Design of Space-Time Bit-Interleaved Coded Modulation and Threaded Algebraic Space-Time Codes with Message Passing Decoding

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Abstract — **We consider bit-interleaved coded modulation (BICM) space-time codes (STC) and the recently proposed threaded algebraic STC (TAST). In particular, we address belief-propagation iterative decoding and interference cancellation with linear interfaces. We study the interplay between different design parameters and the impact they have in the considered receiver strategies.**

I. System Model, BICM STC and TAST

We consider a multiple-input multiple-output (MIMO) system with N_T transmit antennas and N_R receive antennas in a block-fading environment with N_B blocks. The received signal matrix corresponding to the b-th block $\mathbf{Y}_b \in \mathbb{C}^{N_R \times L_B}$ is, $\mathbf{Y}_b = \sqrt{\gamma}\mathbf{H}_b\mathbf{X}_b + \mathbf{Z}_b, \quad b = 1, ..., N_B$, where $\mathbf{H}_b \in \mathbb{C}^{N_R \times N_T}$, is the fading channel matrix over block b (assumed perfectly known at the receiver), with $\mathbf{H} = [\mathbf{H}_1 \cdots \mathbf{H}_{N_B}]$, $\mathbf{X}_b \in \mathcal{X}^{N_T \times L_B}$ is the b-th block of the transmitted signal matrix $\mathbf{X} = [\mathbf{X}_1 \cdots \mathbf{X}_{N_B}] \in \mathcal{X}^{N_T \times L}$ with $L = N_B L_B$, $\mathbf{Z}_b \in \mathbb{C}^{N_R \times L_B}$ is a matrix of noise samples i.i.d. $\sim \mathcal{N}_{\mathbb{C}}(0,1)$, and γ is the average signal to noise ratio (SNR) per transmit antenna. The elements of $\mathbf{H}_b$ are assumed to be i.i.d. complex Gaussian random variables (Rayleigh fading).

We consider STCs defined by a binary block code $\mathcal{C} \subseteq \mathbb{F}_2^N$ of length N and rate r and a spatial modulation function $\mathcal{F} : \mathcal{C} \to \mathcal{S} \subseteq \mathcal{X}^{N_T \times L}$, such that $\mathcal{F}(\mathbf{c}) = \mathbf{X}$. In the case of BICM STCs, $\mathcal{F}$ is obtained as the concatenation of a parsing function that partitions a codeword $\mathbf{c} \in \mathcal{C}$ into sub-blocks, and block-by-block bit-interleaved modulation over the signal set $\mathcal{X}$ according to a labeling rule $\mu : \mathbb{F}_2^M \to \mathcal{X}$, such that $\mu(b_1, \ldots, b_M) = x$, where $M = \log_2 |\mathcal{X}|$. In this case, $N = N_T L M$. The transmission rate of the resulting STC is $R = r N_T M$ bit/s/Hz. BICM STCs are designed assuming a genie aided decoder that produces observables of the transmitted symbols of one antenna, assuming that symbols from all other antennas are known. In this way , the maximum achievable transmit diversity d of STC BICM is given by the Singleton bound (SB) on the block diversity of $\mathcal{C}$ $\delta_\beta = d \leq 1 + \lfloor N_T N_B (1-r) \rfloor$. Therefore, BICM STC code design reduces to design of powerful SB-achieving binary codes. We are currently investigating SB achieving turbo-like constructions with iterative decoding. For BICM STC, we show that traditional worst case design based on the rank diversity is not effective for typical frame error rate (FER) of interest. BICM STCs have been shown to be a simple, pragmatic and flexible approach to construct smart and greedy STCs over MIMO block-fading channels [2].

Generalized layered STCs are defined by N_C component codes $\mathcal{C}_c$, $c = 1, \ldots, N_C$, such that $\mathcal{C} = \mathcal{C}_1 \times \cdots \times \mathcal{C}_{N_C}$, and the space-time modulation function $\mathcal{F} : \mathcal{C}_1 \times \cdots \times \mathcal{C}_{N_C} \to \mathcal{S} \subseteq \mathcal{X}^{N_T \times N}$, that maps the component codewords $\mathbf{c}_c \in \mathcal{C}_c$, $c = 1, \ldots, N_C$ onto the $\mathbf{X}$ according to $\mathcal{F}(\mathbf{c}_1, \ldots, \mathbf{c}_{N_C}) = \mathbf{X}$. In the case of the recently proposed threaded algebraic STC (TAST) layered architecture [1], the encoding functions corresponding to $\mathcal{C}_c, \ c = 1, \ldots, N_C$ are $\gamma_c = \phi_c \mathbf{M}$, where $\mathbf{M}$ is a rate one full-diversity linear algebraic rotation and $\phi_c, \ c = 1, \ldots, N_C$ are the algebraic scalings (Diophantine numbers) that ensure that TAST codes achieve full diversity with maximum-likelihood (ML) decoding. In TAST codes, the component codewords $\mathbf{c}_c \in \mathcal{C}_c$ are mapped onto the array $\mathbf{X}$ following the (layer, antenna, time) indexing triplet $(c, |n + c - 1|_{N_T}, n)$ for $1 \leq n \leq L$ and $c = 1, \ldots, N_C$, thus having full spatial and temporal spans. By construction TAST have full block diversity, i.e., $\delta_\beta = N_T N_B$. We study the concatenation of nonbinary coding schemes and TAST.

II. Message Passing Decoding

ML decoding of such codes is only possible by exhaustive search and one generally resorts to sub-optimal iterative strategies whose goal are to achieve satisfactory performance. For BICM STC, the decoder should emulate a genie aided decoder in order to achieve diversity δ_β. Several iterative decoding techniques are possible. By applying the belief propagation (sum-product) algorithm to the STC dependency graph (in analogy to iterative multiuser receivers for CDMA), the decoder reduces to a maximum-a-posteriori (MAP) soft-input soft-output (SISO) bitwise MIMO demodulator and a MAP SISO decoder of $\mathcal{C}$ that exchange extrinsic information soft messages through the iterations. MAP SISO bitwise MIMO demodulation can be efficiently approximated by sphere decoding techniques. In order to reduce decoding complexity we consider interference cancellation (IC) approximations with linear interfaces: 1) the *Matched Filter* (MF), which performs MRC of the receive antennas with soft IC; 2) The *Unbiased Minimum Mean Squared Error* (MMSE) filter, which minimizes the MSE between the transmitted symbol and the output of the filter given the estimate of the interfering symbols.

Numerical results illustrate the performance tradeoffs of the studied STCs under these decoding strategies, and show that iterative receivers with linear interfaces show evident performance limitations in some important scenarios. Given the fact that exhaustive search SISO demodulation gives very satisfactory results, further reaserch should concentrate on low-complexity SISO sphere-decoding techniques.

References

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