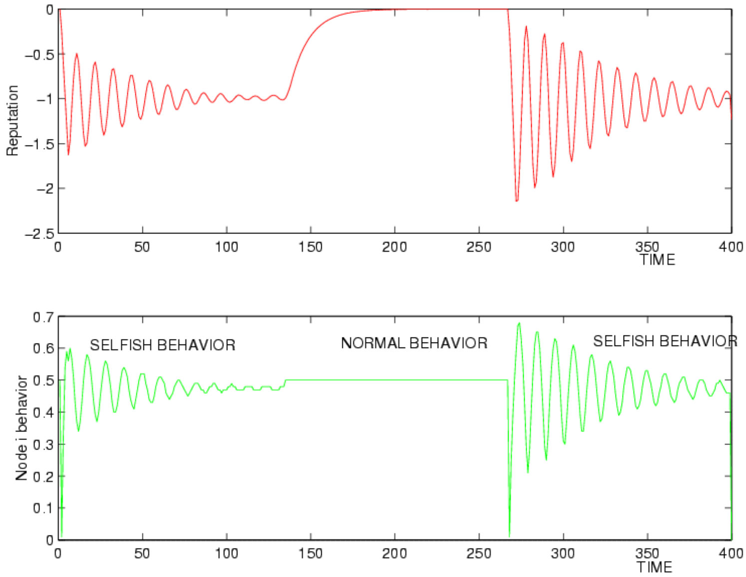

Figure 1. Node behavior when CORE is adopted in the network.

4. Conclusions

In this paper we presented two alternative approaches based on game theory in order to come up with a formal assessment of the properties of the cooperation enforcement mechanism presented in [9, Appendix 2]. We also accomplished the difficult task of validating the use of reputation as an effective tool for stimulating cooperation between the nodes of an ad hoc network.

Using the cooperative game approach, we were able to validate the ERC theory as a suitable alternative to the classic definition of utility function: the ERC theory suggests that players evaluate the outcome of a game in terms of both absolute and relative payoff. It has been demonstrated that the ERC preference structure can improve the identification of equilibrium points for games in coalitional structure and, when applied to a two-stage coalition game, ERC allows for the dynamic formation of even the grand coalition. The key issue that has to be addressed in order to find coalitions composed by a significant number of cooperating nodes is the identification of specific ERC-types: it is necessary that the nodes taking part in the coalition formation process put enough weight in the relative part of the utility function. This can be done in a dynamic setting by rewriting the ERC-types as a function of the reputation value computed in CORE. We suggest using reputation as a corrective factor that stimulates nodes to give more or less relevance to the relative part of the utility function definition. However we still have to provide a formal solution (numerical or analytical) to the dynamic coalition formation process: we believe that a more elegant approach based on a kernel-oriented algorithm (as opposed to the two-stage game approach) can ease our task by assuring a polynomial cost.

The non-cooperative game approach, on the other hand, provides an analytical proof that an ad hoc network with no cooperation enforcement mechanism cannot work: indeed we demonstrated that the best strategy for a selfish node interacting with the rest of the network is to defect. Moreover, by introducing the reputation mechanism defined in CORE as a pricing factor we demonstrated numerically that it is possible to have an asymptotically cooperative behavior. Rational nodes will always choose the strategy that maximizes their utility function: the pricing factor is used to move the maximum to a more suitable operating point from a cooperative point of view. Further insight to the parameters of CORE were obtained through MATLAB simulations: by modifying those parameters it has been possible to analyze the convergence speed towards a cooperative behavior, to determine a cooperation rate required from the nodes of the network and to study the influence of the sampling frequency of the watchdog mechanism used by CORE.

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Appendix 1. Proof of proposition 2.

We have to show that $\delta(k) > 0$ for *k>N/2*.

Remember that in (4) the denominator of $\delta(k)$ is positive due to assumption 1. That is, $\delta(k) > 0$ if the numerator of (4) is positive. Remember also that we assumed $r(\frac{1}{N} - x) \le r(\frac{1}{N} + x), \forall x \in \left[0, \frac{1}{N}\right]$.

The numerator in (4) is positive if *r(cooperate) > r(defect)*. This is the case when equation (7) is satisfied.

Let's proceed by showing that $\delta(k) < 0$ for for *k<N/2-1*.

