Design of Space-Time Bit-Interleaved Coded Modulation and Threaded 
Algebraic Space-Time Codes with Message Passing Decoding 

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Abstract — We consider bit-interleaved coded modulation (BICM) space-time codes (STC) and the recently 
proposed threaded algebraic STC (TAST). In particular, we address belief-propagation iterative de-
coding and interference cancellation with linear interfaces. We study the interplay between different design 
parameters and the impact they have in the considered receiver strategies.

I. SYSTEM MODEL, BICM STC AND TAST 
We consider a multiple-input multiple-output (MIMO) system with \( N_T \) transmit antennas and \( N_R \) receive antennas in a block-fading environment with \( N_B \) blocks. The received signal matrix corresponding to the \( b \)-th block is \( Y_b \in \mathbb{C}^{N_R \times L} \), 
\[ Y_b = \sqrt{H_b} X_b + Z_b, \quad b = 1, \ldots, N_B, \] 
where \( H_b \in \mathbb{C}^{N_R \times N_T} \), is the fading channel matrix over block \( b \) (assumed 
perfectly known at the receiver), with \( H = [H_1 \ldots H_{N_B}] \), \( X_b \in \mathbb{C}^{N_T \times L} \) is the \( b \)-th block of the transmitted signal matrix 
\( X = [X_1 \ldots X_{N_B}] \in \mathbb{C}^{N_T \times NL} \) with \( L = N_B L_H \), \( Z_b \in \mathbb{C}^{N_R \times L} \) is a matrix of noise samples i.i.d. \( \sim \mathcal{C}(0,1) \), and \( \gamma \) is the average signal to noise ratio (SNR) per transmit antenna. The elements of \( H \) are assumed to be i.i.d. complex Gaussian random variables (Rayleigh fading).

We consider STCs defined by a binary block code \( C \subseteq F_2^N \) of length \( N \) and rate \( r \) and a spatial modulation function \( F: C \rightarrow S \subseteq \mathbb{C}^{N_T \times L} \) such that \( F(c) = X \). In the case of BICM STCs, \( F \) is obtained as the concatenation of a parsing function that partitions a codeword \( c \in C \) into sub-blocks, and block-by-block bit-interleaved modulation over the signal set \( X \) according to a labeling rule \( \mu: F_2^N \rightarrow X \), such that \( \mu(b_1, \ldots, b_N) = x \), where \( M = \log_2 |X| \). In this case, \( N = N_T L_M \). The transmission rate of the resulting STC is \( R = r N_T M \) bits/Hz. BICM STCs are designed assuming a genie aided decoder that produces observables of the transmitted symbols of one antenna, assuming that symbols from all other antennas are known. In this way, the maximum achievable transmit diversity \( d \) of STC BICM is given by the Singleton bound (SB) on the block diversity of \( C \) 
\[ d = d_1 = 1 + [N_T N_B (1 - r)]. \] 
Therefore, BICM STC code design reduces to design of powerful SB-achieving binary codes. We are currently investigating SB achieving turbo-like constructions with iterative decoding. For BICM STC, we show that traditional worst case design based on the rank diversity is not effective for typical frame error rate (FER) of interest. BICM STC codes have been shown to be a simple, pragmatic and flexible approach to construct smart and greedy STCs over MIMO block-fading channels [2].

II. MESSAGE PASSING DECODING
ML decoding of such codes is only possible by exhaustive search and one generally resorts to sub-optimal iterative strategies whose goal are to achieve satisfactory performance. For BICM STC, the decoder should emulate a genie aided decoder in order to achieve diversity \( d \). Several iterative decoding techniques are possible. By applying the belief propagation (sum-product) algorithm to the STC dependency graph (in analogy to iterative multi-user receivers for CDMA), the decoder reduces to a maximum-a-posteriori (MAP) soft-input soft-output (SISO) bitwise MIMO modulator and a MAP SISO decoder of \( C \) that exchange extrinsic information soft messages through the iterations. MAP SISO bitwise MIMO demodulation can be efficiently approximated by sphere decoding techniques. In order to reduce decoding complexity we consider interference cancellation (IC) approximations with linear interfaces: 1) the Matched Filter (MF), which performs MRC of the receive antennas with soft IC; 2) The Unbiased Minimum Mean Squared Error (MMSE) filter, which minimizes the MSE between the transmitted symbol and the output of the filter given the estimate of the interfering symbols.

Numerical results illustrate the performance tradeoffs of the studied STCs under these decoding strategies, and show that iterative receivers with linear interfaces show evident performance limitations in some important scenarios. Given the fact that exhaustive search SISO demodulation gives very satisfactory results, further research should concentrate on low-complexity SISO sphere-decoding techniques.

REFERENCES

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