It is possible to rewrite equation (7) as follows:

$B(k+1)C(k)Nk + B(k)C(k+1)N(k+1-N) + C(k)C(k+1)[Nk - 2k(k+1)] < 0$, or

$$B(k+1)C(k)\frac{N}{k+1} + B(k)C(k+1)N\frac{k+1-N}{k(k+1)} + C(k)C(k+1)\left(\frac{N}{k+1} - 2\right) < 0 \tag{16}$$

$$\left[B(k+1)C(k)\frac{N}{k+1} - B(k)C(k+1)\frac{N}{k}\right] + \left[\frac{NB(k)}{k} - C(k)\right]C(k+1)\left(2 - \frac{N}{k+1}\right) < 0 \tag{17}$$

Equation (17) can also be rewritten as:

$$\left[\frac{B(k+1)}{B(k)} - \frac{(k+1)C(k+1)}{kC(k)}\right] + (k+1)C(k+1)\left[\frac{1}{kC(k)} - \frac{1}{NB(k)}\right]\left(2 - \frac{N}{k+1}\right) < 0 \tag{18}$$

Now, from the monotonicity and concavity of *B()* it follows that $\frac{B(k+1)}{(k+1)} < \frac{B(k)}{k}$.

Furthermore, the total cost of cooperation increases *kC(k)* in *k*. Therefore:

$\frac{B(k+1)}{B(k)} - \frac{(k+1)C(k+1)}{kC(k)} \le \frac{k+1}{k} - 1 = \frac{1}{k}$

Since it has also been assumed that payoffs are non negative, $B(k) \ge C(k)$. Thus:

$$(k+1)C(k+1)\left[\frac{1}{kC(k)} - \frac{1}{NB(k)}\right] \ge \frac{(k+1)C(k+1)}{kC(k)}\frac{N-k}{N} \ge \frac{N-k}{N} \tag{19}$$

We therefore obtain:

$$\left[\frac{B(k+1)}{B(k)} - \frac{(k+1)C(k+1)}{kC(k)}\right] + (k+1)C(k+1)\left[\frac{1}{kC(k)} - \frac{1}{NB(k)}\right]\left(2 - \frac{N}{k+1}\right) \le$$
$$\le \frac{1}{k} + \frac{N-k}{N}\left(2 - \frac{N}{k+1}\right) = \frac{N(k+1) + 2(N-k)k(k+1) - (N-k)Nk}{Nk(k+1)}$$

The numerator equals: $-2k^3 + (3N-2)k^2 - N(N-3)k + N$ which can be shown to be negative for $1 \le k < \frac{N}{2} - 1$, as long as *N>8*.

Hence for *N>8* we have that the general conditions for a Nash equilibrium of the ERC-PD game $\delta(k^* - 1) > 0$ are satisfied for *k>N/2*.

NOTE: the condition N>8 can be removed if we assume that the total cost of cooperation increases more than the total benefits gained by defecting, i.e. : $\frac{(k+1)C(k+1)}{kC(k)} > \frac{NB(k+1)}{NB(k)}$.

Appendix 2. The CORE mechanism

The security scheme proposed by Michiardi and Molva [9], stimulates node cooperation by a collaborative monitoring technique and a reputation mechanism. Each node of the network monitors the behavior of its neighbors with respect to a requested function and collects observations about the execution of that function: as an example, when a node initiates a Route Request (e.g., using the DSR routing protocol) it monitors that its neighbors process the request, whether with a Route Reply or by relaying the Route Request. If the observed result and the expected result coincide, then the observation will take a positive value, otherwise it will take a negative value.

Based on the collected observations, each node computes a reputation value for every neighbor using a sophisticated reputation mechanism that differentiates between subjective reputation (observations), indirect reputation (positive reports by others), and functional reputation (task-specific behavior), which are weighted for a combined reputation value. The formula used to evaluate the reputation value avoids false detections (caused for example by link breaks) by using an aging factor that gives more relevance to past observations: frequent variations on a node behavior are filtered. Furthermore, if the function that is being monitored provides an acknowledgement message (e.g., the Route Reply message of the DSR protocol), reputation information can also be gathered about nodes that are not within the radio range of the monitoring node. In this case, only positive ratings are assigned to the nodes that participated to the execution of the function in its totality.

The CORE mechanism resists to attacks performed using the security mechanism itself: no negative ratings are spread between the nodes, so that it is impossible for a node to maliciously decrease another node's reputation. The reputation mechanism allows the nodes of the MANET to gradually isolate selfish nodes: when the reputation assigned to a neighboring node decreases below a pre-defined threshold, service provision to the misbehaving node will be interrupted. Misbehaving nodes can, however, be reintegrated in the network if they increase their reputation by cooperating to the network operation.

As for the other security mechanism based on reputation the CORE mechanism suffers from the spoofing attack: misbehaving nodes are not prevented from changing their network identity allowing the attacker to elude the reputation system. Furthermore, no simulation results prove the robustness of the protocol even if the authors propose an original approach based on game theory in order to come up with a formal assessment of the security properties of CORE.